Optimal Parameters of Linear Dynamic Vibration Absorber for Reduction of Torsional Vibration

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Abstract

This paper presents three analytical methods to determine optimal parameters of the passive mass-spring-disc dynamic vibration absorber (DVA), such as the ratio between natural frequency of DVA and shaft, damping ratio of DVA. The original model presented by Den Hartog, Luft and Warburton are solved and has shown in good agreement. Three analytical methods is then adopted for torsional shaft model. The simulation results indicate that the effectiveness in torsional vibration could be reduced. Finally, the optimal parameters of DVA were applied to decrease the shaft torsional vibration considering the vibration duration and stability criterion.

Keywords: Dynamic vibration absorber, Torsional vibration, Fixed-points theory.

1. Introduction

The dynamic vibration absorber (DVA) or tuned-mass damper (TMD) is a widely used passive vibration control device. When a mass-spring system, referred to as primary system, is subjected to a harmonic excitation at a constant frequency, its steady-state response can be suppressed by attaching a secondary mass-spring system or DVA. This idea was pioneered by Watts in 1883 and Frahm in 1909. However, a DVA consisting of only a mass and spring has a narrow operation region and its performance deteriorates significantly when the exciting frequency varies. The performance robustness can be improved by using a damped DVA that consists of a mass, spring, and damper. The key design parameters of a damped DVA are its tuning parameter and damping ratio.

The optimization technique for original model that is described in detail by Den Hartog [1]. The optimum tuning ratio of the neutralizer was found as a function of the neutralizer's mass given by

$$\alpha_{opt} = \frac{1}{1+\mu} \tag{1}$$

and the damping ratio of the absorber

$$\xi_{opt} = \sqrt{\frac{3\mu}{8(1+\mu)}} \tag{2}$$

where μ is the ratio of the absorber's mass to the primary structure's mass.

Since then, the fixed-points theory and DVA structures have become one of the design laws used in optimizing design of the damped and undamped primary system [6-8].

Luft proposed methodology MEVR (maximum of equivalent viscosity resistance) for the original model [2]. Later, Warburton used minimum of quadratic torque method (MQT) and found that the damping in the neutralizer can also be optimized [3]. The results was given by

$$\alpha_{opt} = \frac{\sqrt{1+\mu/2}}{1+\mu};\ \xi_{opt} = \sqrt{\frac{\mu(1+3\mu/4)}{4(1+\mu/2)(1+\mu)}} \tag{3}$$

This paper presents three analytical methods such as FPM, MQT and MEVR to determine optimal parameters of the dynamic vibration absorber (DVA) for new shaft model such as the ratio between natural frequency of DVA and shaft, the damping ratio of absorber. Since then we compare and evaluate optimal effectiveness of those methods. Based on the main idea is to build a program that calculates to prove optimal analytic solution of the original model, which applies to torsion shaft model. Optimal parameters are presented as very neat analysis. The simulation results indicate that the effectiveness in torsional vibration reduction.

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2. Shaft modeling and equations of vibration

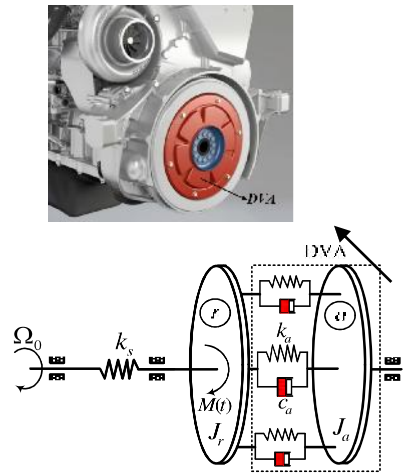

Fig.1. Modeling of the shaft system with DVA.

In this study, the shaft model shown in Figure 1 is considered. The shaft is modeled as a torsion spring which has stiffness k_s and a disc which has moment of inertia is J_r and rotating at the constant angular velocity Ω_0 is disturbed by harmonic torque *M(t)*. The passive mass-spring-disc dynamic vibration absorber (DVA) is attached on the shaft to minimize the torsional vibration of the shaft.

