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Kaniadakis Holographic Dark Energy by Using Hybrid Scale Factor

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Abstract

We carry out the work on Kaniadakis holographic dark energy KHDE with hybrid expansion law by taking the scale factor containing both exponent and exponential form. Therby revealing the present state of accelerating and expanding universe in the flat Friedmann-Robertson-Walber universe. The deceleration parameter q portrays whether he universe is decelerating or accererating. q greater than zero suggests the universe is slowing down or getting decelerated and q less than zero indicates that universe is speeding up or accelerated..Since in present work the value of q lies between -1<q<1, so we conclude that the universe which was decelerating in past is currently expanding as well as accelerating.The deceleration parameter q obtained in the present model of Kaniadakis suggests that universe's expansion from decelerating to an accelerated phase of the present universe.The present work is well consistent with the thermal history of the universe as well as compatible with the observations. The positive value of EoS parameter signifies matter dominated universe at large red shift z >0 i.e. EoS parameter lies in the quintessence region. In the present model EoS parameter do not cross the phantom line even at future z= -1, which is considered to be the dark energy dominated phase and is responsible for the current accelerated phase of the universe. The equation of state parameter EoS replicate the important cosmological behaviour where ω_{kD} *can be quintessence-like, phantom-like or cross the phantom divide before or after the present epoch.*

Keywords: KHDE , FLRW universe, quientessence, phantom, EoS parameter,scale factor.

INTRODUCTION

Cosmology is the conclusion of study of the time interval (spaces) of fixed curvature [1]. The universe is considerably isotropic and homogeneous, this principle is utilized to construct the most fundamental and basic models of cosmology. The present trend of the universe in the field of cosmology is to analyse it from the point of expanding as well as accelerating cosmos [2, 3]. By merely considering the fact that our universe is brimming with matter and radiation, it would not be suffice to explain the accelerated expansion of the universe. To explain this fact one needs extra degree of freedom. The accelerated universe can be justified by atleast by two ways. Firstly, considering an exotic energy-momentum source named as dark energy(DE) [4–7] and secondly, modifying GR(general relativity) theory [8–12]. The whole universe is essentially occupied with three ingredients namely noticeable matter (4.9%) containing radiations and baryonic matter, dark matter (DM) (26.7%) and and dark energy(DE) (68.3%) having their respective densities ρ_{vis}, ρ_{DM} and ρ_{DE} as pointed out in various experimental observations [13]. The sum of these densities is called critical density ρ_{crit}. Here, $\rho_{vis}/\rho_{crit} = 0.05$, $\rho_{DM}/\rho_{crit} = 0.25$ and $\rho_{DE}/\rho_{crit} = 0.70$, where around 95% of its constituting is unseen. Almost 73% of the total energy densities of the present universe is the dark energy (DE) amidst negative pressure. In

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order to provide an authentic illustration of massive and long-term phenomena, various constituents are desired. The Λ cold DM usually referred as (ΛCDM) model based on (GR) provides very good explanation of the massive structured universe and its evolution. Implicit late-time acceleration of the universe can be explained by the dark energy(DE) constituent [14–15]. Additional particle can be interpreted by analysing DM [16] , however, mostly DM is the outcome in the modification of gravity [17]. The theoretical value assigned to cosmological constant (CC) Λ, which is a notable constituents of DE, is contrary to the actual observations value. Even at the present epoch there is confusion whether DE is the consequence of any new field or failure of GR or some unspecified thing [18].

