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Dynamics and Control of a Four-Bar Mechanism with Relative Longitudinal Vibration of the Coupler Link

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Abstract

In the mechanisms and machines operating at high speeds, the elastic vibration of links is inevitable. In this paper the dynamic modeling and controller design for a flexible four-bar mechanism are studied. The fully coupled non-linear equations of motion are obtained by using the Lagrange's equations with multipliers for constrained multibody systems. The resulting differential-algebraic equations are solved using numerical methods. A simple PD controller is designed to reduce the influence of the elastic link on the desired motion.

Keywords: Dynamic analysis, control, Four-bar mechanism, Ritz-Galerkin method, Vibration.

1. Introduction

Traditionally, dynamic analysis and control of mechanisms have been based on the assumption that the links behave as rigid bodies. The demand for high speed lightweight machinery requires a redesign of the current mechanisms. Unfortunately, reducing the weight of four-bar mechanisms and/or increasing their speed may lead to the onset of elastic oscillations, which causes performance degradations such as misfeeding in the case of the card feeder mechanism in Sandor et al [1]. Therefore, the dynamic analysis and control vibration of flexible mechanisms are required. Although dynamic analysis of flexible mechanisms has been the subject of numerous investigations [1-6], the control of such systems has not received much attention [2-4]. Most of the work available in the literature which deals with vibration control of flexible mechanisms employ an actuator which acts directly on the flexible link. The effect of the control forces and moments on the overall motion is neglected. An alternative method would be to control the vibrations through the motion of the input link.

The current study deals with the control of a four-bar mechanism with a flexible coupler link. An actuator is assumed to be placed on the input link which applies a control torque. A simple PD control is designed which requires measurements of the position and angular velocity of the input link only.

2. Equations of motion of the four-bar mechanism

A four–bar mechanism OABC is shown in Figure 1. The mechanism consists of the rigid crank OA of length l_1, the flexible rod AB of length l_2 and the rigid rod BC of length l_3, the distance OC is l_0, τ is the external torque acting on the crank joint. $\mathbf{e}_1^{(0)}$ and $\mathbf{e}_2^{(0)}$ are the unit vectors of the fixed coordinate system Ox_0y_0. $\mathbf{e}_1$ and $\mathbf{e}_2$ are the unit vectors of the reference coordinate system Axy which is rotated with an angle φ_2 to the fixed one. Three variables φ_1, φ_2 and φ_3 are the angles between the x_0-axis and crank OA, the x_0-axis and flexible link AB, the x_0-axis and output link BC, respectively.

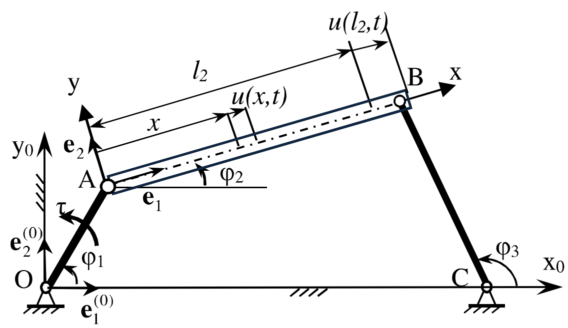

Fig. 1. Diagram of the four-bar mechanism

It should be noted that in general the constraint equations depend on the elastic deformations. In this study, the transverse deformations are neglected. Therefore, the constraint equations here depend on the longitudinal deformations:

$$f_1 = l_1 \cos\varphi_1 + \left(l_2 + u(l_2,t)\right)\cos\varphi_2 - l_3 \cos\varphi_3 - l_0 = 0$$
$$f_2 = l_1 \sin\varphi_1 + \left(l_2 + u(l_2,t)\right)\sin\varphi_2 - l_3 \sin\varphi_3 = 0 \quad (1)$$

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Ritz-Galerkin method is applied to the coupler link. The axial deformation of the coupler link is written as [7, 8]

$$u(x,t) = \sum_{i=1}^{N} X_i(x).q_i(t) \tag{2}$$

where $q_i(t)$ are the modal coordinates and $X_i(x)$ are the mode shapes of the rod. The boundary conditions are as follows:

$$u(0,t) = 0; \quad EA\frac{\partial u(l_2,t)}{\partial x} = 0 \tag{3}$$

The mode shapes are given as [7]:

$$X_i(x) = B_i \sin\left(\frac{2i-1}{2}\frac{\pi x}{l_2}\right) \tag{4}$$

To simplify the equation (4) take $B_i = 1$.