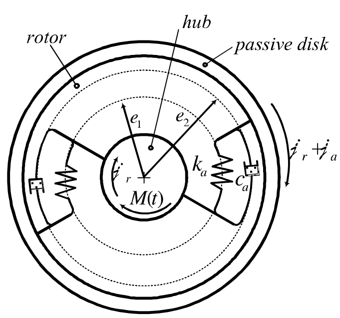

Fig. 2. Modeling of the DVA

Figure 2 shows the model of the DVA used in this study. The DVA contains a passive disk and springs-dampers system. The radius and moment of inertia of the passive disk are *R*, J_a, respectively. The shaft and the passive disc are linked together by springs and dampers system. The stiffness of each spring is k_a. The viscous coefficient of each damper is c_a. *n* is the number of springs-dampers. The angular displacement of the rotor is φ_r and the torsional vibration of shaft can be written as $\theta(t) = \varphi_r - \Omega_0 t$.

The relative angular displacement between the rotor and passive disk as φ_a.

The system equations of motion can be expressed by

$$(J_r + J_a)\ddot{\theta} + J_a\ddot{\varphi}_a + k_s\theta = M(t) \tag{4}$$

$$J_a\ddot{\theta} + J_a\ddot{\varphi}_a + nc_a e_2^2\dot{\varphi}_a + nk_a e_1^2\varphi_a = 0 \tag{5}$$

The natural frequency of the DVA and the shaft, respectively

$$\omega_a = \sqrt{\frac{k_a}{m_a}} \qquad \Omega_s = \sqrt{\frac{k_s}{J_r}} \tag{6}$$

Introducing the dimensionless parameters

$$\mu = \frac{m_a}{m_r}; \eta = \frac{r_a}{r_r}; \gamma = \frac{e_1}{r_r}; \lambda = \frac{e_2}{r_r} \tag{7}$$

$$\alpha = \frac{\omega_a}{\Omega_s}; \beta = \frac{\omega}{\Omega_s}; \xi = \frac{c_a}{m_a\omega_a} \tag{8}$$

where ω is the frequency of excitation torque

Therefore, Eqs.(4) and (5) can be expressed by

$$\left(1+\mu\eta^2\right)\ddot{\theta} + \mu\eta^2\ddot{\varphi}_a + \Omega_s^2\theta = \frac{M}{J_r} \tag{9}$$

$$\begin{aligned} &\mu\eta^2\ddot{\theta} + \mu\eta^2\ddot{\varphi}_a + n\xi\alpha\Omega_s\mu\lambda^2\dot{\varphi}_a + \\ &+ n\alpha^2\Omega_s^2\mu\gamma^2\varphi_a = 0 \end{aligned} \tag{10}$$

The matrix form of Eqs.(9,10) are expressed as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F} \tag{11}$$

where $\mathbf{q} = \{\theta \quad \varphi_a\}^T$

The mass matrix, viscous matrix, stiffness matrix and excitation force vector can be derived as

$$\mathbf{M} = \begin{bmatrix} 1+\mu\eta^2 & \mu\eta^2 \\ \mu\eta^2 & \mu\eta^2 \end{bmatrix} \mathbf{C} = \begin{bmatrix} 0 & 0 \\ 0 & n\xi\alpha\Omega_s\mu\lambda^2 \end{bmatrix} \tag{12}$$

$$\mathbf{K} = \begin{bmatrix} \Omega_s^2 & 0 \\ 0 & n\alpha^2\Omega_s^2\mu\gamma^2 \end{bmatrix} \quad \mathbf{F} = \begin{Bmatrix} \dfrac{M(t)}{J_r} \\ 0 \end{Bmatrix} \tag{13}$$

3. Determine optimal parameters of the DVA

3.1. Fixed-points theory for optimal design

The forced vibration of this system will be of the form

$$M(t) = \hat{M}\,e^{I\omega t} \tag{14}$$

Thus, the stationary response of this system which can be written as

$$\theta(t) = \hat{\theta}\,e^{I\omega t}, \quad \varphi_a(t) = \hat{\varphi}_a\,e^{I\omega t} \tag{15}$$

where $\hat{\theta}$ and $\hat{\varphi}_a$ are complex amplitude vibration of the primary system and DVA, respectively.

