Negative Index of Refraction

Girish Gupta

School of Physics and Astronomy

University of Manchester

BSc Dissertation

October-November 2008

Abstract

Victor Veselago brought about current thinking about the consequences of both the permeability and permittivity of a material being less than zero in 1968, long before it was practically realised. Since then scientists and engineers have worked hard to bring the Russian scientist's ideas to non-theoretical use.

Most notably in the field, David Smith and John Pendry have begun work in bringing about the reality of a 'perfect' lens.

This report aims to give the reader an idea of the theories behind the concept and of the work conducted by scientists and engineers in the latter half of the last century and the beginnings of this, on the realisation of these, specifically focusing on the perfect lens.

1. Introduction and Theory

The index of refraction, n, is an incredibly fundamental concept in the fields of electromagnetism and (arguably) its offshoot, optics. Its most simple definition is as a measure of the speed of electromagnetic radiation in a material (v) compared with the speed of light (c) in a vacuum, $n = \frac{c}{v}$. A more useful definition, however, relates n to the relative permeability (ε_r) and permittivity (μ_r) of a material,

$$n = \pm\sqrt{\varepsilon_r \mu_r}$$

Equation 1 – The index of refraction defined in terms of the relative permeability (ε_r) and permittivity (μ_r) of a material. In this paper we will be taking the negative square root of this equation. This definition of n is more fundamental as the speed of light is defined alongside the vacuum—rather than relative—forms of these two quantities itself, $c = \sqrt{\varepsilon\mu}^{-1}$.

The first major paper published on the topic was in 1968 by Victor Veselago[i]. Back then, there was no evidence that materials with negative refractive indices were naturally occurring or could be artificially produced, yet, Veselago wrote his 1968 paper on a completely theoretical basis, hoping that its rigorously-argued conclusions would one day become viable.

Veselago was not the first scientist to hypothesise about the implications of a negative index of refraction[ii]. In 1904, Horace Lamb suggested the idea of "backward waves" in reference to pressure waves[iii]. In the same year, Arthur Schuster went on to quote Lamb's work and extend the idea to electromagnetic waves in his book *Theory of Optics* and drew up a diagram similar to Figure 2. However, the mention of negative index of refraction by Schuster was a minimal ending to a chapter entitled 'Transmission of Energy'[iv]. It was Veselago's work in 1968 that delved deepest into the repercussions of negative index materials[1,v].

In the paper, the Russian noted that rather than taking the positive square root of Equation 1, he would like to work through the implications of taking the negative root. He admitted that there could be three possibilities arising from doing this:

[1] Materials with negative refractive index were called 'left-handed' by Veselago, the reason for which is explained in the caption for Equation 3. John Pendry and David Smith[v] decided to go with the name 'negative index materials' so as to not confuse this phenomenon with chirality. This is the name that this paper will continue to use.

- No difference will be made to a material's properties. The negative root will have the same physical effect as the positive root.

- The negative root is physically impossible.

- "It could be admitted that substances [such as this] have some properties different from those … with positive ε and μ."

It was the third possibility that spurred Veselago on to continue his research—and eventually the one that was realised to be the case. He worked through Maxwell's equations of electromagnetism and was soon forced to bring in the wave propagation vector **k**. The wave vector, as it is commonly called, represents the direction of wave propagation, similarly—but not identically—to the Poynting vector, **S**. The vector form of the wave vector is not always required and its mathematics is quite useful in scalar form, *k*, the wave number. The wave number is defined as,

$$k = n\frac{\omega}{c}$$

Equation 2 – The wave number is important in electromagnetism. Its vector part appears to point in the wrong direction when the index of refraction is taken to be negative.

where *n* is, again, the material's refractive index and ω is the angular frequency of the radiation.

This quantity is a major player in the propagation of electromagnetic waves. It can be found in all fundamental equations of the topic so when, as Veselago realised, its sign is reversed as that negative square root (of Equation 1) is taken, some interesting things will happen.