The kinetic energy of the four-bar mechanism shown in Figure 1 is given by

$$T = T_{OA} + T_{BC} + T_{AB} = \frac{1}{2}I_O\dot{\varphi}_1^2 + \frac{1}{2}I_C\dot{\varphi}_3^2 + \frac{1}{2}\int_0^{l_2}\mu\dot{\mathbf{r}}_M^2\,dx \tag{5}$$

where I_O and I_C are the mass moments of inertia of the input and output links with respect to the joint axes, respectively, μ is mass per unit length of the coupler link, and $\mathbf{r}_M$ is the position vector of a point M on the coupler link given as

$$\mathbf{r}_M = \mathbf{r}_A + (x+u)\mathbf{e}_1 \tag{6}$$

The coordinates of the point M in the fixed coordinate system are given as:

$$\begin{aligned} x_M &= l_1\cos\varphi_1 + (x+u)\cos\varphi_2 \\ y_M &= l_1\sin\varphi_1 + (x+u)\sin\varphi_2 \end{aligned} \tag{7}$$

By differentiating the equations (7) combined with equation (4), and then substituting into the equation (5), the kinetic energy is obtained as

$$\begin{aligned} T &= \frac{1}{2}\left(I_O + \mu l_1^2 l_2\right)\dot{\varphi}_1^2 + \frac{1}{2}I_C\dot{\varphi}_3^2 + \frac{\mu l_1 l_2^2}{2}\dot{\varphi}_1\dot{\varphi}_2\cos(\varphi_1-\varphi_2) \\ &+ \frac{\mu l_2^3}{6}\dot{\varphi}_2^2 + \frac{\mu}{2}\dot{\varphi}_2^2\left(2\sum_{i=1}^{N}D_i q_i + \sum_{i=1}^{N}\sum_{j=1}^{N}m_{ij}q_i q_j\right) \\ &+ \frac{\mu}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}m_{ij}\dot{q}_i\dot{q}_j + \mu l_1\dot{\varphi}_1\dot{\varphi}_2\cos(\varphi_1-\varphi_2)\sum_{i=1}^{N}C_i q_i \\ &- \mu l_1\dot{\varphi}_1\sin(\varphi_1-\varphi_2)\sum_{i=1}^{N}C_i\dot{q}_i \end{aligned} \tag{8}$$

where $C_i = \int_0^{l_2} X_i dx$, $D_i = \int_0^{l_2} xX_i dx$; $m_{ij} = \int_0^{l_2} X_i X_j dx$

The strain energy according to Reddy [9] is given by

$$\mathcal{P} = \frac{1}{2}EA\int_0^{l_2}\left(\frac{\partial u}{\partial x}\right)^2 dx = \frac{1}{2}EA\sum_{i=1}^{N}\sum_{j=1}^{N}k_{ij}q_i q_j \tag{9}$$

where $k_{ij} = \int_0^{l_2} X'_i X'_j dx$

The Lagrange's equations for constrained holonomic systems are [8]:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\eta}_j}\right) - \frac{\partial T}{\partial\eta_j} = -\frac{\partial\mathcal{P}}{\partial\eta_j} - \left(\sum_{k=1}^{2}\lambda_k\frac{\partial f_k}{\partial\eta_j}\right) + Q_j \tag{10}$$

where η_j are the generalized coordinates which include rigid body coordinates φ_1, φ_2, φ_3 as well as elastic modal coordinates q_j; f_k are the constraint equations, λ_1 and λ_2 are Lagrange multipliers; and Q_j are the generalized forces. By substituting equations (1, 8, 9) into equation (10), we obtained the equations of motion of the system as:

$$\begin{aligned} &\left(I_O + \mu l_1^2 l_2\right)\ddot{\varphi}_1 + \frac{\mu l_1 l_2^2}{2}\ddot{\varphi}_2\cos(\varphi_1-\varphi_2) \\ &+ \mu l_1\ddot{\varphi}_2\cos(\varphi_1-\varphi_2)\sum_{i=1}^{N}C_i q_i - \mu l_1\sin(\varphi_1-\varphi_2)\sum_{i=1}^{N}C_i\ddot{q}_i \\ &+ \frac{\mu l_1 l_2^2}{2}\dot{\varphi}_2^2\sin(\varphi_1-\varphi_2) + \mu l_1\dot{\varphi}_2^2\sin(\varphi_1-\varphi_2)\sum_{i=1}^{N}C_i q_i \\ &+ 2\mu l_1\dot{\varphi}_2\cos(\varphi_1-\varphi_2)\sum_{i=1}^{N}C_i\dot{q}_i = l_1\sin\varphi_1\lambda_1 - l_1\cos\varphi_1\lambda_2 + \tau \end{aligned} \tag{11}$$

$$\begin{aligned} &\frac{\mu l_1 l_2^2}{2}\ddot{\varphi}_1\cos(\varphi_1-\varphi_2) + \mu l_1\ddot{\varphi}_1\cos(\varphi_1-\varphi_2)\sum_{i=1}^{N}C_i q_i + \frac{\mu l_2^3}{3}\ddot{\varphi}_2 \\ &+ \mu\ddot{\varphi}_2\left(2\sum_{i=1}^{N}D_i q_i + \sum_{i=1}^{N}\sum_{j=1}^{N}m_{ij}q_i q_j\right) - \frac{\mu l_1 l_2^2}{2}\dot{\varphi}_1^2\sin(\varphi_1-\varphi_2) \\ &- \mu l_1\dot{\varphi}_1^2\sin(\varphi_1-\varphi_2)\sum_{i=1}^{N}C_i q_i + 2\mu\dot{\varphi}_2\left(\sum_{i=1}^{N}D_i\dot{q}_i + \sum_{i=1}^{N}\sum_{j=1}^{N}m_{ij}\dot{q}_i q_j\right) \\ &= (l_2+u_B)\sin\varphi_2\lambda_1 - (l_2+u_B)\cos\varphi_2\lambda_2 \end{aligned} \tag{12}$$

$$I_C\ddot{\varphi}_3 + l_3\sin(\varphi_3)\lambda_1 - l_3\cos(\varphi_3)\lambda_2 = 0 \tag{13}$$

$$\begin{aligned} &-\mu l_1\ddot{\varphi}_1\sin(\varphi_1-\varphi_2)C_i + \mu\sum_{j=1}^{N}m_{ij}\ddot{q}_j \\ &-\mu l_1\dot{\varphi}_1^2\cos(\varphi_1-\varphi_2)C_i - \mu\dot{\varphi}_2^2\left(D_i + \sum_{j=1}^{N}m_{ij}q_j\right) \\ &= -EA\sum_{j=1}^{N}k_{ij}q_j - \left(\lambda_1\cos\varphi_2 + \lambda_2\sin\varphi_2\right)\alpha_i \end{aligned} \tag{14}$$

where i = 1,2,..,N;

$$\alpha_i = \begin{cases} 1 & \text{when } i = 2k+1,\ k = 1,2,\ldots \\ -1 & \text{when } i = 2k,\ k = 1,2,\ldots \end{cases}$$

3. Dynamic analysis

A set of (3+N) differential equations (11-14) and two algebraic constraint equations (1) are the motion equations of mechanism with an elastic link. Therefore, we have (5+N) differential–algebraic equations with (5+N) variables given as

$$\boldsymbol{M}(s,t)\ddot{s} + \boldsymbol{\Phi}_s^T(s,t)\boldsymbol{\lambda} = \boldsymbol{p}_1(s,\dot{s},t) \tag{15}$$

$$\boldsymbol{f}(s,t) = \boldsymbol{0} \tag{16}$$

where $\mathbf{s} = [\varphi_1\ \ \varphi_2\ \ \varphi_3\ \ q_1 \ldots q_N\ \ \lambda_1\ \ \lambda_2]^T$; $\boldsymbol{\lambda}$ is vector of Lagrange multipliers; $\mathbf{f}$ is the vector of constraint equations. In this paper, the Lagrange multipliers partition method is used to solve the system of differential-algebraic equations (15) and (16) numerically. Some other algorithms can be found in Khang [8].

