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FRW Universe in R^2 Gravity

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Abstract

We investigate a model of the Friedmann Robertson Walker metric (FRW) universe governed by hybrid expansion law (HEL) $a = t^{\alpha} e^{\beta t}$ in R^2 gravity. By Applying, energy and stability conditions in derived model, we check its physical viability. This model is based on $f(R, T) = f(R) + f(T)$; $f(R) = R + \alpha R^2$ and $f(T) = T$ and we have assumed $T = 3p > 0$. It is important to note that our model gives the current universe observable properties, including acceleration, without introducing dark energy or the cosmological constant. The following are some key features of the resulting model: Throughout the expansion of the cosmos, the jerk parameter has a positive sign, but the deceleration parameter has a negative sign. The current universe's late-time acceleration is confirmed by this. Finally, we may say that $f(R, T)$ gravity can give a considerable amount of information. There is no need for dark energy or the cosmological constant in a cosmic scenario with late time acceleration. As a consequence, one may argue that $f(R) = R + \alpha R^2$ is a dark energy model alternative that must be considered when defining the universe. We also discuss the jerk, lerk and snap parameters.

Keywords: R^2 gravity, accelerating universe.

INTRODUCTION

The accumulating astronomical evidence over the last two decades has shown without any reasonable doubt that the expansion of the universe is speeding up. The data includes the finding of the fading of distant Supernovae, measurements of the angular variations of the CMB radiation, and the age of the universe [1]. While the models suggested in [2–4] are stable [5], they have another problematic trait in that they need evolution from a solitary state in the past in a cosmic scenario [6]. In addition, it was discovered in references [7, 8] that in the existence of matter, a singularity may develop in the future if the matter density grows with time; such a future singularity is inevitable, independent of the beginning circumstances, and is attained in a period which is considerably shorter than the cosmic one. Several scenarios (phantom cosmology, quintessence models., etc.) are possible in the normal Friedmann cosmology, in which the energy density falls with time, but not in models [2–4], as demonstrated in ref. [8]. This claim stands in stark contrast to previous works [9]. As of right now, there are around 10^3 articles on the subject of changed gravity; as a result, it is challenging to list many noteworthy works. As a refresher, one may look at the books that were referenced [10–12].

Adding an R^2-term to the action, which precludes the solitary behaviour in both the past and the future, solves the aforementioned issues. While other factors have been incorporated to create the accelerated expansion in the modern cosmos, we focus here on the cosmological development of the cosmos in a theory with merely an extra R^2 term in the action. In the case of sufficiently enormous curvature, $|R|>>|R_0|$, where R_0 is the current cosmological curvature, the effect of such terms is vanishingly small. In ref. [13], the first cosmological model with a quadratic action in the curvature tensors was introduced. Radiative corrections to the standard Einstein-Hilbert action lead to the appearance of such higher-order terms

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when calculating the expected value of the matter's energy-momentum tensor against a curved backdrop. Some theories postulate an exponential (inflationary) expansion of the cosmos in the very early universe without requiring phase transitions, such as the Starobinsky model [14].

The shift to a Friedmann-like universe occurs during the warming caused by gravitational particle creation from scalaron oscillations. For example, see refs. [15–17] for extensive discussions of these model properties. Several papers [18] explored the gravitational dynamics of the universe at the fourth order. A similar investigation was conducted using a version of huge Brans-Dicke (BD) theory in which the kinetic component is removed (i.e., the BD parameter is set to $\omega = 0$) in ref. [19]. While the fluctuating behaviour seen in the Hubble parameter and curvature is similar to that discovered in our study (and previously in many others), the quantitative aspects are substantially different. Radiative corrections not only produce R^2-terms, but also $R_{\mu\nu}R^{\mu\nu}$-terms, which include the Ricci tensor squared. However, such radiatively generated terms are often fairly tiny in size. Since the characteristic mass parameter is on the order of the Planck mass in both situations, this scenario is not very relevant to the applications we will be discussing in the next sections. However, R^2 cosmology (without $R_{\mu\nu}R^{\mu\nu}$) with considerably bigger magnitude of R^2 than the natural value from radiative corrections has been proposed in the literature. In order to create a model that may, for example, eliminate singularities, an ad hoc assumption of big R^2 terms is established. We adhere to the tenets of those writings. However, it could be instructive to investigate the effects of more intricate models including R^2 and $R_{\mu\nu}R^{\mu\nu}$ variables. Possible research directions include looking at this.

