A Chromatic Aberration-Measuring Method for Refracted Converging Lens based on Foucault Knife - Edge Test

Nguyen Thanh Dong**, Vu Toan Thang

Hanoi University of Science and Technology, No. 1, Dai Co Viet, Hai Ba Trung, Hanoi, Viet Nam

Received: August 10, 2016; Accepted: November 03, 2017

Abstract

The paper proposes a new chromatic-aberration-measuring method for geometric optics of a single converging lens based on the Foucault knife-edge test. An opto-mechatronics measurement system aided by a computer vision was built to automatically determine displaced positions of the knife-edge for discovering chromatic aberration components in the measured lenses. The evaluation results of its chromatic aberration elements measured by this system are appropriated with theoretical calculations.

Keywords: Chromatic aberration in a lens, Foucault knife-edge test, Abbe index, computer vision

1. Introduction

The Foucault knife-edge test is one of the oldest and most conventional techniques for sensing optical aberrations. It has become widely used for the observation and measurement of chromatic aberration, which is one of essential aberration components of a single converging lens or a convex mirror, reported by [1-6]. In optical manufacturing and metrology methods, the measuring method capable of quantitatively and efficiently measuring the deformation of aspherical mirrors is built by using the useful and economical technique as the Foucault knife-edge test [3]. Besides, the Foucault tester is improved by the combination of an electronic detector array and a digital microprocessor to estimate quantitative of optical surfaces in comparing to its results with that of interferometry-measuring techniques [4]. Furthermore, the Foucault tester was used to detect the longitudinal chromatic aberration in achromatic lenses as an objective lens in a telescope. Thus, a pair of the reasonable achromatic crown and flint lens was suitably chosen to remove the chromatic aberration in achromatic lenses [5, 7].

In this paper, a new shadow-measuring method is proposed to exactly position dark regions and color-brightness regions for quantitative assessment of both longitudinal and transverse chromatic aberration domains in a single converging lens. An automatic Foucault knife-edge tester is setup to verify the method principle. The Foucault-measuring system can find the best position to perform the test with the vision-aided feedback system using a webcam. Even this model can be operated efficiently under normal illumination conditions in a laboratory and not need for a dark room. In addition, it serves to eliminate in the loop human vision operation that is susceptible to human judgment and subjectivity.

2. Chromatic aberration-measuring method

a. Chromatic aberrations of a single convex lens

For lenses, the refraction index varies as a function of the wavelength of light. Axial *longitudinal chromatic aberration* is the dispersing light by the lens with the different colors (wavelengths) coming to focus at various positions along the axis of symmetry, LCA = $F_{B1}F_{R1}$. F_{B1} and F_{R1} are the relative blue and red focus points of marginal refractive rays, respectively. Fig. 1 illustrates that the image of an axial point in the presence of chromatic aberration is a center bright dot surrounded by a halo. The bright dot includes the focused lights and the other nearly focused lights. The out-of-rays form the halo. Consequently, the image would have a yellowish dot (formed by the orange, yellow and green rays) and a purplish halo (including the red and blue rays) at the position (II). If the observed screen on which the image is formed is moved away toward the lens (at the position (I)), the central dot will become blue, and the boundary halo will become red. Nevertheless, if it is moved away (at the position (III)), the central dot will become red, and the boundary halo will become blue. In this research, the perpendicular distance of MN which is located at position (II) is called to the *transverse chromatic aberration* (TCA), TCA = MN.

* Corresponding author: Tel.: (+84) 983.639.488
Email: dong.nguyenthanh@hust.edu.vn

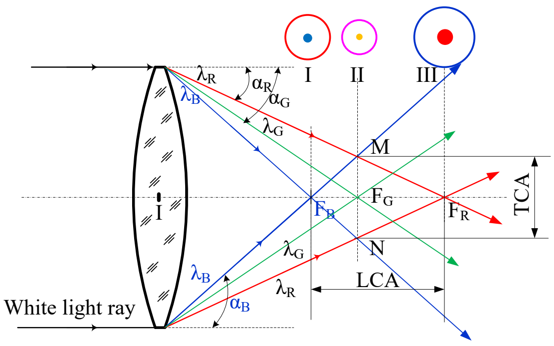

Fig. 1. A single converge lens with under-corrected chromatic aberrations is due to the blue rays undergoing a greater refraction than the red rays

The *longitudinal chromatic aberration (LCA)* in a lens can be given by [8]

$$LCA = \frac{f_Y}{\nu_{Abbe}} \tag{1}$$

where f_Y is the focal length of a lens at the yellow wavelength, $\lambda = 546{,}074$ nm; ν_{Abbe} is the Abbe index:

$$\nu_{Abbe} = \frac{n_Y - 1}{n_B - n_R} \tag{2}$$

where n_Y, n_B, and n_R are refracted indexes of yellow, blue, and red rays, respectively.