For comparison purposes, another set of simulations is carried out by assuming that the coupler link behaves as a rigid body (i.e., neglecting the longitudinal deformations). The values of the parameters used in the simulations are given in Table 1. The torque on the input link is given by:

$$\tau(t) = \begin{cases} \tau_0 \sin(2\pi t / T_m) & t \le T_m \\ 0 & t > T_m \end{cases} \tag{17}$$

where τ_0 is the peak torque and T_m is the duration of the torque.

The initial conditions are selected as follows: angle of input $\varphi_{10} = \pi/2$, angular velocity of input link $\dot{\varphi}_{10} = 0$, elastic deformations $q_{i0} = 0$ and elastic deformations velocity $\dot{q}_{i0} = 0$. By using the Newton - Raphson method for solving constraint equations (1a, b) with the initial conditions as above, we obtain the initial positions of the other links as:

$$\varphi_{20} = 0.6752\,rad, \varphi_{30} = 2.1564\ rad, \dot{\varphi}_{20} = 0, \dot{\varphi}_{30} = 0.$$

Figures 2 and 3 compare responses of rigid and flexible mechanism with a peak torque magnitude of τ_0 = 0.03 Nm and T_m = 1s. As mentioned in section 1, because of the fully coupled nature of the equations, the rigid body coordinates (e.g., input and output link angular displacements) are affected by the elastic deformation of the coupler link. However, this effect is negligible when the peak of torque is small.

Another set of simulations is carried out with a peak torque magnitude of τ_0 = 0.1 Nm, T_m = 1s and the simulations are performed during the period from 0 to 3s. The responses for the flexible and rigid models of the four-bar mechanism are shown in Figures 4 through 6. The effect of flexibility is now clearer, since the larger torque causes larger elastic deformations.

Table 1. Parameters of four-bar mechanism

Constant [unit]	Description	Value
l_0[m]	Length of the ground link	0.4064
l_1[m]	Length of the input link	0.0635
l_2[m]	Length of the coupler link	0.3048
l_3[m]	Length of the output link	0.3048
I_O[kgm^2]	Moment of inertia of the input link	7.466 x10^{-6}
I_C[kgm^2]	Moment of inertia of the output link	2.002 x10^{-3}
μ[kg/m]	Mass per unit length	0.2237
E[N/m^2]	Modulus of elasticity	2.06x10^{11}
A[m^2]	Cross-sectional area of the coupler link	8.19 x10^{-6}
m_1[kg]	Mass of the input link	0.0142
m_2[kg]	Mass of the coupler link	0.0682
m_3[kg]	Mass of the output link	0.0682

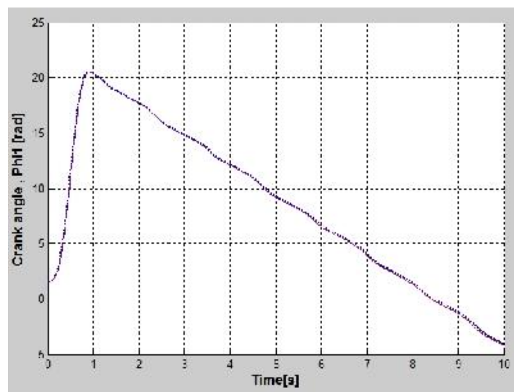

Fig. 2. Crank angle, τ_0 = 0.03 Nm.
rigid, flexible.

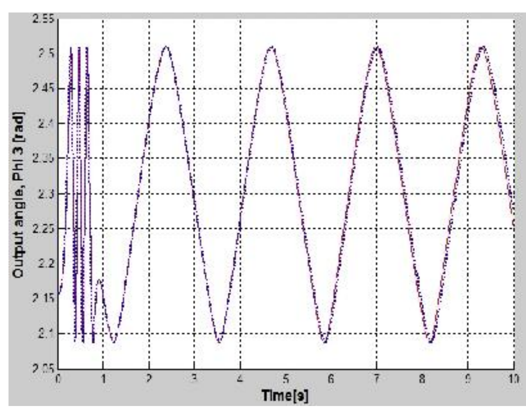

Fig. 3. Output angle, τ_0 = 0.03 Nm.
rigid, flexible.