Despite the fact that $f(R)$ theories have been examined at cosmic sizes to describe the accelerated expansion of the universe, they should also be verified at lower scales, namely at the astrophysical level. As a result, it's critical to look at the physical properties of compact stars within the context of $f(R)$ gravity theories. The Starobinsky model [20], which is given by $f(R) = R + \alpha R^2$, is one of the simplest variations of general theory of relativity in the strong gravity domain. The structure can be investigated by using two perturbative approaches [21–26] (here, $f(R)$ is regarded a minor disturbance from General Relativity *GR*, compact stars can be found and non-perturbative [27–30]. The gravitational mass (constructed on the star's surface) decreases with term a, according to both perturbative and nonperturbative approaches. Although a "gravitational sphere" develops beyond the star in a non-perturbative way, the astrophysical mass (as computed by distant observers) of compact stars grows with a [31, 32] for a study of the many definitions of mass in R^2 gravity. The introduction is effective with a bonus, draw mass-radius diagrams for neutron and quark stars in equilibrium. We know that the lowest normal-mode frequency of any physical condition must be real in order for it to be stable [33].

THE METRIC AND FIELD EQUATIONS

The Metric and $f(R, T) = f(R) + f(T)$; $f(R) = R + \alpha R^2$, $f(T) = T$, Gravity

The FRW space-time is written as follows:

$$ds^2 = -c^2dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \tag{1}$$

Here, the scaling factor is represented by $a(t)$, and it is a function of only t. By simply subtracting the Einstein-Hilbert action from the action representing this version of General Relativity, the action characterising this modification of General Relativity is obtained. R is exchanged for a generic nonlinear function [34–36]

$$S = \frac{c^4}{16\pi\, G_n} \int d^4x\, \sqrt{-gf(R)}\, S_m \tag{2}$$

Where *Gn and Sm* are the Newton's constant and the action for the matter fields. In this work we follow this metric conformity [37, 38]), according to action (2) the metric tensor and yield the modified Einstein equations:

$$f_R(R)R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} f_R - \nabla_\mu \nabla_\nu\, f_R + \, g_{\mu\nu} \Box f_R = kT^m_{\mu\nu} \tag{3}$$

here $T^m_{\mu\nu}$ is the energy momentum tensor for matter field, $f_R(R)$ is derivative of *f(R)* with respect to R that is *fR(R) = d f(R)/dR*, is shows the covariant derivative and $\Box = \nabla_\mu \nabla^\mu$ is the d'Alembert operator in the curved spacetime. The dynamics of R for a given matter source are determined by the trace of the above equation

$$3f_R(R) + \, Rf_R(R) - \, 2f_R(R) = 8\pi\, G_n T^m \tag{4}$$

The Einstein field equations can be written as

$$2\frac{\ddot{a}}{a} + \frac{\dot{a^2}}{a^2} - 6\frac{\alpha}{a^4}\left(5a^4 - 12a\dot{a}^2\ddot{a} + a^2\dot{a^2} + 4a^2\dot{a}\ddot{a} + 2a^2\dddot{a}\right) = -\beta\rho \tag{5}$$

$$3\frac{\dot{a^2}}{a^2} - 18\frac{\alpha}{a^4}\left(a^2\ddot{a}^2 + 2a\dot{a}\ddot{a} - 5\dot{a}^4 + 2a\dot{a}^2\ddot{a}\right) = 8\pi\rho + 2\beta - \beta\rho \tag{6}$$

We can define the Hubble's parameter (H) in the term of scale factor

$$H = \frac{\dot{a}}{a} \tag{7}$$

Applying the hybrid expansion law

$$a = t^\alpha e^{\beta t} \tag{8}$$

where α and β are constants.