From Figure. 1, the *transverse chromatic aberration (TCA)* in a lens can be calculated according to the formula hereafter:

$$TCA = 2f_Y(\tan\alpha_B - \tan\alpha_Y) \tag{3}$$

or it can be obtained as:

$$TCA = 2f_Y(\tan\alpha_Y - \tan\alpha_R) \tag{4}$$

where α_R, α_Y, and α_B which are formed by the peripheral white rays making refractive rays, are dispersed angles of red, yellow, and blue rays, respectively. The blue ray, having the highest refractive index, is strongly refracted more than other color rays in a simple positive lens. Otherwise, the red beam is less refractive than other color rays.

Adding the equation (3) to the equation (4), it can be obtained:

$$TCA = f_Y(\tan\alpha_B - \tan\alpha_R) \tag{5}$$

On the other hand, $\tan\alpha_R$ and $\tan\alpha_B$ are written by:

$$\tan\alpha_R = \frac{D}{2f_R}; \tan\alpha_B = \frac{D}{2f_B} \tag{6}$$

where D is the diameter size of the lens, which is received incident light beam.

The equation (5) can be rewritten as:

$$TCA = Df_Y\frac{f_R - f_B}{2f_Rf_B} \tag{7}$$

where f_R and f_B are the relative focal lengths of a lens at the red and blue wavelength, respectively.

It can be considered that multiplication of f_Rf_B is approximately equal to f_Y^2 and LCA is equal to the subtraction of (f_R - f_B). Therefore, the equation (7) can be rewritten by:

$$TCA = D\frac{LCA}{2f_Y} \tag{8}$$

Substituting the equation (1) into the equation (8)

$$TCA = \frac{D}{2\nu_{Abbe}} \tag{9}$$

From the equation (9) the TCA in a converging lens only depends upon its diameter and Abbe index. In the presence of chromatic aberration, the LCA and TCA can be calculated if some of the specifications of the lens are known, including ν_{Abbe}, f_Y, and D.

b. Measurement of chromatic aberrations in single convex lens based on the Foucault knife – edge test

Figure 2 provides a more detailed view of the knife-edge arrangement in front of the tested lens, as well as what observers or photographs closely behind the knife-edge. When the knife-edge comes inside far from the focal point, the reddish halo (formed by the red, orange and yellow rays) enlarges on the upper half of 'shadow' image of the knife-edge. In contrary, the bluish halo (due to the purple, blue and green rays) extends on the lower half of the shadow when the knife-edge goes outside far from the focal point. The imaginal shadow of the knife becomes dark when its position is located nearly in the focal zone. The couples of positions (A, A'), (B, B'), …, and (P, P') allows the couples of the smallest red and blue halos on the screen, respectively, as shown in Figure 2. If one can determine these positions, two linear equations of AP and A'P' in the Oxy system can be given by

$$\begin{aligned} y &= a_1x + b_1 (AP) \\ y &= a_2x + b_2 (A'P') \end{aligned} \tag{10}$$