It is worth now bringing in a relationship between the magnetic (**B**) and electric (**E**) fields that, quite evidently, pervade electromagnetism. The relationship neatly incorporates our wave vector, **k**,

$$\mathbf{B} = \frac{1}{c}\mathbf{k} \times \mathbf{E}$$

Equation 3 – It was an equation similar to this that spurred Veselago into describing his hypothesised materials as 'left-handed'. The equation, when **k** is negative, forms a left-handed set of vectors.

The relationship between the three most fundamental vectors in electromagnetism can now be seen[2]. As can be seen by Equation 2, changing the sign of the refractive index will do the same to the wave number and vector. It can be seen by Equation 3 that if **k** is positive, **B**, **k**, and **E** will form a right-handed set of vectors. However, if is it negative, a left-handed set will be formed.[3]

The Poynting vector was mentioned earlier as being similar to the wave vector. The vector describes the direction in which energy is carried by the wave. It is defined as,

$$\mathbf{S} = \frac{1}{\mu}\mathbf{E} \times \mathbf{B}$$

Equation 4 – The Poynting vector points in the direction of energy flux. Surprisingly, this appears to be negative too when the material has a negative index of refraction.

Again, the three vectors in Equation 4 will form a right-handed set with each other unless μ is negative, in which case you will again get a left-handed set of vectors.

Here is where things begin to get interesting. If the direction of the Poynting vector is reversed, the implication is that the energy is flowing the 'wrong' way. The energy flow represented by **S** is in the opposite direction to **k**. It is easier to see this by bringing in the wave velocity,

$$\mathbf{v_p} = \frac{\omega}{\mathbf{k}}$$

Equation 5 – The phase velocity appears to be negative. Veselago's genius was in noting that there are different forms of velocity so the idea is not as paradoxical as it sounds.

It is clearly seen by Equation 5 that the wave velocity and wave vector will always point in the same direction (assuming the angular frequency is positive). With a negative wave vector, this brings about a startling effect—the velocity of the wave will appear to go backwards, from receiver to source.

---

[2] In his 1968 paper, Veselago actually does the maths with the magnetic field strength vector, **H**. However, there is no need to introduce this quantity here so the algebra has been worked through to remove it.
[3] Veselago went on to introduce a matrix of which he called the determinant the 'rightness' of a material, **p**. If the set of **E**, **B**, and **k** was positive, **p** would equal +1; if the set were negative, **p** would equal -1.

Veselago's genius was in noting that there are many different methods of defining the velocity of a wave. Equation 5 is in fact the phase velocity of the wave, hence the subscript **p**. This is the most common form of the velocity and simply denotes the rate at which any point on the wave (a peak or a trough, for example) passes a point in space. However, as waves become more complex than a coherent, monochromatic beam, which is, of course, the case in nature, other velocities need to be brought in. The fact that dispersion occurs, i.e., the refractive index being a function of wavelength, is also important.

The group velocity is slightly different and brings in more realism. It takes into account the envelope that carries the wave that the phase velocity defines. It is defined by,

$$\mathbf{v_g} = \frac{d\omega}{d\mathbf{k}}$$

Equation 6 – The group velocity describes the velocity of the wave packet. If the wavelength is constant, the group velocity is equal to the phase velocity.

This group velocity will not have the same sign as the phase velocity. The Poynting vector will by definition have opposite sign and the rest of this report will work through the implications of the fact that the energy flux (direction given by the Poynting vector) has an opposite sign to the phase velocity.

2. Physical Consequences

There are a number of consequences of the above theories. Any equation featuring those variables we are interested in making negative should be of interest to us. Fortunately, those variables, μ, ε, n and their derivatives, pop up all over electromagnetism and optics. I will describe the theory behind some of the physical consequences and go on to describe how they have been investigated since Veselago's 1968 paper.