Substituting Eqs.(14-15) into Eq.(11), this becomes

$$\left(-\beta^{2}\begin{bmatrix}1+\mu\eta^{2} & \mu\eta^{2}\\ \mu\eta^{2} & \mu\eta^{2}\end{bmatrix}+ 2i\beta\begin{bmatrix}0 & 0\\ 0 & \nu\xi\alpha\Omega_s\mu\lambda^{2}\end{bmatrix}+ \begin{bmatrix}\Omega_s^{2} & 0\\ 0 & \nu\alpha^{2}\Omega_s^{2}\mu\gamma^{2}\end{bmatrix}\right)\begin{bmatrix}\hat{\theta}\\ \hat{\varphi}_a\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}\frac{\hat{M}}{k_s} \tag{16}$$

Hence the stationary response of the primary system is expressed as

$$\hat{\theta}=\frac{A_1+iA_2\xi}{A_3+iA_4\xi}\left|\frac{\hat{M}}{k_s}\right| \tag{17}$$

where

$$A_1=-\alpha^{2}\gamma^{2}\nu+\beta^{2}\eta^{2};\quad A_2=-\alpha\beta\nu\lambda^{2};$$

$$A_3=\alpha^{2}\beta^{2}\mu\nu\eta^{2}\gamma^{2}+\alpha^{2}\beta^{2}\gamma^{2}\nu-\beta^{4}\eta^{2}-\alpha^{2}\gamma^{2}\nu+\beta^{2}\eta^{2}$$

$$A_4=\alpha\beta^{3}\eta^{2}\lambda^{2}\mu\nu+\alpha\beta^{3}\lambda^{2}\nu-\alpha\beta\lambda^{2}\nu$$

After short caculation the Eq.(17) we obtained the real amplitude of the vibration response, which can be written as

$$\left|\hat{\theta}(t)\right|=\sqrt{\frac{A_1^{2}+A_2^{2}\xi^{2}}{A_3^{2}+A_4^{2}\xi^{2}}}\left|\frac{\hat{M}}{k_s}\right|=A\left|\frac{\hat{M}}{k_s}\right| \tag{18}$$

where A is called the amplifier function that is defined by

$$A=\sqrt{\frac{A_1^{2}+A_2^{2}\xi^{2}}{A_3^{2}+A_4^{2}\xi^{2}}} \tag{19}$$

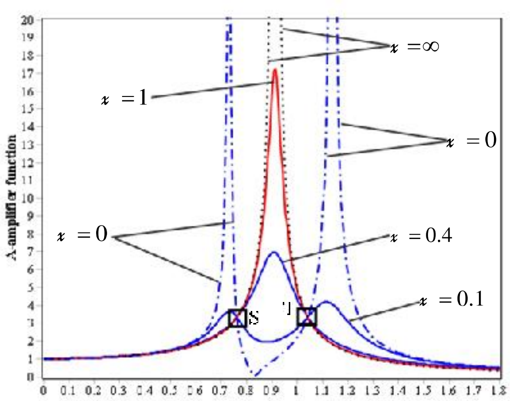

Fig. 3 The graph of amplifier function

Figure 3 shows a plot of the amplifier fuction with some of damping ratio. For $c = 0$ or c becomes infinite so the amplifier fuction curve becomes infinite. That means some where in between there must be a value of damping ratio for which the peak becomes a minimum. Two other curves are draw in Fig. 3, for $\xi = 0.1$ and 0.4.

The first step of this method is to specify two fixed points. Suppose that two points (S and T) with horizontal coordinates as a β_1, β_2. The conditions for A does not depend on the damping ratio is expressed as follows

$$\frac{\partial A}{\partial \xi}=0 \tag{20}$$

Substituting Eq.(19) into Eq.(20), this becomes

$$-\frac{\xi\left(A_1^{2}A_4^{2}-A_2^{2}A_3^{2}\right)}{\left(A_4^{2}\xi^{2}+A_3^{2}\right)^{2}\sqrt{\dfrac{A_1^{2}+A_2^{2}\xi^{2}}{A_3^{2}+A_4^{2}\xi^{2}}}}=0, \tag{21}$$

$$\Rightarrow A_1^{2}A_4^{2}-A_2^{2}A_3^{2}=0 \tag{22}$$

From Eq.(22), we have

$$\left|\frac{A_1}{A_3}\right|\Big|_{\beta=\beta_1}=\left|\frac{A_2}{A_4}\right|\Big|_{\beta=\beta_1} \tag{23}$$

$$\left|\frac{A_1}{A_3}\right|\Big|_{\beta=\beta_2}=\left|\frac{A_2}{A_4}\right|\Big|_{\beta=\beta_2} \tag{24}$$