By analysing the results of background progress and also late time growth of structure [19] as well as the advancement of gravitational waves [20] many workers have done work on arising DE by employing cosmological features with the help of equation of state (EoS) parameter and its evolution, dark energy EoS, modified dark energy models. Additionally a new cosmological aspect, exotic DE in phantom mode DE [21], dark matter radiation interacts between DE and DM or vacuum energy density [22] CC [23] with negative value are assumed to be responsible for the measurement of the Hubble constant [13, 24], H_0 tension [25–26] between early time and late time universe. Moreover, according to the recent observations with a flat FLRW universe [27] there found some discrepencies particularly in the Planck temperature and polarisation anisotropies. Especially in the last decade the holographic DE attributed on holographic principle was also included in analysing the DE complications which is a suitable procedure and assists in procuring cosmological aspects [28–29] of the vacuum energy density where ρ_Λ performs the function energy density of DE (ρ_{DE}). The usual existing holographic energy density expressed as $\rho_D = 3c^2M^2L^{-2}$, where c is a constant, is used for the interpretation of the accelerated expansion of the universe [30–31]. Recently a new DE model has been suggested known as Tsallis holographic dark energy (THDE) [32] by modifying the regular holographic DE as $S_\alpha = \gamma A^\alpha$, where γ ia a constant and α characterize the non-extensive parameter,which is dependent upon the holographic hypothesis and Tsallis entropt [33] accompanying energy density $\rho_D = BL^{2\alpha -4}$, here B is unidentified parameter and IR cutoff is L. As long as one uses the suitable limits of α = 1 and γ =1/4G (where in units, h = c = k_B =1) the power law distribution of probability fails its usefulness and the Bekenstein entropy is revived and the system fulfilling the normal probability distribution [34]. Recently authors [35] have studied THDE by using hybrid expansion law and able to explain successfully the the present accelerating phase from the decelerated phase of the universe.

Upon employing the energy density of quantum fields in free space bring about a well known model termed as holographic dark energy model (HDE) [28–29], HDE model is employed to interpret the present acceleration of the universe. Entropy of a system is associated with geometrical entities like its radius which is one of the basic equation in the evolvement od HDE. Bekenstein-Hawking entropy emerges as a black hole and cosmological application of the fundamental Boltzmann-Gibbs entropy. Recently, Kaniadakis entropy-based expansion of the HDE outline has been proposed [36–38] accompanying Hubble horizon and future event horizon as IR cutoff in flat as well as in non-flat universe. Moreover, Kaniadakis has suggested a parameter generalizing the Boltzmann-Gibbs entropy, is termed as Kaniadakis entropy [39]. Thus the basic features of standard statistical theory has been achieved which is the outcome of self-consistent and coherent relativistic statistical theory. Accepted statistical theory has been redeemed in a specific limit by continuous change of distribution function of extended statistical theory therby distorting the basic Maxwell-Boltzmann ones. Recently it has been shown that Kaniadakis entropy of a black hole is obtained as

$$S_{kD} = \frac{1}{k} Sinh(k\, S_{BH}) ,$$

here k is a free parameter. Inspired by the thermodynamic interest of the apparent horizon (Hubble horizon) authors Moradpour et al. [36] applied this horizon as the IR cutoff , and suggested a new HDE model named as Kaniadakis holographic dark energy (KHDE) looks to be able to describe the present accelerated expanding state of the universe. The relevance of KHDE model in non-flat universe in addition to other IR cutoffs have also been discussed in Refs. [29, 37]. Essentially, as the non-extensive aspect of gravity, and also the behaviour of DE are mysterious, so there is no boundation on the values of k at the present moment, and more accurate investigation may help to impose some restrictions on its values. Hence it is expected to observe that researchers may take different intervals for k to cope with the observational need, based on the primary presupposition like the IR cutoff [36–38]. Further, the workers proposed that KHDE is also in compliance with observations of all datasets apart ups [40–41]. Very recently, KHDE has been studied by many workers in diverse astrophysical and cosmological set ups [42–49].

METRIC AND FIELD EQUATIONS FOR KANIADAKIS HOLOGRAPHIC DARK ENERGY

As discussed in introduction, a new parameter k was introduced by Kaniadakis and named it as Kaniadakis entropy just by generalizing the Boltzmann-Gibbs entropy, which is consequence of coherent and self-consistent relativistic ststistical approach. According to authors Moradpour et al. [36]

a single free generalized Kaniadakis entropy parameter k for a black hole is given as

$$S_{kD} = \frac{1}{k} Sinh(k\, S_{BH}) \tag{1}$$

A state of the art DE model was introduced by including the above entropy in HDE hypothesis and the resulting model is called as Kaniadakis holographic dark energy (KHDE) in Cosmology and Astronomy. The GR and the cosmic principles are most basic principles required for thorough study of the universe.