Doppler Shift

Doppler shift is a well known phenomenon that occurs for both electromagnetic and pressure (sound) waves. Though we are dealing with electromagnetism, the more obvious and ubiquitous example is of the emergency vehicle with sirens wailing coming towards and then shooting off passed, the observer. As the vehicle's sirens approach, the frequency of the sound increases. As it passes you, the frequency again decreases as the wavelength of the wave 'stretches out'. This is very basic but will suffice here. The equation governing the phenomenon is,

$$\frac{f'}{f} = \frac{c}{c + v_p \cos\theta}$$

Equation 7 – Doppler shift is entirely analogous to the red and blue shift that is observed in astronomy as stellar objects come towards or travel away from each other. In negative index materials, the phase velocity is negative so forcing the frequency change to be in the opposite direction to 'normal'.

where f' is the frequency of the wave observed, f is the frequency of the wave emitted by the siren (constant in above example), c is as always the speed of the wave and v_p is the speed of the source (ambulance, for example). The angle θ is the angle between the observer and source[4].

The relevance to our case of negative index of refraction is that the velocity in Equation 7 is in fact the phase velocity, that which changes sign. This has the simple consequence of reversing Doppler shift. This means that as an ambulance's sirens approach, the frequency will decrease rather than increase.

Proof that Doppler shift does work 'backwards' in a negative index material came in 2002 as an ending note when rigorous testing was conducted on some of the first examples of negative index materials[xvii]. More will be spoken about this later.

Čerenkov Radiation

Čerenkov radiation, like Doppler shift, has an electromagnetic and pressure analogue. The sound wave analogue is commonly seen in supersonic flight as a sonic boom is heard. We will this time concentrate on the electromagnetic instant.

The radiation is emitted when a charged particle travels faster than the speed of electromagnetic radiation in a particular material, given by $v_p = \frac{c}{n}$. A shockwave, i.e., a cone of light (that is 'forward-pointing', important later), is produced, similar to that in a sonic boom. The intensity of the light produced is proportional to its

[4] For a simple one dimensional problem, possible values of θ are 0° and 180° and all that these do is change the sign preceding the v_p in the denominator of Equation 7 and make the magnitude of the cosine term equal to one.

frequency so, when visible, Čerenkov radiation appears to the blue end of the spectrum. It is for this reason that the spent fuel rods in nuclear reactors are characterised by a blue glow.

An interesting application of Čerenkov radiation is in neutrino detection. Mount Ikenoyama in Japan houses a 50,000 ton tank of pure, transparent water through which light can travel huge distances. As a neutrino passes through, it will release a tiny amount of radiation which is detected by some of 11,000 photomultiplier tubes surrounding the tank[vi,vii].

In the case where the medium through which the object travels faster than the speed of light is of negative refractive index, the radiation cone produced will be backward-pointing. Luo, et. al.,[viii] showed this effect in 2003 using photonic crystals; they observed a "definite emission angle in … the backward direction".

Snell's Law and its Implication to Lensmakers

Snell's Law of Refraction is fundamental in the field of optics. Regarding the refraction of a light beam as it passes between media of differing refractive indices, the equation is easily derived from Fermat's Principle of Least Time—the idea that an electromagnetic wave will take the path that takes the least time.

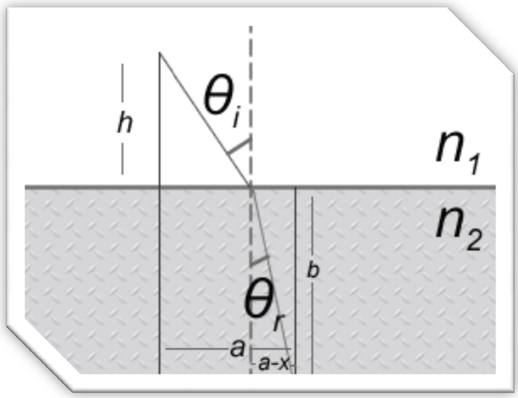

Figure 1 – Snell's Law can be easily derived from Fermat's Least Time Principle, as well as more simply with similar triangles.

