ejpmr, 2020,7(5), 206-208

SJIF Impact Factor 6.222

EUROPEAN JOURNAL OF PHARMACEUTICAL AND MEDICAL RESEARCH

www.ejpmr.com

Review Article

ISSN 2394-3211

EJPMR

SATELLITES: LINEAR OSCILLATION OF THE SYSTEM (EQUILIBRIUM FOR SMALL ECCENTRICITY)

***Dr. Sushil Chandra Karna**

Department of Mathematics M.B.M. Campus Rajbiraj, Saptari (Nepal).

***Corresponding Author: Dr. Sushil Chandra Karna**

Department of Mathematics M.B.M. Campus Rajbiraj, Saptari (Nepal).

Article Received on 24/02/2020 **Article Revised on 14/03/2020** **Article Accepted on 04/04/2020**

ABSTRACT

This paper deals with the linear oscillation of the system about the positions of equilibrium for small eccentricity. We will try to find the condition of equilibrium position.

KEYWORDS: Eccentricity; Oscillation; Equilibrium.

INTRODUCTION

The effect of Earth's oblateness and magnetic force on the motion of a system of two artificial satellites connected by light, flexible and in extensible string. We come to know that

$$\rho = \frac{1}{1+e\cos v}$$

Where

$e =$ Eccentricity of the orbit

$v =$ True anomaly of the centre of mass

By determining the motion of the other satellite, we apply the identity as

$$m_1\overrightarrow{\rho_1} + m_2\overrightarrow{\rho_2} = 0$$

Where

$$\overrightarrow{\rho_1}, \overrightarrow{\rho_2} = \text{Radius vector of } m_1, m_2$$

There are three types of motions are give by

(i) Free Motion i.e (λα=0)

(ii) Constrained Motion i.e (λα≠0)

(iii) Evolutional Motion

(Combination of free and constrained motion)

Mathematical Approach

In the case of constrained motion

We apply

$$x^2 + y^2 = \frac{1}{\rho^2} \qquad \text{.......... (1)}$$

We transform the polar form by replacing

$$\left.\begin{aligned} x &= (1+\cos v)\cos\psi \\ y &= (1+e\cos v)\sin\psi \end{aligned}\right\} \qquad \text{.......... (2)}$$

Diff ^{H}- (2) w.r. to v we obtain

$$x^1 = -\frac{\psi^1 \sin\psi}{\rho} - e\cos\psi.\sin v$$

$$x^{11} = -\frac{\psi^{11}\sin\psi}{\rho} - \frac{\psi^{1^2}\cos\psi}{\rho} + 2e\psi^1\sin\psi\sin v$$
$$-e\cos\psi.\sin v$$

II$_y$

$$y^1 = \frac{\psi^1\cos\psi}{\rho} - e\sin\psi.\sin v$$

$$y^{11} = \frac{\psi^{11}\cos\psi}{\rho} - \frac{\psi^{1^2}\sin\psi}{\rho} - 2e\psi^1\cos\psi\sin v$$
$$-e\sin\psi.\cos v$$

$$\left.\begin{aligned} \rho^1 &= \rho^2 e\sin v \rho = \frac{1}{1+e\cos v} \\ i.e. e\sin v &= \frac{\rho^1}{\rho^2} \end{aligned}\right\} \qquad \text{.......... (3)}$$

We have the system; when centre of mass moves along keplerian elliptical orbit in Nechvile's co-ordinate then

$$\left.\begin{aligned} x^{11} - 2y^1 - 3x\rho &= \lambda \propto^{.x} - \frac{4A_{0.x}}{\rho} - \frac{A\cos i}{\rho} \\ y^{11} + 2x^1 &= \lambda \propto^{.y} + \frac{A_{0Y}}{\rho} - \frac{A\cos ie^1}{\rho^2} \end{aligned}\right\} \qquad \text{......... (4)}$$

Replacing the value of $\rho, \rho^1 and\ x, y$ in (4) we have

$$-\lambda \propto \frac{1}{\rho}\cos\psi - \frac{4A_0\cos\psi}{\rho^2} - \frac{A\cos i}{\rho} \qquad \text{……… (5)}$$

II$_y$

$$\frac{\psi^{11}\cos\psi}{\rho} - \frac{\psi^2\sin\psi}{\rho} - 2e\psi^1\cos\psi\sin v - e\sin\psi\cos v$$
$$-\frac{2\psi^1\sin\psi}{\rho} - 2e\cos\psi\sin v = \lambda \propto \frac{1}{\rho\sin\psi} + \frac{A_0\sin\psi}{\rho^2}$$
$$-A\cos i.e\sin v \qquad \text{……… (6)}$$

Multiplying (5) by $\sin\psi$ and (6) by $\cos\psi$ and the subtracting first from the second

$$(1+e\cos v)\psi^{11} - 2e\psi^1\sin v - 2e\sin v + 3\sin\psi\cos\psi$$
$$= 5A_0(1+e\cos v)^2\sin\psi\cos\psi + A\cos i[(1+e\cos v)\sin\psi - e\sin v\cos\psi] \qquad \text{…….. (7)}$$