To explain the expansion of the universe, an isotropic and homogeneous Friedmann-Lamaitre Robertson Walker metric is is expressed as

$$ds^2 = -dt^2 + a^2\,(\mathrm{t})\left({dr^2}\Big/{1-K\,r^2} + r^2\, d\Omega^2 \right), \tag{2}$$

weather to increase or decrease the homogeneous and the isotropic spatial parts is recommended by a time varying function a(t) known as scale factor. The value of variable parameter K gives the spatial curvature. K=1 represents open universe, K=0 gives flat universe and K=1 serves as open universe models. According to Eq.(1), if DE is assumed to regulate the present accelerated expansion of the universe then a box of volume L^3 must accumulate vacuum energy not greater than its equal size of black hole [29]. One therefore gets

$$\Lambda^4 \cong \rho_{kD} \propto {S_k}\Big/{L^4}, \tag{3}$$

here, ρ_{kD} is the vacuum energy. Now if considering the Hubble horizon of flat FRW universe as the IR cut off (eg. L=1/H → A= $4\pi/H^2$), one can write [29, 36]

$$\rho_{kD} = \frac{3C^2 H^4}{8\pi k} Sinh\left(\pi k / H^2\right), \tag{4}$$

where C^2 is an exotic constant [29]. It is clear that for k →0, one has $\rho_{kD} = 3C^2H^2/8$, which is the popular Bekenstein entropy based on HDE. Assuming that there is no mutual interaction among the cosmic categories, collectively with pressure-less fluid having matter density ρ_m and the DE candidate having KHD'2E energy density ρ_{kD} and its corresponding pressure p_{kD} , the Friedmann equations and the energy-momentum conservation law get to the below resulting expressions as

$$\dot{\rho}_m + 3H\rho_m = 0, \tag{5}$$

$$p_{kD} = -\left(\frac{\dot{\rho}_{kD}}{3H} + \rho_{kD}\right), \tag{6}$$

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_{kD}\right), \tag{7}$$

$$H^2 + \frac{2}{3}\dot{H} = -8\pi G/3\left(p_{kD}\right), \tag{8}$$

$$\dot{\rho}_{kD} + 3(1+\omega_{kD})H\rho_{kD} = 0, \tag{9}$$

here dot represents for derivative w.r.t. time, $H = \dot{a}/a$ and KHDE equation of state parameter (EoS) is expressed as

$$\omega_{kD} = {p_{kD}}/{\rho_{kD}}. \tag{10}$$

The corresponding density parameters and spatial curvature of fractional densities are expressed as

$$\Omega_m = \frac{8\pi G}{3H^2}\rho_m, \; \Omega_{kD} = \frac{8\pi G}{3H^2}\rho_{kD}. \tag{11}$$

Using Eq. (7) above expression can be xxpressed as

$$\Omega_m + \Omega_{kD} = 1 \tag{12}$$

(by using Eq. 11).

By differentiating Eq.(4) w.r.t. cosmic time t , we have

$$\dot{\rho}_{kD} = \frac{3C^2}{8\pi k}\left[4H^3\dot{H}\,Sinh\left({\pi k}/{H^2}\right) - 2\pi kH\dot{H}\,Cosh\left({\pi k}/{H^2}\right)\right] \tag{13}$$

Now, by differentiating Eq.(7) and by the help of Eqs.(5), (9), (12) and (13) we obtain