The optical path length is defined as $L = nl$ where n is the refractive index of the material and l is the actual length traversed. Mathematically, Fermat's principle requires us to minimise L by finding its derivative with respect to distance, x. On Figure 1, the optical path is obviously from top to bottom via the two angles θ. So, $L = n_1\sqrt{x^2 + h^2} + n_2\sqrt{(a-x)^2 + b^2}$. In order to find the minimum, this needs differentiating with respect to x, $\frac{dL}{dx} = n_1\frac{x}{\sqrt{x^2+h^2}} + n_2\frac{x-a}{\sqrt{(a-x)^2+b^2}}$, and then setting this to zero. Doing this reveals,

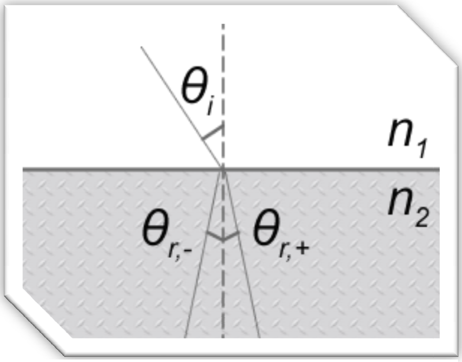

Figure 2 – The angle of the incoming ray is θ_i. If the material has positive refractive index (which is greater than n_1), then the ray will follow the path with angle $\theta_{r,+}$. However, if the material has a negative index of refraction (again, with modulus greater than n_1), the ray will follow the path of $\theta_{r,-}$ —still obeying Snell's Law, Equation 8.

$$n_1 \sin\theta_i = n_2 \sin\theta_r$$

Equation 8 – Snell's Law of Refraction, applicable for both positive and negative *n*.

The variables are self-explanatory from Figures 1 and 2: n_1 and n_2 are the refractive indices of the two materials while θ_i and θ_r are the angles of incidence and refraction with respect to a normal that is perpendicular to the material's surface. It is curious that Equation 8 also applies to materials with negative refractive index.

In the traditional situation where n_1 is less than n_2, and both are positive, the ray path will take the route shown by the angle $\theta_{r,+}$ in Figure 2. However, in the situation where n_2 is negative, the path will be that denoted by $\theta_{r,-}$. As Snell's Law is so fundamental to lenses and optical instruments, the equations governing those must be worked through with the possibility of negative *n*.

A derivation of the Lensmaker's Equation here is slightly long-winded and off topic. The equation turns out as

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Equation 9 – The Lensmaker's Equation

The focal length of the lens is *f*, *u* and *v* are the object and image distances respectively, *R* is the radius of curvature of the lens' surfaces while n_2 is again the refractive index of the lens and n_1 the refractive index of the material surrounding the lens. If *n* is positive, we have the normal situation where a converging (convex) lens converges parallel light rays to a focus at positive *v* and a diverging (concave) lens diverges them forming a focus at negative *v*. Making *n* negative, as always, swaps this around and instead, a converging lens diverges light and a diverging lens converges light, as shown in Figure 3.

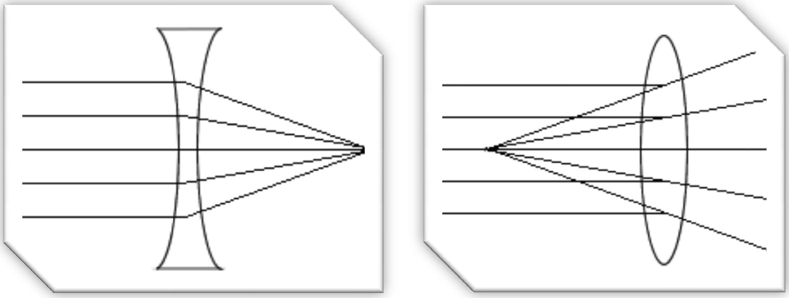

Figure 3 – On the left is a diverging lens that is actually converging parallel light beams to a focus. On the right is a converging lens that actually diverges parallel light beams and forms a focus to the left of the lens. This is in stark contrast to traditional lens diagrams where *n* is positive.

This startling result was proven in 2004 as shown later in Figure 5. It was John Pendry who, a year earlier, had hypothesised that this could lead to a 'perfect' lens—by which he meant one that did not have an upper limit on resolution of the wavelength of light travelling through it[ix].

Pendry hypothesised that the negative index lens used could have parallel sides and still focus light. This seems very odd until you remember that lenses work due to Snell's Law on each surface. If that law is reversed (as it is above) then the refraction angle is at the same angle direction it would be with a converging lens (Figure 4). This was proven in 2003 and 2005 to be the case[xviii, xix].

