Fatigue Life Prediction under Multiaxial Variable Amplitude Loading Using a Stress Invariant Based Criterion

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Abstract

Predicting fatigue damage for structural components subjected to variable amplitude loadings is a complex issue. In order to estimate fatigue life under those loading conditions, a multiaxial fatigue criterion must gather with a fatigue damage accumulation rule that allows capturing different damage mechanisms when they are activated. In this paper, combinations of a stress invariant based fatigue criterion with some damage accumulation rules are carried out to deal with variable amplitude loadings. An approach combining three tools, including multiaxial criterion, multiaxial S-N curves and cumulative damage rule are used in this study. Results show good correlations of fatigue life between experimental and predicted results for 1045 steel.

Keywords: Variable amplitude loading, multiaxial fatigue criterion, Damage accumulation.

1. Introduction

Damage evaluation of variable amplitude loadings is a challenge in multiaxial fatigue characterization. Under the variable amplitude loadings, different damage mechanisms can be activated. To predict fatigue life, a multiaxial fatigue criterion must gather with a fatigue damage accumulation rule that allows capturing the different damage mechanisms when they are activated. In general, four categories of fatigue criteria can be distinguished: critical plane approaches, integral approaches, approaches based on the stress invariants and energy approaches [1]. Among multiaxial fatigue criteria, the approaches based on stress invariants have an obvious advantage of computation time compared to the critical plane and the integral approaches. A recent stress based criterion proposed by Vu et al. [1] shows a very good prediction quality for a wide range of experimental data conducted on various steels. Regarding damage accumulation rules, a comprehensive review of many approaches can be found elsewhere [2]. In this paper, combinations of the Vu et al. criterion and some damage accumulation rules are carried out to deal with variable amplitude loading conditions.

2. Fatigue life prediction methodology

2.1. Prediction approach

In order to predict the fatigue life under multiaxial variable amplitude loading, an approach combining three tools (multiaxial criterion, multiaxial S-N curves and cumulative damage rule) are used in this study. The approach includes following steps:

Step 1: choose a reference S-N curve (under fully reversed torsion, for instance) and identify the function of the reference S-N curve (power law function) from experimental data:

$$\Sigma_{xya} = \frac{t_{-1}}{1 - \mathscr{k}\, N^{-\ell}} \qquad (1)$$

where $\mathscr{k}$ and ℓ are material parameters, Σ_{xya} is shear stress amplitude and N is number of cycles.

Step 2: predict the fatigue limit values of different constant amplitude multiaxial loading cases by a multiaxial fatigue criterion.

Step 3: build the S-N curve of each constant amplitude multiaxial loading case from its fatigue limit value and the function of reference S-N curve [3].

Step 4: estimate the cumulative damage and the fatigue life of the multiaxial variable amplitude loading (many blocks) by using a cumulative damage rule.

In this study, a fatigue criterion based on stress invariants is used in step 2 and different cumulative rules (Miner, DCA) will be applied in step 4. More details about step 1 and step 3 may be found in [3].

2.2. A fatigue criterion based on stress invariants

Vu et al. [1] proposed a multiaxial criterion for high cycle fatigue based on invariants of macroscopic stress tensor (denoted as Vu criterion). By

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introducing quantity $J_{2,mean}$, the multiaxial fatigue endurance criterion is established as follows:

$$f = \sqrt{\gamma_1 J'_2(t)^2 + \gamma_2 J^2_{2,mean} + \gamma_3 I_f\left(I_{1,a}, I_{1,m}\right)} \le \beta \tag{2}$$

where γ_1, γ_2, γ_3 and β are material parameters; $J'_2(t)$ and $J_{2,mean}$ capture shear stress effect and phase shift effect. $J'_2(t)$ is the second invariant of stress amplitude part defined from the deviator of the amplitude of the stress tensor $\underline{\underline{S}}^a(t)$ and $J_{2,mean}$ is the mean value of $J'_2(t)$ during period T:

$$J'_2(t) = \sqrt{\frac{1}{2}\underline{\underline{S}}^a(t) : \underline{\underline{S}}^a(t)} \tag{3}$$