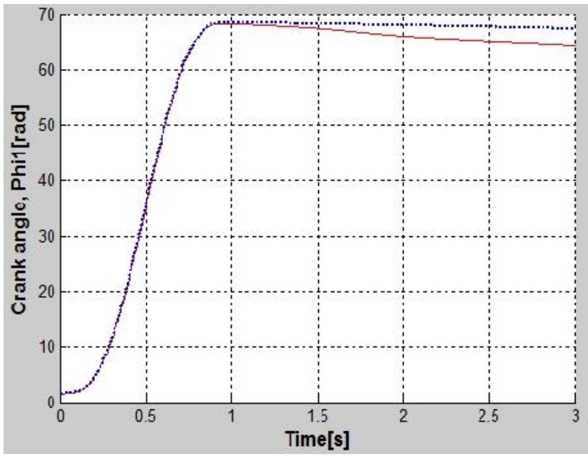

Fig. 4. Crank angle, τ_0 = 0.1 Nm
········· rigid, —— flexible.

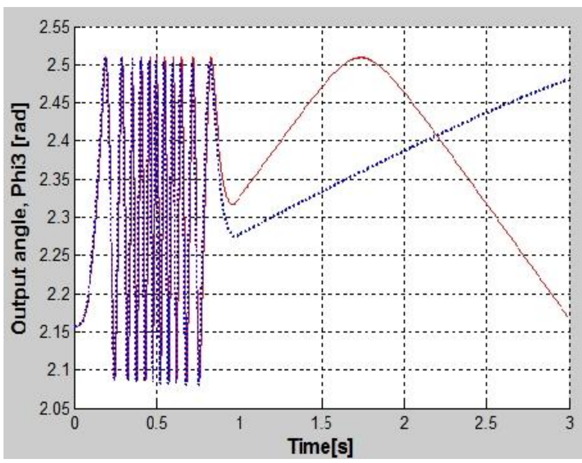

Fig. 5. Output angle, τ_0 = 0.1 Nm.
········· rigid, —— flexible.

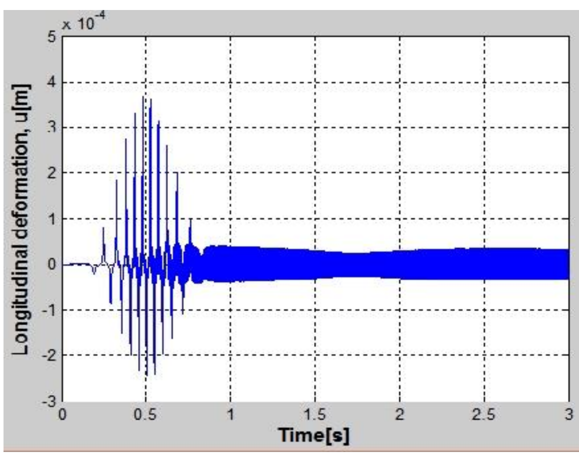

Fig. 6. Longitudinal deformation of flexible coupler link, τ_0 = 0.1 Nm.

4. Control flexible four - bar mechanism

The governed equations are highly non-linear differential equations. Most control designs require a linear unconstrained model. In this case a straightforward linearization is not possible since it may be difficult to find an operating point. However, the equations can be linearized around a rigid body trajectory. Since the main goal of the controller is to suppress vibrations, a rigid body trajectory can be used as the nominal trajectory and the deviations from this can be assumed small. The real trajectory of flexible mechanism can be obtained as follows

$$\begin{cases} \varphi_1 = \varphi_{1d} + y_1 \\ \varphi_2 = \varphi_{2d} + y_2 \\ \varphi_3 = \varphi_{3d} + y_3 \\ y_{3+i} = q_i \end{cases} \tag{18}$$

where φ_{1d}, φ_{2d}, φ_{3d} represent the rigid mechanism trajectory; y_1, y_2, y_3 are deviation of flexible trajectory versus rigid trajectory. It is assumed that y_1, y_2, y_3 , y_{3+i} are small.