The scientist then went on to talk about the impedance of the material. If both ε and μ were equal to –1 then the impedance would match that of free space implying that there would be no reflection. Things became more interesting when he noted that a lens made from a negative index material would cancel the decay of evanescent waves—waves that decay in amplitude but not in phase as they propagate away from an object. Pendry's 2000 paper showed that "evanescent waves emerge from the far side of the medium enhanced in amplitude by the transmission process".

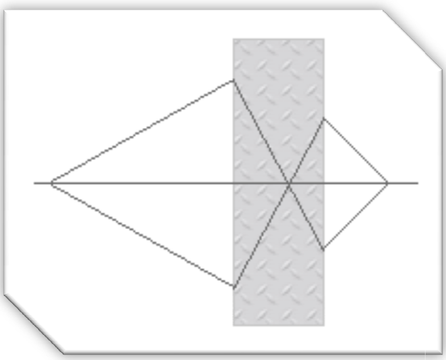

Figure 4 – Light rays are coming in from the left here. Due to the lens being of a negative index material, it refracts slightly differently to the method expected and so shows that a material with parallel sides can focus light.

"In other words," stated Anthony Grbic and George Eleftheriades in 2002, "these perfect lenses achieve diffraction-free, near-field focusing."[x]

A break should be taken to explain the nature of evanescent waves. They are commonly formed as the product of total internal reflection, when sinusoidal waves are reflected off a surface at an angle greater than the material's critical angle. The term evanescent means "tending to vanish" as this intensity of the waves decays exponentially rather than sinusoidally.[xi] An evanescent wave has a so-called "imaginary velocity"[xii].

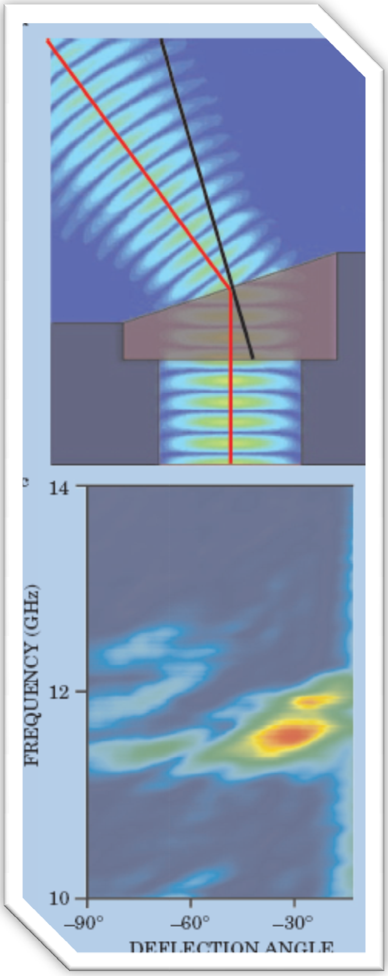

Figure 5 – This shows graphical proof of the situation in Figure 2 when the material has a negative refractive index. The deflection angle is also negative.[v]

Grbic and Eleftheriades were able to show the effect using a form of negative index material and ended their 2003 paper, "The … medium is a likely candidate for microwave subwavelength focusing and imaging applications."

3. The Reality of Negative Index Materials

It was in the mid-nineties, nearly a century after Lamb and Schuster first thought about the prospect of "backward waves"[iii], that scientists and engineers began to look into the possibility of "engineering artificial materials to have a tailored electromagnetic response"[v].

Figure 6 – "Photograph of [a] left-handed metamaterial (LHM) sample. The LHM sample consists of square copper split ring resonators and copper wire strips on [fibre] glass circuit board material. The rings and wires are on opposite sides of the boards, and the boards have been cut and assembled into an interlocking lattice." [xiv]

The method was simple yet ingenious: put together a collection of repeated elements that would have a strong response to electromagnetic fields. The idea was that incident radiation would not be able to distinguish between the elements and would instead 'see' a homogenous material. It would be easier to tailor the elements to give what appeared to be a uniform material with the parameters they wanted, negative ε and μ.