$$J_{2,mean} = \frac{1}{T}\int_0^T J'_2(t)dt \tag{4}$$

$I_f\left(I_{1,a}, I_{1,m}\right)$ is a function of $I_{1,a}$ and $I_{1,m}$ reflecting respectively the effects of amplitude and mean value of the hydrostatic stress. The values of $I_{1,a}$ and $I_{1,m}$ are defined from $I_1(t) = tr(\sigma)$ that is the first invariant of the stress tensor:

$$I_{1,a} = \frac{1}{2}\left\{\max_{t\in T} I_1(t) - \min_{t\in T} I_1(t)\right\} \tag{5}$$

$$I_{1,m} = \frac{1}{2}\left\{\max_{t\in T} I_1(t) + \min_{t\in T} I_1(t)\right\} \tag{6}$$

Vu criterion is identified from two fatigue limits, under fully reversed torsion (t_{-1}) and under fully reversed tension (f_{-1}), for instance. The value of γ_1, γ_2, γ_3 and β are determined from the two fatigue limits:

$$\beta = t_{-1} \tag{7}$$

$$\gamma_3 = \frac{t^2_{-1} - f^2_{-1}/3}{f_{-1}} \tag{8}$$

$$\gamma_1 + \frac{4}{\pi^2}\gamma_2 = 1 \tag{9}$$

In order to capture the effects of phase shift and mean stress, Vu et al. [1] proposed to distinguish two categories of metals based on ultimate strength R_m:

Low-strength metals (R_m < 750 MPa): $\gamma_1 = 0.65$ and $\gamma_2 = 0.8636$; $I_f\left(I_{1,a}, I_{1,m}\right) = I_{1,a} + I_{1,m}$

High-strength metals (R_m > 750 MPa): $\gamma_1 = 0.65$ and $\gamma_2 = 0.8636$; $I_f\left(I_{1,a}, I_{1,m}\right) = I_{1,a} + \frac{f_{-1}}{t_{-1}} I_{1,m}$

The presence of $J_{2,mean}$ quantity allows capturing accurately effects of phase shift and frequency on fatigue limit of material under multiaxial loading. The prediction capacity of the criterion is tested on 119 iso-frequency axial–torsion experiments and some other more complex loadings: biaxial loading and asynchronous loading and the results show that the criterion is in good accordance with the experimental data [1]. Under proportional loading, the assessment of the criterion can be carried out from an analytical solution. Under others loading cases, the numerical implementation of the criterion is very simple and can be easily integrated in a damage model or in a finite element code.

2.3. Cumulative damage rules

Damage accumulation models aim to account accumulated damage in material based on damage parameters. Basically, damage parameters are used to estimate fatigue strength under certain stress level and loading path. Then, a relation between the estimated fatigue life and the loading cycles is performed. Among the cumulative damage rules, the simplest and well-known is the linear accumulation rule proposed by Miner [4] as follows:

$$D = \sum_{j=1}^{\#blocks}\left(\sum_{i=1}^{\#cycles}\frac{n_i}{N_{i,f}}\right)_j \tag{10}$$

where D is the accumulated damage, n_i is the applied number of cycles at the life level $N_{i,f}$. Miner's rule predicts that the failure occurs when $D \ge 1$.

Damage Curve Approach rule (DCA) is a nonlinear damage accumulation [5]. The DCA concept is that damage accumulation proceeds along the curve associated with the life level at which a cycle ratio is applied. In general case when K block loadings are applied before failure occurs, the equation for DCA becomes:

$$\{[(n_1/N_1)^{(N_1/N_2)^{0.4}} + n_2/N_2]^{(N_2/N_3)^{0.4}} + \ldots + \\ + n_{K-1}/N_{K-1}\}^{(N_{K-1}/N_K)^{0.4}} + n_K/N_K = 1 \tag{11}$$

Note that the subscripts 1, 2…, K-1, K are the sequence numbers of the loadings as they occurs. A particular interest in (11) is the exponent 0.4. It is shown that this value is reasonable for many materials.