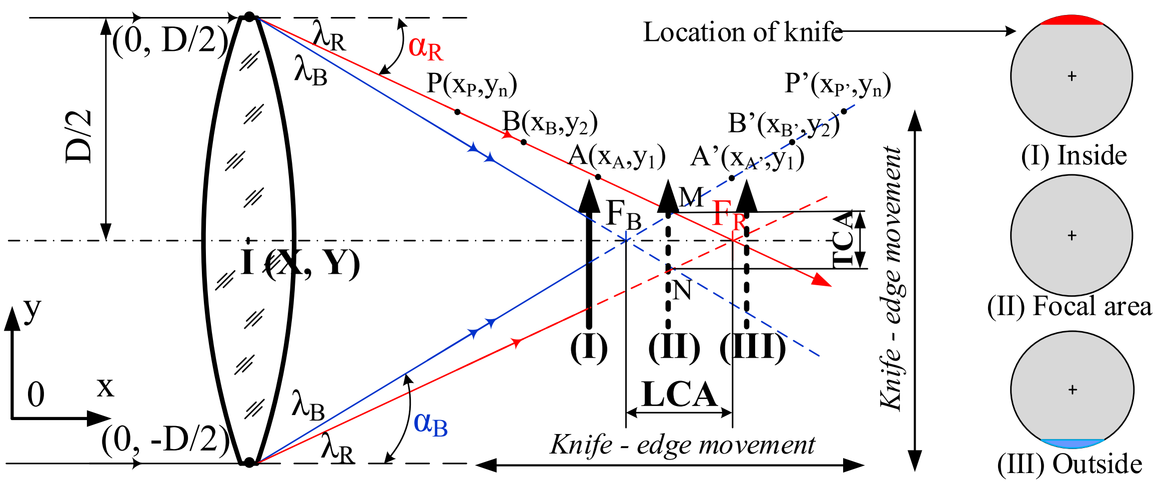

Fig. 2. Detailed schematic view of the Foucault knife-edge measurement principle

It can be seen that it is essential to determine two components from the equations (10) including $a_1 = -\tan\alpha_R$ and $a_2 = \tan\alpha_B$. Accordingly, two refracted angles α_R and α_B of the red and blue colors can be easily derived from the two equations above. As a consequence, the LCA of a converge lens with its diameter of D can be taken account of as

$$LCA = \frac{D}{2}\left(\cot\alpha_R - \cot\alpha_B\right) \qquad (11)$$

To determine the TCA of the lens, the original point O (0,0) in the Oxy system is converted to the central point I in a new IXY system shown in Fig. 2. From Fig. 1, if the red marginal ray with a constant slope of α_R and the blue marginal ray with a constant slope of α_B cross two constant points (0, D/2) and (0, -D/2), respectively, two new linear equations of AP and A'P' will be obtained in the form as

$$\begin{aligned} Y &= -\tan\alpha_R X + \frac{D}{2} (AP) \\ Y &= \tan\alpha_B X - \frac{D}{2} (A'P') \end{aligned} \qquad (12)$$

The intersection M (X_M, Y_M) of the two rays AP and A'P' can be calculated by Eq. (12). As a result, if monochromatic rays are refracted at the lens periphery, the TCA of a lens can be TCA = $2Y_M$ = MN.

3. Design and experiments of the Foucault knife – edge test using a webcam sensor

a. Design of the Foucault knife-edge test using a webcam

Fig. 3 shows the schematic diagram of the experiment system and Fig. 4 describes how to set up the experiment-measured system. The components were set up on a long bench. A long ruler of 1.5 m on belong to the edge of the bench served as an alignment reference. A standard white light source with a color temperature of 2854 K was put at the focal point of a parabolic reflector with a focal length of 900 mm and a diameter of 114 mm. The direction of the reflected light must be parallel to the optical axis of the measured lens. A diaphragm which was nearly placed in front of the source to limit the passage of light coming into the lens Fig. 5(a). The size of beams is adjusted from 2 to 30 mm. When the diaphragm hole is the smallest size, the light source becomes a standard-point source.

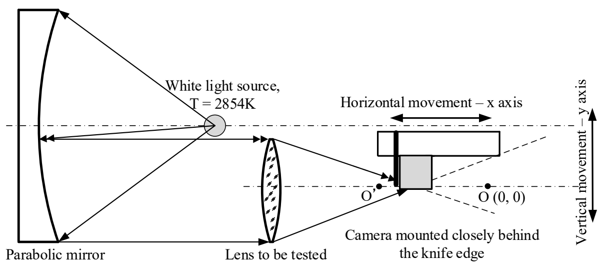

Fig. 3. Schematic diagram of the Foucault test

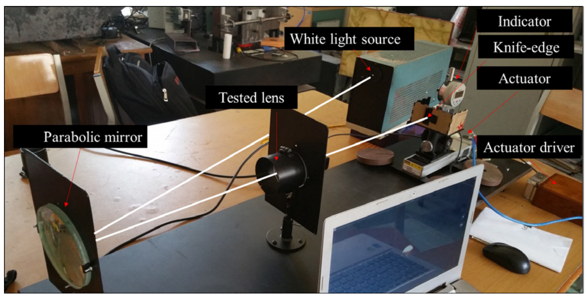

Fig. 4. The configuration of the test system as Fig. 3

In the measurement system, a thin knife-edge was vertically mounted on the stage, which is driven by a computer-aided precision actuator (Orientalmotor Co., Vexta PK245: an itinerary and a resolution of 50 mm and 10 µm, respectively). The fine-straight edge of the knife was employed to cut the color light beam near the focal point.