These so-called metamaterials—materials which gain their properties not from their structure but from their chemical composition[xiii]—are so commonly associated with the topic of negative index of refraction, that the term 'metamaterial' is now synonymous with negative index materials. In 2001, a paper was published by scientists from the University of California in which they claimed to have created a negative index material by this method and had performed experiments which "directly confirm[ed] the predictions of Maxwell's equations that n is given by the negative square root of $\varepsilon\mu$ for the frequencies where both the permittivity (ε) and the permeability (μ) are negative"[xiv].

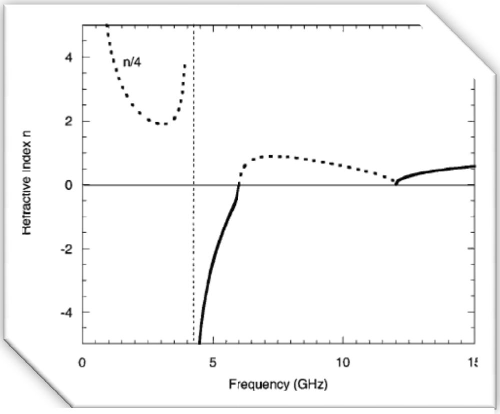

Figure 7 – "Real (solid) and imaginary (dashed) branches of $n(\omega)$ versus frequency. The region where $n(\omega)$ is negative occurs when the permeability and permittivity are both negative." [xi]

Figure 6 shows an example of the material used. It consists of a two-dimensional array of split ring resonators and wires (both copper) on a fibre-glass circuit board. It was shown in beam-deflection experiments that this material did indeed behave as if it had a negative refractive index at certain frequencies[xiv,xv].

Later that year, the same authors (along with a new collaborator) published a paper which continued the research and manufactured a "two-dimensional isotropic [metamaterial which] will now permit experiments to verify some of the explicit predictions of reversed electromagnetic-wave properties including negative index of refraction as analyzed by Veselago."[xvi] Figure 7 shows the frequencies at which the materials exhibited this negative index behaviour.

To confirm that the materials did work as expected, the split-ring resonator device was rigorously tested by scientists from Washington DC's Naval Research Laboratory the following year[xvii]. The agreement was "excellent" and the scientists went on to show that the expected Doppler effect did indeed take place. What took place in their experiments was "the opposite of what occurs in a conventional positive index material", just as Veselago had predicted.

Those Washington scientists were not the only ones to remain unconvinced by the experimental data. Three scientists from MIT and Harvard, in 2003, performed similar experiments and acquired similar results, "refuting alternatives posed in the criticisms"[xviii]. They found the same results as in Figure 3 and were also able to test the hypothesis given in Figure 4, that parallel-sided slabs were able to focus light.

They concluded, "Taken as a whole, these data indicate focusing behaviour associated with a negative index of refraction." This parallel-sided lens idea was also tested in 2005 using photonic crystals, another method of creating what seems to be a negative index material[xix].

There were more attempts to disprove the ideas of Pendry and Smith in those years but experimental data was generally on their side. A news article in a 2002 issue of *Science* reports of a "spat" between Pendry and other scientists about whether Veselago or indeed Pendry had done their calculations and experimentation correctly[xx]. Prashant Valanju claimed that Veselago had made a fundamental error in a ray diagram in his 1968 paper and also insists that if true, the theory would violate Einstein's 1905 postulate that no signal could travel faster than light. Valanju also claimed that what experimenters saw were not negative index materials refracting in the sense Veselago had predicted but solely "diffraction effects". However, many more experiments were carried out in this period by a huge number of scientists and their number overwhelmed those that disagreed.

Pendry accused those who disagreed of letting "emotion" cloud their judgement. He and David Smith went on to submit a number of papers to more formally answer their critics. In one, in 2004, they described their theoretical and practical work on the topic of negative index materials but focused specifically on the 'perfect' lens. They also mentioned the ideas about using photonic crystals—"structures whose refractive index is periodically modulated"[xxi]. This periodicity is of the order of the wavelengths used so the "distinction between refraction and diffraction is blurred"[v]. This would later be worked on by many scientists around the world to prove that parallel-sided lenses formed of a negative index material would in fact focus light[xv,xxii,xxiii].