We obtain the value of A at two points (S, T) these are expressed as follows

$$A\big|_S=\left|\frac{A_2}{A_4}\right|\Big|_{\beta=\beta_1} \tag{25}$$

$$A\big|_T=\left|\frac{A_2}{A_4}\right|\Big|_{\beta=\beta_2} \tag{26}$$

Den Hartog [1] reported that the graph of amplifier function does not change in between the two peaks (S, T) when the vertical coordinates of the S and T must be equal. In this condition, we have

$$A\big|_S=A\big|_T \tag{27}$$

The optimal parameter of α and β are specifed by solving Eqs.(23-27) which can be written as

$$\alpha=\alpha^{*}=\frac{\eta}{\gamma\sqrt{\nu}\left(\mu\eta^{2}+1\right)} \tag{28}$$

$$\beta_{1,2}^{2}=\beta_{1,2}^{*2}=\pm\eta\sqrt{\frac{\mu}{\mu\eta^{2}+2}}+\frac{1}{\mu\eta^{2}+1} \tag{29}$$

Then, the optimum absorber damping can be identified as follows

$$\frac{\partial A}{\partial \beta} = 0 \tag{30}$$

Eq. (19) gives

$$A^2\left(A_3^2 + A_4^2\xi^2\right) = A_1^2 + A_2^2\xi^2 \tag{31}$$

Taking derivative of Eq.(31) with respect to β, this becomes

$$\xi^2 = -\frac{-A^2A_3\frac{\partial A_3}{\partial \beta} - AA_3^2\frac{\partial A}{\partial \beta} + A_1\frac{\partial A_1}{\partial \beta}}{-A^2A_4\frac{\partial A_4}{\partial \beta} - AA_4^2\frac{\partial A}{\partial \beta} + A_2\frac{\partial A_2}{\partial \beta}} \tag{32}$$

Substituting Eq.(30) into Eq.(32) we obtain

$$\xi^2 = -\frac{-A^2A_3\frac{\partial A_3}{\partial \beta} + A_1\frac{\partial A_1}{\partial \beta}}{-A^2A_4\frac{\partial A_4}{\partial \beta} + A_2\frac{\partial A_2}{\partial \beta}} \tag{33}$$

Substituting Eqs.(28-29) into Eq.(33), this becomes

$$\xi_1^2 = -\left.\frac{A_1\frac{\partial A_1}{\partial \beta} - A^2A_3\frac{\partial A_3}{\partial \beta}}{A_2\frac{\partial A_2}{\partial \beta} - A^2A_4\frac{\partial A_4}{\partial \beta}}\right|_{\beta=\beta_1} \tag{34}$$

and

$$\xi_2^2 = -\left.\frac{A_1\frac{\partial A_1}{\partial \beta} - A^2A_3\frac{\partial A_3}{\partial \beta}}{A_2\frac{\partial A_2}{\partial \beta} - A^2A_4\frac{\partial A_4}{\partial \beta}}\right|_{\beta=\beta_2} \tag{35}$$

Brock [5] reported that the optimal value of damping as follows

$$\xi_{opt} = \xi^* = \sqrt{\frac{\xi_1^2 + \xi_2^2}{2}} \tag{36}$$

Substituting Eqs.(34-35) into Eq. (36) we obtain the optimal value of damping ratio as following

$$\xi_{opt} = \xi^* = \frac{\gamma\eta^2}{\lambda^2}\sqrt{\frac{3\mu}{2n(1+\mu\eta^2)}} \tag{37}$$

3.2. Minimum of quadratic torque (MQT) for optimal design

The state equations of Eqs.(9,10) are expressed as [3,9]:

$$\dot{\mathbf{y}}(t) = \mathbf{B}\mathbf{y}(t) + \mathbf{H}_f M(t) \tag{38a}$$

where

$$\mathbf{y} = \left\{\theta \quad \varphi_a \quad \dot{\theta} \quad \dot{\varphi}_a\right\}^T \tag{38b}$$