3. Application to multiaxial variable amplitude loading

3.1. Multiaxial S-N curves

This paragraph introduces the application of the mentioned above approach for 1045 carbon steel under multiaxial variable amplitude loading. The main mechanical characteristics of this steel are: Young's modulus E = 205 (GPa), yield stress $R_{p0.2}$ = 280 (MPa) and ultimate strength R_m = 580 (MPa). The fatigue limits in fully reversed torsion (t_{-1}) and fully reversed tension (f_{-1}) have been reported in [6] as: $t_{-1} = 169$ (MPa), $f_{-1} = 240$ (MPa). The parameters k and ℓ in equation (1) are identified using fully reversed torsion test data and the results are: $k = 1206.39$, $\ell = 0.7805$. With these values of k, ℓ and the fatigue limits of different multiaxial constant amplitude loading cases, the functions of S-N curves are as follows:

- Under fully reversed torsion:

$$N = 1206.39^{1.2812}\left(\frac{\Sigma_{xya}}{\Sigma_{xya} - 169}\right)^{1.2812} \quad (12)$$

- Under fully reversed tension:

$$N = 1206.39^{1.2812}\left(\frac{\Sigma_{xa}}{\Sigma_{xa} - 240}\right)^{1.2812} \quad (13)$$

where Σ_{xa} is normal stress amplitude.

- Under in phase tension – torsion (stress ratio k = $\Sigma_{xya} / \Sigma_{xa}$ = 0.5):

$$N = 1206.39^{1.2812}\left(\frac{\Sigma_{xa}}{\Sigma_{xa} - 190}\right)^{1.2812} \quad (14)$$

- Under in phase tension – torsion (stress ratio k = 1):

$$N = 1206.39^{1.2812}\left(\frac{\Sigma_{xa}}{\Sigma_{xa} - 132}\right)^{1.2812} \quad (15)$$

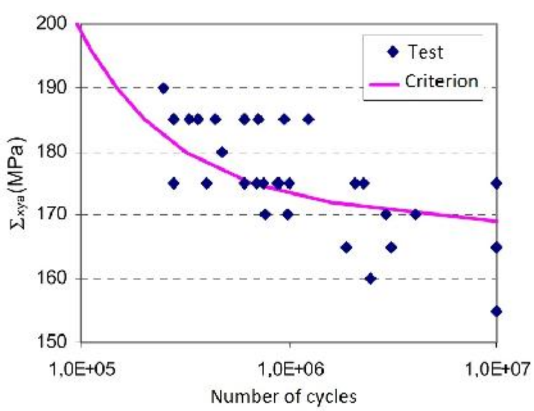

Fig. 1. S-N curve under reversed torsion

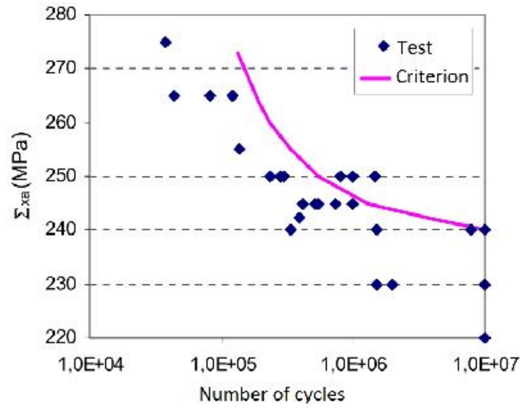

Fig. 2. S-N curve under reversed tension

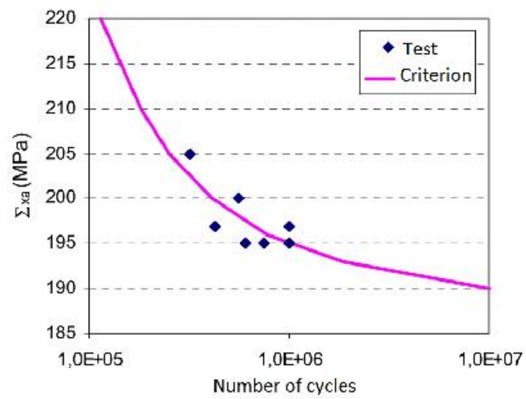

Fig. 3. S-N curve under in phase tension – torsion (k=0.5)