PHYSICAL BEHAVIOR OF THE MODEL

After the solving field equations, the density ρ expression is

$$\rho = \frac{t^2\alpha - 36\alpha^2 - t^2\alpha^2 + 48\alpha^3 - t^4\beta - 2t^3\alpha\beta + 36t\alpha^2\beta + 6t\alpha^3\beta - t^4\beta^2 + 48t^2\alpha^2\beta^2 - e^{t\beta}\, t^{2+\alpha}\,(\alpha + t\beta)^2}{4\pi\, t^4} \tag{9}$$

The Ricci scalar can be defined as

$$R = 6H + 16\dot{H}^2 \tag{10}$$

$$R = 6\, t^{-2}(\alpha(-1 + 2\alpha) + 4t\alpha\beta + 2t^2\beta^2) \tag{11}$$

$$\frac{\dot{R}}{R} = \frac{-2\alpha(-1+2\alpha) + 2t\beta}{t(2\alpha^2 + 2t^2\beta^2 + \alpha(-1 + 4\beta t)} \tag{12}$$

JERK, SNAP AND LARK PARAMETERS

Because the jerk parameter has a third order derivative of scale factor with respect to time, it is a kinematical term that measures the expansion rate of the cosmos more precisely than the Hubble parameter. A positive jerk parameter accelerates the expansion of the cosmos [39, 40]. Figure shows that the developed model evolves with positive jerk parameter values that are not equal to 1.

The jerk parameter (*J*) for FRW model is given by [41, 42]

$$J = 1 - (1+z)\frac{\dot{H}(z)}{H} + \frac{1}{2}\,(1 + z)^2\,\frac{[H\ddot{}(z)]^2}{[H(z)]^2} \tag{13}$$

where $\dot{H}(z)$ and $\ddot{H}(z)$ represents the first and second order derivative of *H(z)* with respect to z respectively.

Jerk, Snap and Lerk parameters are

$$J = 1 + \frac{\alpha(2 - 3\alpha - 3t\beta)}{(\alpha + t\beta)^3}$$

$$S = 1 + \frac{3\alpha(-2+\alpha)}{(\alpha + t\beta)^4} + \frac{8\alpha}{(\alpha + t\beta)^3} - \frac{6\alpha}{(\alpha + t\beta)^2}$$

$$L = 1 + \frac{\alpha(4(6-5\alpha)+15(-2+\alpha)(\alpha+\beta t)+20(\alpha+\beta t)^2-10(\alpha+\beta t)^3)}{(\alpha+t\beta)^5}$$

Figure 1 Shows the variation of density and jerk parameter with respect to time.

Figure 2 Shows the variation of Ricci scalar and snap parameter with respect to time. Both figures are shows the accelerating expansion of the universe.