The system matrix **B** is derived in [4,9] and has the form

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} & \mathbf{E} \\ \mathbf{-M^{-1}K} & \mathbf{-M^{-1}C} \end{bmatrix} \tag{38c}$$

where **E** is matrix unit, $\mathbf{E} \in \mathbb{R}^2$

In this study, the **B** matrix can be obtained as

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\Omega_s^2 & \nu\alpha^2\Omega_s^2\mu\gamma^2 & 0 & \nu\xi\alpha\Omega_s\mu\lambda^2 \\ \Omega_s^2 & -\frac{\left(1+\mu\eta^2\right)\nu\alpha^2\Omega_s^2\gamma^2}{\eta^2} & 0 & -\frac{\left(1+\mu\eta^2\right)\nu\alpha^2\Omega_s^2\lambda^2}{\eta^2} \end{bmatrix} \tag{39}$$

Matrix of excitation force is obtained as [4,9]

$$\mathbf{H}_f = \frac{1}{M(t)}\left[\mathbf{0} \quad \mathbf{M}^{-1}\mathbf{F}\right]^T = \left[0 \quad 0 \quad J_r^{-1} \quad -J_r^{-1}\right]^T \tag{40}$$

The quadratic torque matrix **P** is solution of the Lyapunov equation [3]

$$\mathbf{BP} + \mathbf{PB}^T + S_f\mathbf{H}_\mathrm{f}\mathbf{H}_\mathrm{f}^T = \mathbf{0} \tag{41}$$

where S_f is the white noise spectrum of the excitation torque. The quadratic torque for vibration of shaft is determined by solving the Eq.(41)

$$P_{11} = \frac{1}{2}S_f\frac{\left[\begin{array}{l} n^2\alpha^4\gamma^4\left(1+\mu\eta^2\right)^2 + \\ +n\left(\begin{array}{l}\nu\xi^2\lambda^4(1+\mu\eta^2) - \\ \eta^2\alpha^2\gamma^2(2+\mu\eta^2)+\eta^4\end{array}\right)\end{array}\right]}{\Omega_s^3\nu\xi\alpha\mu\lambda^2 m_r^2\rho_r^2\eta^4} \tag{42}$$

Minimum condition are expressed as

$$\left.\frac{\partial P_{11}}{\partial \alpha}\right|_{\alpha=\alpha^*} = 0; \left.\frac{\partial P_{11}}{\partial \xi}\right|_{\xi=\xi^*} = 0 \tag{43}$$

The optimal parameters of the DVA for design that was determined by solving the Eqs.(42,43)

$$a_{opt} = \frac{\sqrt{2}}{2}\frac{h\sqrt{n(2+mh^2)}}{ng(1+mh^2)} \quad (44)$$

$$x_{opt} = \frac{gh^2}{2\ell^2}\sqrt{\frac{2m(4+3mh^2)}{n(1+mh^2)(2+mh^2)}} \quad (45)$$

3.3.Maximum of equivalent viscous resistance (MEVR) for optimal design

The first step of this method is to specify these quadratic torques. By solving the Eq.(41) these quadratic torques for vibration of shaft were obtained as

$$P_{32} = -\frac{S_f}{2h^2 r_r^4 m_r^2 m\Omega_s^2} \quad (46)$$

$$P_{33} = \frac{S_f\begin{bmatrix} a^4h^2g^4mn^2 + a^4g^4n^2 + \\ +a^2\ell^4n^2x^2 - 2a^2h^2g^2n + h^4 \end{bmatrix}}{2\Omega_s n x a m\ell^2 m_r^2 r_r^4 h^4} \quad (47)$$

$$P_{34} = \frac{S_f(a^2g^2n - h^2)}{2nxa\Omega_s\ell^2 r_r^4 m_r^2 mh^2} \quad (48)$$

After short caculation the Eqs.(4,5) we obtained

$$m_r r_r^2\ddot{g} + k_s g = nk_a e_1^2 j_a + nc_a e_2^2 \dot{j}_a + M \quad (49)$$

Hence the equivalent resistance torque on the primary structure which was obtained as

$$M_{eqv} = nk_a e_1^2 j_a + nc_a e_2^2 \dot{j}_a \quad (50)$$

Substituting Eqs.(7,8) into Eq.(50), this becomes

$$M_{eqv} = nm_a a^2\Omega_s^2 g^2 r_r^2 j_a + nx\, m_a a\,\Omega_s \ell^2 r_r^2 \dot{j}_a \quad (51)$$