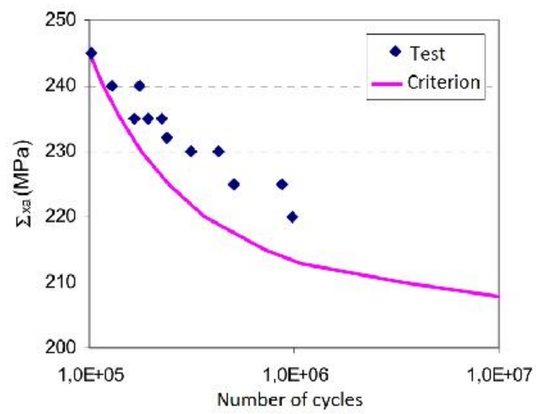

Fig. 4. S-N curve under in phase tension – torsion (k=1)

Fig. 1 to Fig. 4 show the correlation between experimental data [6] and predicted values for S-N curves under fully reversed torsion (Fig. 1), fully reversed tension (Fig. 2), in phase tension – torsion k=0.5 (Fig. 3) and in phase tension – torsion k=1 (Fig. 4). It can be seen that the predicted S-N curves fit well with the experimental points both in values and in trends for all studied loading cases. This reveals that the Vu criterion is suitable for the prediction of multiaxial S-N curves.

3.2. Experiment data

A testing campaign of variable amplitude loading is carried out by Flacelière in [6]. The variable amplitude loading is created by multiple blocks of constant loading. These blocks have different loading natures, including fully reversed torsion (denoted as To), fully reversed tension (denoted as Ta), in phase tension-torsion (denoted as TaTo) (k = 0.5) and in phase tension-torsion (TaTo) (k = 1). Many loading profiles are conducted based on two principal element blocks (denoted as Block A and Block B). All the load cases are resumed in Table 1. One or several specimens are tested for each load case. For more details, with load case 1, the two element blocks are fully reversed tension in 10^5 cycles (Ta) and fully reversed torsion in 10^5 cycles (To). The loading profile of load case 1 includes repeatedly one block A followed by one block B until the failure of specimen. The same loading profiles are applied for load cases 2, 6 and 7. For the loading profile of load cases 3, 4 and 5, the block A is respectively repeated 1, 2 and 3 times before the block B is applied until the failure.

Table 1. Variable amplitude tests on 1045 steel [6]

Load case	Block A (cycles)	Block B (cycles)	Loading profile
1	Ta (10^5)	To (10^5)	(A/B)/(A/B)/...
2	To (10^5)	Ta (10^5)	(A/B)/(A/B)/...
3	To (10^5)	Ta (10^5)	(A)/(B/B/...)
4	To (10^5)	Ta (10^5)	(A/A)/(B/B/...)
5	To (10^5)	Ta (10^5)	(A/A/A)/(B/A/B...)
6	TaTo k=1 (10^5)	TaTo k=0.5 (10^5)	(A/B)/(A/B)/...
7	TaTo k=0.5 (10^5)	TaTo k=1 (10^5)	(A/B)/(A/B)/...

3.3. Results and discussion

For the experimental tests shown in Table 1, fatigue life predictions were carried out based on the methodology mentioned in section 2.1. For each block, the life level $N_{i,f}$ is determined from S-N curve functions (12, 13, 14, 15) depending on the nature of loading (Ta, To, TaTo k = 0.5 or TaTo k =1). The Miner or DCA damage rule is then used to estimate the cumulative damage and the coresponding fatigue life of each load case. Table 2 shows the experimental and the predicted fatigue life of all studied loading cases.

Table 2. Fatigue life of 1045 steel

Case	Speci -men	Test N_f (x10^5) (cycles)	Miner N_f (x10^5) (cycles)	DCA N_f (x10^5) (cycles)
1	1	6,27	6,02	5,94
	2	6,21		
2	1	4,84	6,03	6,10
	2	5,59		
	3	9,54		
3	1	5,65	5,66	5,78
4	1	3.93	5,84	5,99
5	1	5.23	6,24	6,36
6	1	2,58	2,02	1,71
	2	1,27	2,64	2,54
	3	4,18	2,75	2,65
7	1	4,89	5,4	5,32
	2	6,37	3,55	3,62
	3	4,12		