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}\left(\omega_{kD}\Omega_{kD} + 1\right), \tag{14}$$

using above Eq. we can obtain the equation for deceleration paramerer (DP) as $q = -\frac{\dot{H}}{H^2} - 1$ or

$$q = \frac{1}{2}\left(1 + 3\omega_{kD}\Omega_{kD}\right) \tag{15}$$

The form of EoS parameter ω_{kD} can be derived by using Eqs.(4), (9) and (13) as

$$\omega_{kD} = \frac{1}{3H^4}\left(2\pi k\dot{H}\,Coth\left(\pi k/H^2\right) - 4\dot{H}H^2\right) - 1 \tag{16}$$

Moreover, the density parameter of KHDE model can be obtained by Eqs.(4) and (11) as

$$\Omega_{kD} = \frac{C^2H^2}{k}Sinh\left[\frac{\pi k}{H^2}\right] \tag{17}$$

PRESENT MODEL AND DISCUSSIONS

We have considered Hybrid expansion law and the chosen scale factor is

$$a(t) = t^{\alpha} \exp[\beta t] \tag{18}$$

Hence, $\dot{a} = \exp[\beta t] t^{\alpha} \left(\beta + \alpha t^{-1}\right)$ (19)

The Hubble parameter is given as

$$H = \frac{\dot{a}}{a} = \left(\beta + \alpha t^{-1}\right) \tag{20}$$

The cosmic time derivative of Hubble parameter is simply given by

$$\dot{H} = -\alpha t^{-2} \tag{21}$$

Hence the deceleration parameter (DP) is given by

$$q = -\left[1 + \dot{H} / H^2\right]$$

$$q = \left[\frac{\alpha t^{-2}}{(\beta + \alpha t^{-2})^2} - 1\right] \tag{22}$$

The deceleration parameter q depicts whether the universe is slowing down or accelerating. $q > 0$ signifies that the universe is in decelerating phase and q less than zero predicts that the universe is in acceleration stage. Currently the universe is accelerating with the deceleration parameter q likely to be in the range $-1 \leq q < 0$. Figure 1 depicts the nature of deceleration parameter with red shift z for various combinations of constants α and β for this model, which illustrates that universe passes from the previous phase of deceleration to the present phase of acceleration. The transition from deceleration to acceleration is obtained around $0.15 < z < 0.25$ which is according to the present cosmological observations [50].

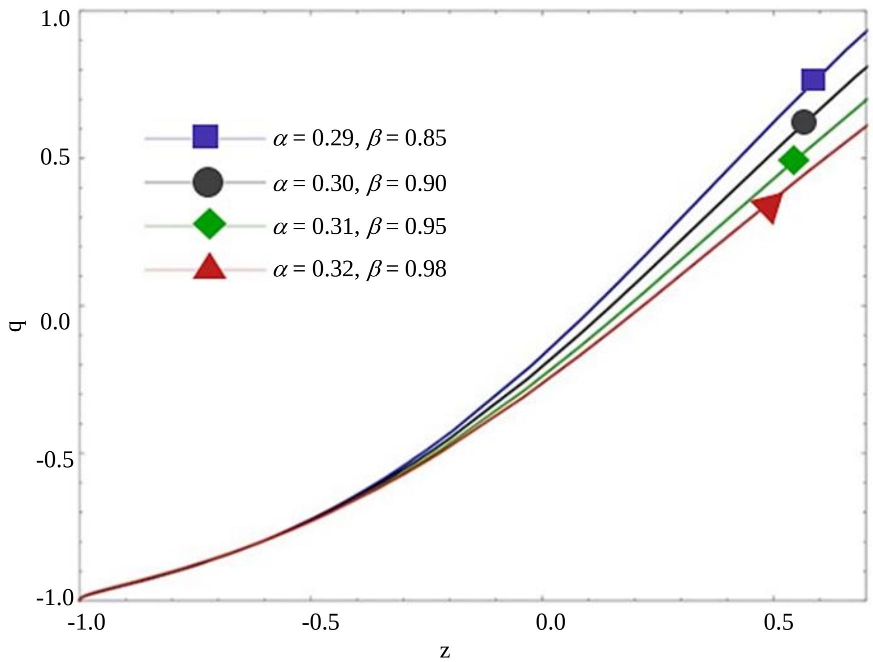

Figure 1. Shows the variation of deceleration parameter q with respect to red shift z for different set of values of constants α and β.