Again (5) is multiply by $\cos\psi$ and (6) by $\sin\psi$ and adding, we get

$$A_0(1+e\cos v)^2(4\cos^2\psi - \sin^2\psi) + \frac{A\cos i}{\rho}(\cos\psi + e\rho\sin v.\sin\psi) - \frac{\lambda\propto}{\rho} \qquad \text{……. (8)}$$

This equation determines undetermined Lagrange's multiplier.
The motion will be constrained as long as

$$\lambda(t) >,0 \text{ i.e } \lambda \propto (t) >,0$$

It means the particle will start moving with in the circle of variable radius

$$x^2 + y^2 = \frac{1}{\rho^2}$$

Now, the equation of motion of the system is given by

$$5A_0(1+e\cos v)^2\sin\psi\cos\psi + A\cos i[(1+e\cos v)\sin\psi - e\sin v.\cos\psi] \qquad \text{……… (9)}$$

This is a second order differential equation with periodic term from equation (9) eccentricity is very small that implies $e = 0$ and there exists stable positions of equilibrium for equatorial orbit $(i = 0)$ given by

(i) $\varphi_0 = 0$, $\quad \sin\psi_0 = \frac{A}{(3-5A_0)}$

(ii) $\varphi_0 = 0$, $\quad \psi_0 = 0$

We focus on first case as the oscillation of the system about the stable position of equilibrium.

$$\varphi_0 = 0; \sin\psi_0 = \frac{A}{3-5A_0}$$

e = to be taken as a small parameter

Replacing

$$\psi = \psi_0 + \partial$$
$$\psi^1 = \partial^1$$
$$\psi^{11} = \partial^{11}$$
$$\sin\psi = \sin(\psi_0 + \partial) = \sin\psi_0.\cos\delta + \cos\psi_0\sin\delta$$
$$= \frac{A}{3-5A_0} + \delta\sqrt{1 - \frac{A^2}{(3-5A_0)^2}} \qquad \text{……… (10)}$$

II$_y$

$$\cos\psi = \cos(\psi_0 + \delta) = \cos\psi_0\cos\delta - \sin\psi_0.\sin\delta$$
$$= \sqrt{1 - \frac{A^2}{(3-5A_0)^2}} - \delta\frac{A}{(3-5A_0)} \qquad \text{……… (11)}$$

There fore we observe that linear sing the equation of motion w.r. to δ and δ^1 in case of equatorial orbit $(i = 0)$ we have

$$(1+e\cos v)\delta^{11} - 2e\delta^1\sin v - 2e\sin v + 3\delta\left[1 - \frac{2A^2}{(3-5A_0)^2}\right] + \frac{3A}{(3-5A_0)}\sqrt{1 - \frac{A^2}{(3-5A_0)^2}} \qquad \text{…... (12)}$$

$$= 2e\sin v - \frac{3A}{(3-5A_0)}\sqrt{1 - \frac{A^2}{(3-5A_0)^2}}$$
$$-Ae\sin v\left[\sqrt{1 - \frac{A^2}{(3-5A_0)^2}} - \delta\frac{A}{(3-5A_0)}\right] \qquad \text{……… (13)}$$

Suppose

$$\delta = \frac{z}{1+e\cos\rho} = ze \qquad \text{……… (14)}$$
$$z = \delta(1+e\cos v)$$
$$z^1 = (1+e\cos v)\delta^1 - e\delta\sin v$$
$$z^{11} = (1+e\cos v)\delta^{11} - 2e\delta^1\sin v - e\delta\cos v$$
$$z^{11} = e\frac{z}{(1+e\cos v)}\cos v = (1+e\cos v)\delta^{11} - 2e\delta^1\sin v$$

$$z^{11} + ez.\rho\cos v = (1+e\cos v)\delta^{11} - 2e\delta^1\sin v \qquad \text{…….. (15)}$$
$$z^{11} + ze\cos v[1 - e\cos v + e^2\cos^2 v] + 3z[1 - e\cos v + e^2\cos^2 v]$$
$$\left[1 - \frac{2A^2}{(3-5A_0)^2}\right] - 5A_0 z(1+e\cos v)\left\{1 - \frac{2A^2}{(3-5A_0)^2}\right\}$$

$$-5A_0(1 + 2e\cos v + e^2\cos^2 v)\frac{A}{3-5A_0}\sqrt{1 - \frac{A^2}{(3-5A_0)^2}}$$

$$-\frac{A^2}{(3-5A_0)}\left(1-e\cos v+e^2\cos^2 v\right)-zA\sqrt{1-\frac{A^2}{(3-5A_0)^2}}$$

$$=2e\sin v-\frac{3A}{(3-5A_0)}\sqrt{1-\frac{A^2}{(3-5A_0)^2}}-Ae\sin v\sqrt{1-\frac{A^2}{(3-5A_0)^2}}$$