For better comparison of the results, fatigue life predictions are illustrated in Fig. 5. These graphs display the predicted life versus the experimental life on log-log coordinates. The solid line represents perfect correlation between experimental and predicted values. The outer bound (dash lines) is related with a factor of two of fatigue life. Data points that fall below the solid line represent conservative estimations and points above represent non-conservative prediction. The Fig. 5a shows the correlation between the estimations using Miner rule and the experimental data while the Fig. 5b shows the results by using DCA rule. As shown in Fig. 5a and Fig. 5b, there is not a significant difference in the predictions using Miner rule and using DCA rule. This reveals that the effect of load sequence is relatively small for the considered loading cases. The predicted results of both Miner rule and DCA rule are quite satisfactory. All the data points are in the limit zone. It can be concluded that the fatigue life prediction method gives good estimation for 1045 steel.

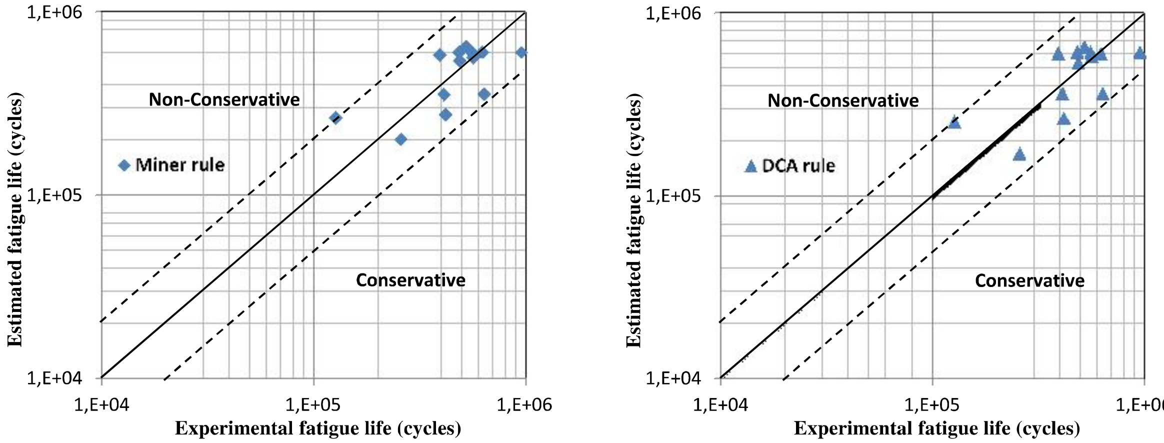

Fig. 5. (a) Miner rule and (b) DCA rule fatigue life correlation of 1045 steel under variable amplitude loading

4. Conclusion

In this paper, the criterion based on stress invariants developed by the present authors, was used to account fatigue damage accumulation under variable amplitude loading conditions. The concluding remarks are as follows:

- The Vu criterion is suitable for the prediction of multiaxial S-N curves.
- Under variable amplitude loading, the prediction approach combining three tools (multiaxial criterion, multiaxial S-N curves, damage accumulation rule) gives good estimation of fatigue life.
- Both Miner and DCA damage rules are appropriate for prediction of fatigue life of 1045 steel.

The prediction methodology used in this study could be modified to deal with more complex loading such as random loading in service. Cycle counting techniques and/or overload effects need to be taken into account in further study.

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References

[1] Q.H. Vu, D. Halm, Y. Nadot, Multiaxial fatigue criterion for complex loading based on stress invariants, International Journal of Fatigue. 32 (2010) 1004–1014.

[2] A. Fatemi, L. Yang, Cumulative fatigue damage and life prediction theories: a survey of the state of the art for homogeneous materials, Int. J Fatigue. 20(1) (1998) 9-34.

[3] V. Papadopoulos, Long life fatigue under multiaxial loading, International Journal of Fatigue. 23 (2001) 839–849.

[4] M. A. Miner, Cumulative damage in fatigue, Journal of Applied Mechanics. 67 (1945) A159-A164.

[5] S.S. Manson, G.R. Halford, Practical implementation of the double linear damage rule and damage curve approach for treating cumulative fatigue damage, Int. Journ. of Fracture. 17 (1981) 169-192.

[6] L. Flacelière, Contribution à la modélisation du dommage en fatigue multiaxiale d'un acier C36 – Confrontation à l'expérience. PhD Thesis, University of Poitiers, France (2004).