In order to investigate whether it is possible to suppress vibrations of the flexible link by a control torque applied to the input link, a simple control strategy, in particular PD controller is used. The target of the controller is that y_1, y_2, y_3 , y_{3+i} approach zero when time approaches infinity (or large enough). The control torque applied to the input link is given by:

$$\tau_c = -K_P y_1 - K_d \dot{y}_1 \tag{19}$$

where K_p, K_d are the PD controller gains, $y_1 = \varphi_1 - \varphi_{1d}$; $\dot{y}_1 = \dot{\varphi}_1 - \dot{\varphi}_{1d}$. Thus, the total torque acting on the input link is $\tau + \tau_C$.

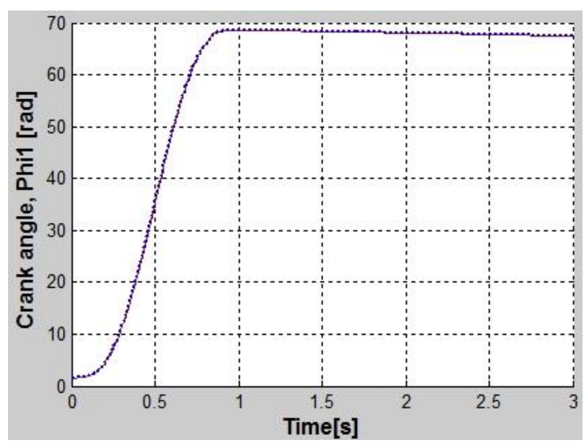

Fig. 7. Crank angle with PD controller, τ_0 = 0.1 Nm
········· rigid, —— flexible.

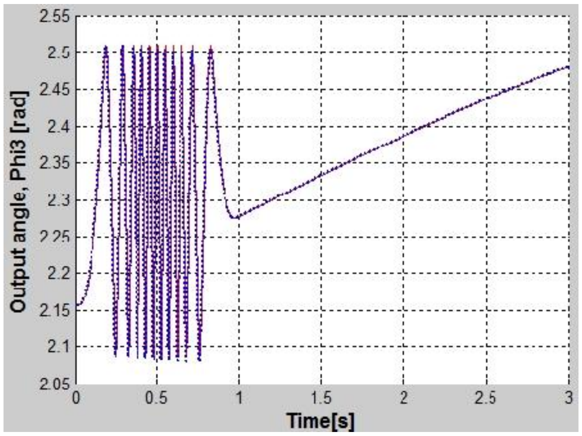

Fig. 8. Output angle with PD controller. τ_0 = 0.1 Nm.
.... rigid, —— flexible.

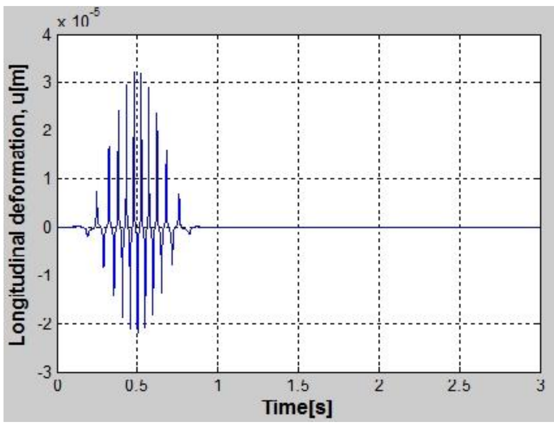

Fig. 9. Longitudinal deformation of flexible coupler link with PD controller, τ_0 = 0.1 Nm.
.... rigid, —— flexible.

Now, we will suppress vibrations in case the peak torque magnitude of τ_0 = 0.1 Nm and T_m = 1s (e.g., Figures 4 – 6). The parameters used in the simulations are given in Table 1. The controller gains are chosen as K_p = 0.5, K_d = 0.2. The calculating results are shown in Figs 7 – 9. These results show that controller is able to suppress the vibrations and control the link angular motions. In Figure 9 the longitudinal deformation is suppressed within 0.85 s.

5. Conclusions

In this study, dynamic modeling and control of a four-bar mechanism with flexible coupler link has been investigated and it was assumed that there is only axial deformation. The non-linear equations of motion are obtained through a constrained Lagrangian approach and described by a set of differential-algebraic equations. The governed differential-algebraic equations are solved numerically to simulate the system behavior. A simple PD controller was designed which does not require measurement of the elastic deformations. The controller has been shown to be efficient in suppressing the vibrations of the flexible link as well as controlling the rigid body motion.

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