$$+\frac{A^2 e\sin vz}{(3-5A_0)}\left(1-e\cos v+e^2\cos^2 v\right)$$

$$z^{11}+z\left[(3-5A_0)-\frac{2(3-5A_0).A^2}{(3-5A_0)^2}\right]$$

$$=e\left[2\sin v-A\sqrt{1-\frac{A^2}{(3-5A_0)^2}}.\sin v+z\sin v.\frac{A^2}{(3-5A_0)}\right.$$

$$+z\cos v\left\{2+5A_0-\frac{2(3+5A_0)A^2}{(3-5A_0)^2}+10A_0\cos v.\right.$$

$$\left.\frac{A}{(3-5A_0)}\sqrt{1-\frac{A^2}{(3-5A_0)^2}}-\frac{A^2}{(3-5A_0)}.\cos v\right]$$

$$+e^2\left[-\frac{z\sin 2vA^2}{2(3-5A_0)}+z\cos^2 v\left\{-2+6.\frac{A^2}{(3-5A_0)^2}\right\}\right.$$

$$\left.+5A_0.\cos^2 v.\frac{A}{3-5A_0}\sqrt{1-\frac{A^2}{(3-5A_0)}}+\frac{A^2\cos^2 v}{3-5A_0}\right]+$$

.... (16)

Suppose

$$(3-5A_0)-\frac{2A^2}{(3-5A_0)}=n_1^{\,2}(say)$$

$$(3-5A_0)^2-2A^2=n_1^{\,2}(3-5A_0)$$

$$2A^2=(3-5A_0)(3-5A_0-n_1^{\,2})$$

$$A^2=\tfrac{1}{2}(3-5A_0)(3-5A_0-n_1^{\,2})$$

$$A=\sqrt{\frac{1}{2}(3-5A_0)(3-5A_0-n_1^{\,2})}$$

Here

$$A\sqrt{1-\frac{A^2}{(3-5A_0)^2}}=\sqrt{\frac{(3-5A_0)(3-5A_0-n_1^{\,2})}{2}}\times$$

$$\sqrt{1-\frac{(3-5A_0)(3-5A_0-n_1^{\,2})}{2}\bigg/(3-5A_0)^2}$$

$=0$ The equation (16) reduce to

$$z^{11}+n_1^{\,1}z=e\left[2\sin v-\tfrac{1}{2}\sin v\sqrt{(3-5A_0+n_1^{\,2})(3-5A_0-n_1^{\,2})}\right.$$

$$+\frac{1}{2}z\sin v.(3-5A_0-n_1^{\,2})+z\cos v\left\{2+5A_0-\frac{(3+5A_0)(3-5A_0-n_1^{\,2})}{(3-5A_0)}\right\}$$

$$\left.+\frac{10A_0\cos v}{2(3-5A_0)}\sqrt{(3-5A_0+n_1^{\,2})(3-5A_0-n_1^{\,2})}-\frac{\cos v.(3-5A_0-n_1^{\,2})}{2}\right]$$

$$+e^2\left[-\frac{z\sin 2v(3-5A_0-n_1^{\,2})}{4}+z\cos^2 v\left\{-2+\frac{3(5A_0-n_1^{\,2})}{(3-5A_0)}\right\}\right.$$

$$\left.+\frac{5A_0.\cos^2\sqrt{(3-5A_0+n_1^{\,2})(3-5A_0-n_1^{\,2})}}{2.(3-5A_0)}+\frac{\cos^2 v.(3-5A_0-n_1^{\,2})}{2}\right]$$

.... (18)

Where,

$$n_1^{\,2}=(3-5A_0)-\frac{2A^2}{3-5A_0}$$

CONCLUSION

We obtained that the linear oscillation of the system about the position of equilibrium.

REFERENCES

1. Bate, R.R Mueller, D.D and white, J.E fundamentals of Astrodynamies Dover, New York, 1671.
2. Danby, J.M.A Fundamentals of celestial Mechanics Mevemillan, New York, 1962.
3. Battin, R.H. Astronautical Guidance Mcgraw-Hill, New York, 1964.
4. Fitzpatrek, P.M. Principles of celestial Mechanics Dcademic press, New York, 1970.
5. Kaplan, M.H. Modern spaceeraft Dynamics and coulrol, Wiley, New York, 1976.
6. Meirovitch, L, Mcthods of Analyfical Dynauics Mcgraw-Hill, New York, 1970.
7. Hughes, Peter, c. Spacecraft Altitude Dynamics wile, New York, 1986.
8. Kane yeoman R, Likens, Peter L, and Evensong, D.A. Spacecraft Dynamics MC-Grew Hill, New-York, 1983.
9. Thomson, W.T. Involution to space Dynamics hilly, New York, 1961.
10. Draper C.S, wrangle, W, and Hookah, H inertial Guidance, paragon, New York 1960.
11. Bruiting, K.R Inertial Navigation System Dialysis Wiley, New York, 1971.
12. Meirovitch L. Methods of Analytical Dynamics MC-Grew –Hill, New York.