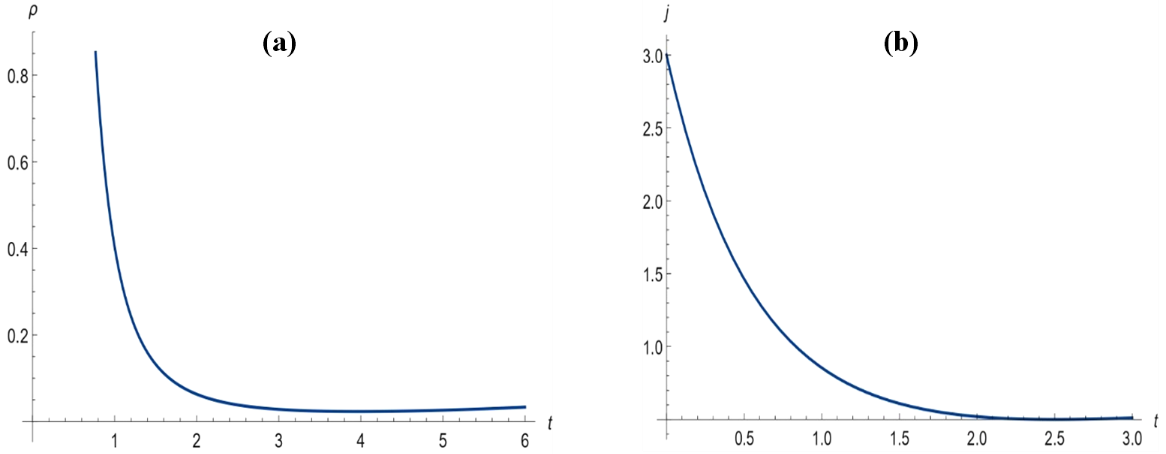

Figure 1. (a) $\alpha = 0.5$ & $\beta = 0.2$ (b) $\alpha = 0.5$ & $\beta = 0.2$.

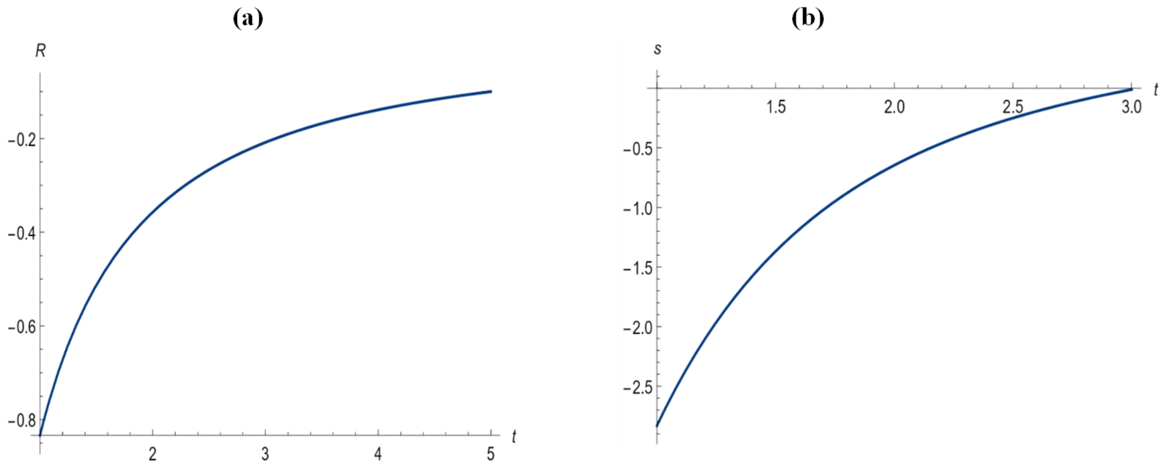

Figure 2. (a) $\alpha = 0.5$ & $\beta = 0.2$ (b) $\alpha = 0.5$ & $\beta = 0.2$.

CONCLUSIONS

We looked at the most basic model of acceleration in this work. We have taken the functional form $f(R) = R + \alpha R^2$ to fit the universe into the framework of the $f(R, T)$ theory of gravity. It is important to note that our model gives the current universe's observable properties, including acceleration, without introducing dark energy or the cosmological constant. The following are some key features of the resulting model: Throughout the expansion of the cosmos, the jerk parameter has a positive sign, but the deceleration parameter has a negative sign. The current universe's late-time acceleration is confirmed by this. Finally, we may say that $f(R, T)$ gravity can give a considerable amount of information. There is no need for dark energy or the cosmological constant in a cosmic scenario with late time acceleration. As a consequence, one may argue that $f(R) = R + \alpha R^2$ is a dark energy model alternative that must be considered when defining the universe.

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