4. The Future of Negative Index Materials

Though very thoroughly researched, and much evidence produced, the subject of negative indices of refraction has become a very controversial one. Pendry and Smith's ideas are still disputed, though sound proof offered against them seems somewhat limited.

Just two months ago, Alexander Ramm in Manhatten wrote a scathing report claiming that negative refraction could never produce a perfect lens, due to "the fluctuations of the refraction coefficient of the [negative index] slab".[xxiv] His paper ends with an admission, however, that his arguments are only valid if quantum effects are ignored.

Scientists in Delhi are currently using negative index photonic crystals to work on a polarisation beam splitter[xxv] while over in Shanghai, scientists have found a new method of creating a negative index material. They have used thin silver film samples and found a "pseudonegative refraction index" with its most negative index at the lowest photon energies[xxvi]. A negative index material was produced last year (2007), by completely new methods, that showed evidence of the effect at visible frequencies[xxvii].

The Lezec, et.al., paper was spun by many science reporters into something that it very much was not but would guarantee public attention and increased government spending—the idea of invisibility due to negative refraction.[xxviii] The *Softpedia* article cited even has the audacity to end with a claim that "in the future … the stealth fighter will literally become the invisible fighter". That is clearly a long way off.

It was just over a hundred years ago that "backward waves"[iii] were first thought about. In that time, mainly thanks to Veselago[i] in 1968 working thoroughly through the physical and theoretical implications, we are now beginning to see a reality in negative index materials. Though still very much at the laboratory level—costing huge amounts of money, in no practical state to be used in real-life applications and only majorly tested with microwaves[5]—the topic has gained much momentum especially in the early years of this century.

Its implications, mainly thanks to Pendry[v]—from the perfect lens to possible invisibility—have forced business and government to feed scientists around the world with research dollars. This has, in turn, spurred most scientists into not disagreeing with Pendry, as few have had the courage to do—rightly or wrongly.

[5] Though as stated earlier, the effect was last year produced for visible light with a new form of negative index material.

The future will undoubtedly require more research into new methods of creating negative index materials or refining the current methods. When this happens, scientists and engineers can work together on bringing the costs involved down. However, this is too far into the future. There is no clear and distinct use for the materials as yet. Though the papers released over the last ten years or so have exciting prospects, not one of them has been able to give good reason or method for following their paper up with a serious attempt at manufacture. The only follow-up is further research, which at the moment is tending to go off on a tangent to the paper(s) on which it was based. Once these tangents become aligned, the future of negative index materials will become focused and a reality.