Thus the equivalent resistant coefficient of the DVA on the primary structure was obtained as

$$c_{td} = -\frac{\begin{bmatrix} nx\, m_a a\,\Omega_s\ell^2 r_r^2\langle \dot{j}_a\dot{g}\rangle + \\ +nm_a a^2\Omega_s^2 g^2 r_r^2\langle j_a \dot{g}\rangle \end{bmatrix}}{\langle \dot{g}^2\rangle} \quad (52)$$

If the primary system is excited by random moment with a white noise spectrum S_f, then the average value of Eq.(52) are the components of the matrix **P** in Eq.(41), Lyapunov equation, this means

$$c_{td} = -\frac{\begin{bmatrix} nx\, m_a a\,\Omega_s\ell^2 r_r^2 P_{34} + \\ +nm_a a^2\Omega_s^2 g^2 r_r^2 P_{32} \end{bmatrix}}{P_{33}} \quad (53)$$

Maximum condition are expressed as

$$\left.\frac{\partial c_{td}}{\partial a}\right|_{a=a^*} = 0; \left.\frac{\partial c_{td}}{\partial x}\right|_{x=x^*} = 0 \quad (54)$$

The optimal parameters of the DVA are determined by solving the equation Eqs.(53-54)

$$a_{opt} = a^* = \frac{h}{g\sqrt{n(1+mh^2)}} \quad (55)$$

$$x_{opt} = x^* = \frac{gh^2}{\ell^2}\sqrt{\frac{m}{n}} \quad (56)$$

4. Numerical simulations and discussions

In this paper, we survey the shaft with the parameters in Table 2. The shaft rotating is disturbed by the harmonic torque *M(t)* of amplitude *5 Nm* and frequency *18.849 rad/s.*

Table 1. Value of optimal parameters

Parameters	FPM	MQT	MEVR
a	0.6670	0.6703	0.6737
x	0.0656	0.0537	0.0541

Table 2. The input parameters for simulation

Parameters	Value	Units
m_r	5.0	kg
m_a	0.1	kg
a	0.1	m
x	0.1	m
e_1	0.06	m
e_2	0.08	m
n	6	-

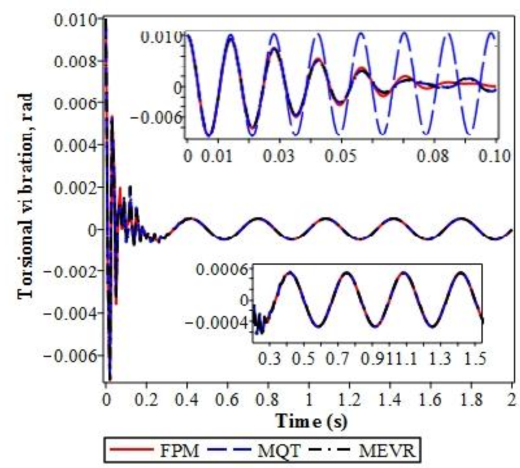

Fig. 4. Torsional vibration with optimized DVA

Table 1 describes the optimum value that corresponding to the input data of Table 2. Simulation results with optimal parameters described in Fig. 4.

These results show that torsional vibration of shaft without DVA has a harmonic form amplitude of about 0.02 rad.

Figure 4 shows that in the first 0.4s, the amplitude of the torsional vibration reduces rapidly. Effectiveness of the optimal DVA using FPM is highest in comparison to the two other methods, however, the difference between the viration response curves are negligible, especially vibration responses of the system with MQT and MEVR are nearly the same. This shows the strength of the fixed-points theory compared to other analytical methods. After the above period, the torsional vibration of the shaft shifts to the steady state with a very small amplitude of about 1.20E-03 rad. At this stage, the vibration responses with optimal DVA determined by all methods are almost identical.

5. Conclusions

In this paper, three analytical methods have been developed and examined for new shaft model. The same procedure as in the conventional analytical theory has been used to derive the optimum tuning and damping ratios of the device. Research results are verified by numerical simulation with high reliability. The optimal parameters were determined in analytical form and furthermore leads to the simple explicit formulas. The results presented in this paper may offer new ways of using the device over the conventional one.

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