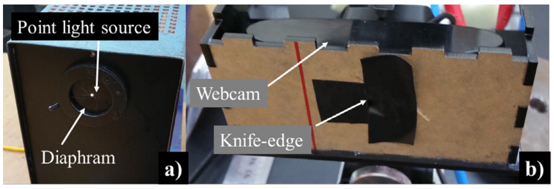

Fig. 5. (a) Light source and diaphragm;
(b) Knife-edge and webcam

In the design, a webcam (Logitech Co., C920: a resolution of 1920 x 1280 pixels) as a sensor was placed closely behind the knife-edge from Fig. 5(b). The webcam captured live-shadow patterns that can project on the screen of a laptop. The captured images which are analyzed will generate commands on how to correctly determine the current position, comparing to the original position and the next motion direction of the knife-edge.

To investigate the relative error of the actuator, the stage was moved back and forth with an itinerary of 1 m by the actuator. The displacement of the stage is compared with the measurement result of a reference indicator (Mitutoyo Co., 543-185) on an anti-vibration table, as plotted in Fig. 6(a). Likewise, we controlled the actuator to incorporate the stage with displacements of 2, 3, …, and 10 mm, respectively. The absolute errors between the displacements and the results of the indicator are shown in Fig. 6(b). The relative error is approximately 0.15% if the displacement is 100 mm.

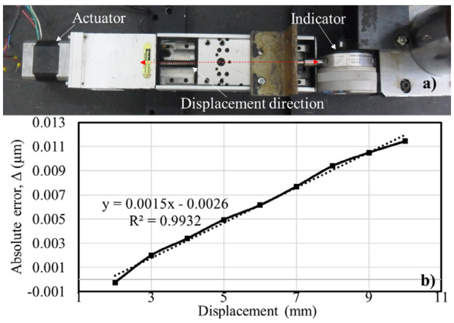

Fig. 6. (a) An experiment setup for investigating the relative of error of the actuator;
(b) Absolute error between displacements of the actuator and results of the indicator

b. Experiments of the Foucault knife-edge test using a webcam

Experimental steps are represented in the following procedures:

1. Due to the point, I (0, 0) in the IXY is a virtual point. Hence we set O (0,0) that is the farthest position of the knife-edge from a tested lens in the displacement range of the actuator. In the beginning, the position of the knife-edge will be calibrated to O (0, 0).

2. In next steps, the knife-edge is horizontally moved forth to the closest position and gone back to O (0, 0) by the computer-aided actuator. The movement direction (x-axis) of the knife-edge is parallel to the optical axis of the tested lens. While the knife-edge is continuously moving, live pictures are continuously being processed by the computer-aided webcam. The characteristics of brightness, color, and halo shapes from each image will be analyzed to find the positions A and A', in which the webcam can detect the smallest red and blue halos, respectively.

3. Likewise, the knife-edge is moved back and forth 10 times by the actuator. As a result, the average coordinates of A and A' ((x_A, 0) and ($x_{A'}$, 0)) are determined.

4. The knife-edge is then vertically displaced according to y-axis, which is perpendicular to the x axis by a hand micrometer, with a step of 20 µm. A measuring loop is repeatedly performed from the steps (1) to (3) for determining two positions B and B' ((x_B, 20) and ($x_{B'}$, 20))).

5. In next steps, the knife will be vertically moved in turn to the positions: y = 40, 60, …, 180 µm by the hand micrometer. The steps from (1) to (4) are continuously repeated. Finally, the Foucault tester can determine a serial of particular positions consisting of A, A', B, B', …, P and P'. It is easy to build two straight line AP and A'P' from those points above, and the refractive angles α_R and α_B will be derived from these linear equations. As a result, LCA and TCA of the measured lens can be determined, based on those expressions (11) and (12).

4. Results

Table 1. Characteristics of lenses

Lens	1st	2nd	3rd
Diameter – D (mm)	67,86	52,36	53,96
Abbe index – ν_{Abbe}	61,68	58,34	64,06
Optic Material	K19	TK 16	K8
Refractive index – n, at λ = 587,56 nm	1,5188	1,6126	1,5163
Focal length – f (mm), at λ = 587,56 nm	426,59	98,83	52,37

Three single-converging lenses marked 1st, 2nd and 3rd (they were made by Z23 company-Vinh Phuc province-Vietnam) were tested by the measurement system. These measured lenses originally received their characteristics in the following table 1 above.