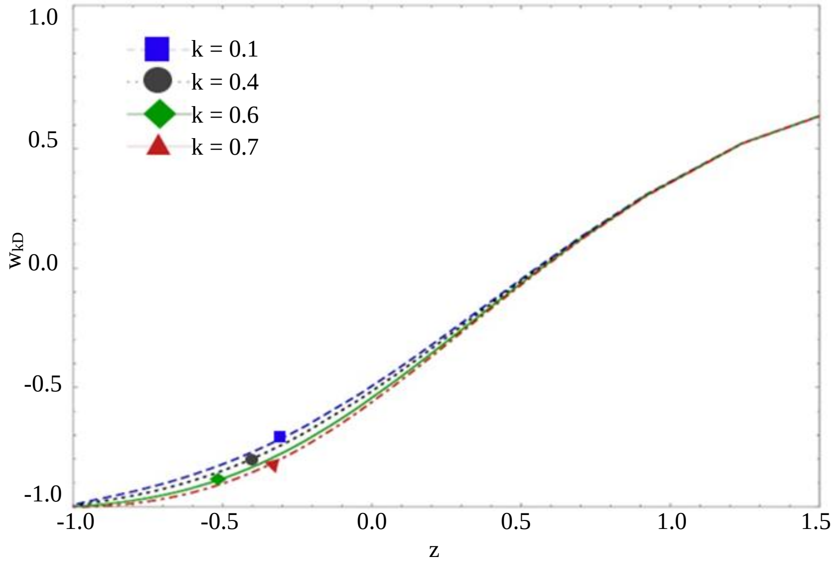

Figure 2. Shows the nature of equation of state parameter ω_{kD} with respect to red shift z for a particular set of value of constants α = 0.31 and β = 0.95 and different values of k viz. k = 0.1; k = 0.4; k = 0.6 and k = 0.7.

The EoS parameter for the present model becomes

$$\omega_{kD} = \frac{C^2(\beta+\alpha t^{-2})^2}{k} Sinh\left(\frac{\pi k}{(\beta+\alpha\, t^{-1})^2}\right) \tag{23}$$

Figure 2 depicts the conventional behaviour of ω_{kD} from positive to negative values with different values of k. As it can be seen, the consequent evolution of DE and the thermal history of the universe, are consistent with the observations, as long as the universe enters a DE dominated phase in the future viz. z→ -1. From this Figure it is clear that the KHDE EoS parameter ω_{kD} lies in the quintessence region in the early time (past) and do not cross the phantom region divide line even at present epoch z=0 and eventually approaches the cosmological constant (CC) Λ i.e. ω_{kD} = -1 at the late time (future). The value of ω_{kD} at present epoch is -1 which is in agreement with observational data. The sources (matter) with ω_{kD} < -1/3 (i.e. - 0.333) accelerate the universe and violates the strong energy condition. A prominent applicant for DE that breaches strong energy condition (SEC) are accountable for accelerating the universe is called as the phantom having the value of EoS parameter ω_{kD} < -1. On the other hand oppositely, quintessence is regarded as matter having the value of EoS parameter as -1< ω_{kD} ≤ -1/3. Also it is evident that EoS parameter ω_{kD} do not lie in the phantom region for different values of k.