References

[i] Veselago, V.G. 1968, ‘The electrodynamics of substances with simultaneously negative values of ε and μ’, *Sov. Phys. Usp.*, vol. 10, no. 4, pp. 509–514.
[ii] Moroz, A. 2003, *Some negative refractive index material headlines...*, viewed 2 November 2008, <http://www.wave-scattering.com/negative.html>.
[iii] Lamb, H. 1904, *Proc. London Math. Soc.*, sec. 2, vol. 1, p. 473.
[iv] Schuster, A. 1904, *Theory of Optics*, Edward Arnold, London, p. 330.
[v] Pendry, J.B. & Smith, D.R. 2004, ‘Reversing light with negative refraction’, *Phys. Today*, pp. 37–43.
[vi] Nave, R 2006, *Super-Kamiokande neutrino detector*, viewed 7 November 2008, <http://hyperphysics.phy-astr.gsu.edu/hbase/particles/neutrino2.html#c1>.
[vii] Kearns, E., Kajita, T. & Totsuka, Y. 1999, ‘Detecting massive neutrinos’, *Sc. Amer.*, vol. 281, no. 2.
[viii] Luo, C., Ibanescu, M., Johnson, S.G. & Joannopoulos, J.D. 2003, ‘Cerenkov radiation in photonic crystals, *Science*, vol. 299, no. 368, pp. 368–371.
[ix] Pendry, J.B. 2000, ‘Negative refraction makes a perfect lens’, *Phys. Rev. Let.*, vol. 85, no. 18, pp. 3966–3969.
[x] Grbic, A. & Eleftheriades, G.V. 2003, ‘Growing evanescent waves in negative-refractive-index transmission-line media’, *App. Phys. Let.*, vol. 82, no. 12, pp. 1815–1817.
[xi] Prieve, D. 2000, *Evanescent waves*, viewed 27 October 2008, <http://www.andrew.cmu.edu/user/dcprieve/Evanescent%20waves.htm>.
[xii] Lipson, S.G. & Lipson, H. 1969, *Optical Physics*, Cambridge Uni. Press, Cambridge, pp. 22–24.
[xiii] Anissimov, M. 2003, *What is a Metamaterial?*, viewed 5 November 2008, <http://www.wisegeek.com/what-is-a-metamaterial.htm>.
[xiv] Shelby, R.A., Smith, D.R. & Schultz, S. 2001, ‘Experimental verification of a negative index of refraction’, *Science*, vol. 292, no. 5514, pp. 77–79.
[xv] Smith, D.R. & Kroll, N. 2000, ‘Negative refractive index in left-handed materials’, *Phys. Rev. Let*, vol. 85, no. 14, pp. 2933–2936.
[xvi] Shelby, R.A., Smith, D.R., Nemat-Nasser, S.C. and Schultz, S. 2001, ‘Microwave transmission through a two-dimensional, isotropic, left-handed metamaterial’, *App. Phys. Let.*, vol.78, no. 4, pp. 489–491.
[xvii] Rachford, F.J., Smith, D.L., Loschialpo, P.F. & Forester, D.W. 2002, ‘Calculations and measurements of wire and/or split-ring negative index media’, *Phys. Rev.*, vol. 66, no. 3.
[xviii] Houck, A.A., Brock, J.B. & Chuang, I.L. 2003, ‘Experimental observations of a left-handed material that obeys Snell’s law’, *Phys. Rev. Let.*, vol. 90, no. 13.
[xix] Lu, Z., Shi, S., Schuetz, C.A. & Prather, D. 2005, ‘Experimental demonstration of negative refraction imaging in both amplitude and phase’, *Optics Exp.*, vol. 13, no. 6.
[xx] Kakaes, K. 2002, ‘Theorists doubt claims for perfect lens’, *Science*, vol. 296, no. 5572, pp. 1380–1381.
[xxi] Notomi, M. 2000, ‘Theory of light propagation in strongly modulated photonic crystals: Refraction-like behaviour in the vicinity of the photonic band gap’, *Phys. Rev. B*, vol. 62, no. 16, pp. 10696–10705.
[xxii] Parimi, P.V., Lu, W.T., Vodo, P., Sokoloff, J., Derov, J.S. & Sridhar, S. 2004, ‘Negative refraction and left-handed electromagnetism in microwave photonic crystals’, *Phys. Rev. Let.*, vol. 92, no.12.
[xxiii] Ozbay, E., Guven, K., Cubukcu, E., Aydin, K. & Alici, B. 2004, ‘Negative refraction and subwavelength focusing using photonic crystals’, *Mod. Phys. Let.*, vol. 18, no. 25, p. 1275.
[xxiv] Ramm, A.G. 2008, ‘Does negative refraction make a perfect lens?’, *Phys. Let.*, vol. 372, no. 43, pp. 6518–6520.
[xxv] Rajput, M. & Snihn., R.K. 2008, ‘Photonic crystal based polarization beam splitter utilizing the phenomenon of negative refraction’, *Proc. SPIE*, vol. 7056, no. 20.
[xxvi] Wu, Y., Gu, W., Chen, Y., Dai, Z., Zhou, W., Zheng, Y. & Chena, L. 2008, ‘Negative refraction at the pure Ag/air interface observed in the visible Drude region’, *App. Phys. Let.*, vol. 93, no. 7, p. 71910.
[xxvii] Lezec, H.J., Dionne, J.A. & Atwater, H.A., 2007, ‘Negative refraction at visible frequencies’, *Science*, vol. 316, no. 430, pp. 430–432.
[xxviii] Dorneanu, L. 2007, *Bending Light Into Invisibility*, viewed 7 November 2008, <http://news.softpedia.com/news/Bending-Light-Into-Invisibility-50194.shtml>.