1. Calculation of chromatic aberrations in lenses

From the Table 1, the equations (1), and (9), the chromatic aberrations of three tested lenses were calculated in the following table (2). It is noted here that because each lens was also reliably fixed by a diaphragm on a bracket stage, its diameter size (D') exposed incident light was reduced partly, being equal to a thickness of the diaphragm.

Table 2. Calculated results of chromatic aberrations by the equations (1) and (9)

Items (mm)	Lens 1st	Lens 2nd	Lens 3rd
D'	65,46	49,96	51,56
LCA	6,92	1,61	1,64
TCA	0,53	0,43	0,40

2. Measurement results

a. For the first lens

Fig. 7 illustrates that the controller would determine two line graphs of two refracted mono-color rays in the Oxy system on the basis of a set of specially recognized points A, A', …, P and P', as shown in the following equations forming

$$y = -0{,}0705x + 7{,}1104 (AP - red)$$
$$y = 0{,}0716x + 3{,}8433 (A'P' - blue)$$

Additionally, two refractive angles α_R and α_B could be obtained as $\tan\alpha_R$ = 0,0705 and $\tan\alpha_B$ = 0,0716. Accordingly, the LCA of the 1st lens was calculated from the equation (11) LCA ~ 7,13 (mm).

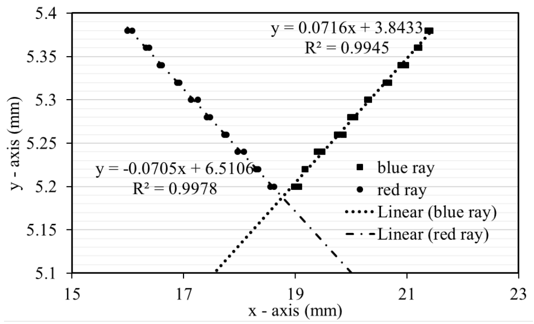

Fig. 7. Diagram of blue and red rays in the 1st lens

On the other hand, the two linear equations in the IXY system from the equations (12) are as follows

$$Y = -0{,}0705X + \frac{65{,}46}{2} (AP - red)$$
$$Y = 0{,}0716X - \frac{65{,}46}{2} (A'P' - blue)$$

The M point which is the intersection of two lines AP and A'P' would be discovered as M (460,66; 0,253). Finally, the TCA in the 1st lens ~ 0,51 (mm).

b. For the second and third lenses

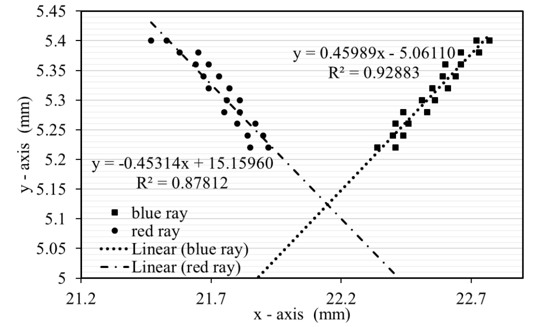

Fig. 8. Diagram of blue and red rays in the 3rd lens

All experimental steps were similarly fulfilled for both two 2nd and 3rd lenses with measured results shown in Table 3. Besides, in the 2nd lens, two refractive angles of the red and blue refracted rays were $\tan\alpha_R$ = 0,26341 and $\tan\alpha_B$ = 0,26806, respectively. Two linear graphs were schematically illustrated in Fig. 8.

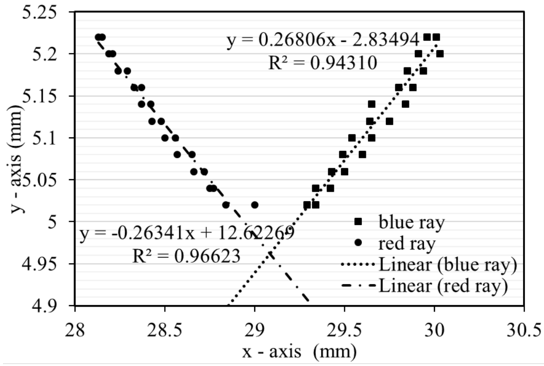

Fig. 9. Diagram of blue and red rays in the 2nd lens

The straight lines of two refracted rays were measured by the Foucault-assembled system for the 3rd lens, as shown in Fig. 9, with $\tan\alpha_R$ = 0,45314 and $\tan\alpha_B$ = 0,45989.