By using Eq.(17) we can get the expression of density parameter for the present model as

$$\Omega_{kD} = \frac{C^2\left(\beta+\alpha t^{-1}\right)^2}{k} Sinh\left[\frac{\pi k}{(\beta+\alpha t^{-1})^2}\right] \tag{25}$$

$$\Omega_m = 1 - \frac{C^2\left(\beta+\alpha t^{-1}\right)^2}{k} Sinh\left[\frac{\pi k}{(\beta+\alpha t^{-1})^2}\right] \tag{26}$$

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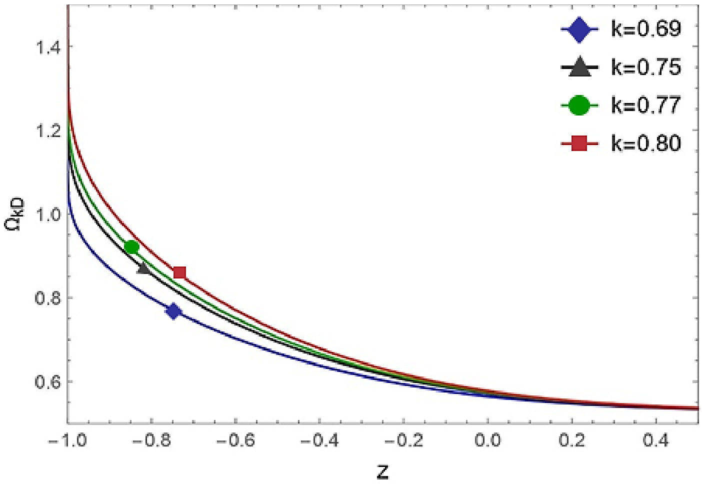

Figure 3. Illustrates nature of the density parameter of the present model of KHDE Ω_{kD} with respect to red shift z for 4 different values of constants k keeping α = 0.31, β = 0.95 and taking C= 0.41.

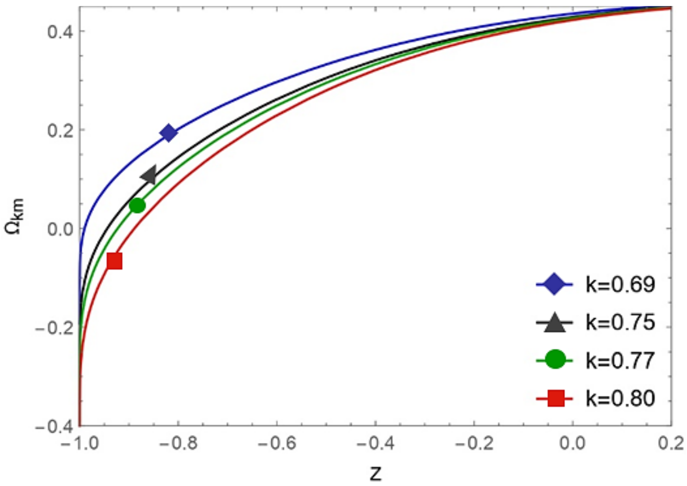

Figure 4. Illustrates nature of the density parameter of the present model of KHDE Ω_{km} with respect to red shift z for 4 different values of constants k keeping α = 0.31, β = 0.95 and taking C= 0.41.

Figure 3 and Figure 4 illustrate the evolution of the dimensionless energy densities Ω_{kD} and Ω_{km} in terms of redshift as obtained by the exact solutions, also $\Omega_{km}(z)=1-\Omega_{kD}$. Figure 3 clearly reveals that at the early times (past) of the universe where z→ ∞, we have $\Omega_{kD}\rightarrow$ 0.5 is in accordance to the observations, whereas at the late time (future) z→ -1, the DE prevails i.e. $\Omega_{kD}\rightarrow 1$. Also it is obvious from Figure 3 that for decreasing values of constant k parameter Ω_{kD} becomes smaller at the late time, which suggests that more energy is transported to the matter component.

CONCLUSIONS

We studied the recently suggested KHDE model in a flat Friedmann-Lamaitre Robertson Walker universe brimmed with KHDE and matter in the framework of GR with the apparent horizon as the IR cut-off. The late-time accelerated expansion of the universe is described by examining the quintessence and phantom scalar model. We utilized the time dependent deceleration parameter q to obtain the solution of our model. In order to anticipate the evolution, extension and acceleration of the present universe from the obtained KHDE model many cosmological parameters have been accomplished.

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