In a comparison of theoretical calculations and experimental results, the relative error of two chromatic aberration kinds in three lenses measured by the Foucault-measuring system was thus performed in the following Table 3 below.

Table 3. Comparison of theoretical and experimental results

		Theory	Experiment	Error Δ (%)
1st Lens	LCA	6,92	7,13	3%
	TCA	0,53	0,51	4%
2nd Lens	LCA	1,61	1,64	2%
	TCA	0,43	0,44	2%
3rd Lens	LCA	0,82	0,84	2 %
	TCA	0,40	0,38	5 %

5. Discussions

Theoretically, both longitudinal and transverse chromatic aberrations in any lens can be achieved by the proposed measurement method. However, because chromatic aberration in an achromatic lens as the objective lens of a telescope [7] or an apochromatic lens is very small, it is difficult for the Foucault system to detect this aberration. This limitation is not intrinsic to this method. The Foucault system can be improved for the induced measurement error by using an automatic micrometer with a pitch of up to 1 µm to realize an accurate measurement of the knife location, at the cost of a complicated configuration.

The maximum difference (Δ) between the results measured by the Foucault-measuring system and the theoretical calculations was about 5% for the TCA in the 3rd-measured lens. The chromatic aberrations of the measured lenses also mainly depend on a series of specially recognized positions, in which the knife-edge cuts blue and red refracted rays coming into focus. Relative error (Δ%) of chromatic aberration types, of course, was affected by value excursion of those refractive angles.

Since the Foucault system used the webcam, which has a high resolution of 1920 x 1280 pixels, play as an automatic sensor to find out the smallest red and blue halos, the chromatic aberration measurements of a single converging lens can be experimented advantageously in normal illumination condition without compelling to measure in full dark space. In other words, this tester can entirely eliminate human-in-the-loop operation which is easily susceptible to human judgment and subjectivity, is known as an opto-mechatronics system.

6. Conclusions

In summary, this paper proposed a new chromatic-aberration-measuring method of a refractive converge lens based on the Foucault-assembled knife-edge test. The viability of the proposed method and its ability to measure chromatic aberration elements in a single converging lens were demonstrated. The percentage error in measuring chromatic aberration was about 5% of the maximum TCA in the 3rd-measured lens. Besides, the Foucault-assembly-measuring system was educationally built in the laboratory not only to be a measurement instrument for chromatic aberration of lenses but also to assist students in understanding more how the appearance of chromatic aberrations in a lens. In future, this assembled tester using the proposed measurement method will also be developed and improved to determine and analyze chromatic aberrations in an achromatic lens or an apochromatic lens in a next report.

Acknowledgments

I would like to thank the HUST Project for financial support to do this research.

Reference

[1] D. Malacara, "Foucault or knife-edge test,' Chapter 8 in Optical Shop Testing, Wiley, New York (1978), pp. 231-246.

[2] H. G. Conrady, “Study of the significance of the Foucault knife-edge test when applied to refracting systems”, IOPscience, Vol.24, No.4, 1924, pp.219-226.

[3] H. B. Cheng, Y. Yam and H. Tong, “A Quantitative Knife-edge Testing Method for Local Deformation”, Proceeding of the 3rd Annual IEEE Conference on Automation Science and Engineering, Scottsdale, AZ, USA, Sept 22-25, 2007, pp. 818-822.

[4] Donald E. Vanderberg, William D. Humbel, Alan Werthelmer, “Quantitative evaluation of optical surfaces by means of an improved Foucault test approach”, Optical Engineering, Vol 32, No. 4, August 1993, pp. 1951 – 1954.

[5] Duane H. Jaecks, “An investigation of the eighteenth-century achromatic telescope”, Annals of Science, Vol. 67, No. 2, 2010, pp. 149 – 186.

[6] W. D. Furlaw, L. M. Escriva, A. Pons, M. M. Corral, “Optical aberrations measurement with a low cost optometric instrument”, Am. J. Phys. 70(8), August 2002.

[7] N. T. Đông, N. T. P. Mai và L. H. Hưng, “Thiết kế, chế tạo và xác định sắc sai cho vật kính ghép đôi tiêu sắc của kính viễn vọng”, Tạp chí KH & CN các Trường ĐHKT, số 98-2014, tr. 46-51.

[8] Joseph M. Geary, Axial Color and Achromats”, Chapter 16 in Introduce to Lens Design with Practical ZEMAX Examples, Willmann-Bell Inc., USA, September 2002, pp. 175-177.