Different Models for LMP Calculation in Wholesale Power Markets Considering Active Power Reserves: a Comparison

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Abstract

Locational marginal price (LMP) is an important element in the operation of electricity markets. LMP is used to determine payments in the electricity markets, to derive bidding strategies of market participants, and to make plan for new transmission lines and power plants. This paper compares the DC optimal power flow (DCOPF) model and AC optimal power flow (ACOPF) that are used to calculate LMP in the wholesale electricity market. The study takes into account the price-sensitive loads and active power reserves. DCOPF model has 2 forms: DCOPF without losses and iterative DCOPF with losses. Fictitious nodal demand (FND) is used to calculate marginal loss component of LMP. In addition, branch flow limits are also adjusted in the iterative DCOPF model. LMPs, active power outputs and reserves of generators are illustrated on a 3 bus system.

Keywords: Locational marginal prices (LMP), wholesale power markets, active power reserves, DCOPF, ACOPF, fictitious nodal demand (FND).

Nomenclature

λ_{Gib}	Price of the energy block b offered by generating unit i (constant)
P_{Gib}	Power of the energy block b offered by generating unit i (variable)
λ_{Gi}^{RR+}	Price of Up Regulation Reserve (RR) offered by generating unit i (constant)
λ_{Gi}^{RR-}	Price of Down Regulation Reserve offered by generating unit i (constant)
λ_{Gi}^{SR}	Price of Spinning Reserve (SR) offered by generating unit i (constant)
λ_{Gi}^{XR}	Price of Supplemental Reserve (XR) offered by generating unit i (constant)
P_{Gi}^{RR+}	Up Regulation Reserve Power offered by generating i (variable)
P_{Gi}^{SR}	Spinning Reserve Power offered by generating i (variable)
P_{Gi}^{XR}	Supplemental Reserve Power offered by generating i (variable)
λ_{Djk}	Price of the energy block k bid by demand j (constant)
P_{Djk}	Power block b bid by demand j (variable)
λ_{b}^{RR+}	Price of Up Regulation Reserve block b bid by Area (constant)
λ_{b}^{CR}	Price of Contingency Reserve (CR) block b bid by Area (constant)
λ_{b}^{OR}	Price of Operation Reserve (OR) block b bid by Area (constant)
A_{b}^{RR+}	Up Regulation Reserve Power block b bid by Area (variable)
A_{b}^{CR}	Contingency Reserve Power block b bid by Area (variable)
A_{b}^{OR}	Operation Reserve Power block b bid by Area (variable)
P_{Dj}^{E}	Elastic power of demand j
P_{Dj}^{F}	Constant power of demand j
P_{Dj}	Total power of demand j
SR%	Percentage of spinning reserve in contingency reserve
SF_{ij-m} (SF_{l-i})	Sensitivity of branch power flow ij (l) with respect to injected power m (i)
P_l	Active power flow on branch l
R_l	Resistance of branch l
P_{ij} (P_l)	Active power flow on branch ij (l)
Q_{ij}	Reactive power flow on branch ij
S_{ij}^{max}	Power flow limit on branch ij
LMP_E	Marginal Energy Price
LMP_L	Marginal Loss Price
LMP_C	Marginal Congestion Price
LF_i	Loss factor for node i
μ_l	Shadow price of transmission constraint on line l
$R_i^{(1)}$	Revenue of generating i from DCOPF without losses or FND-based DCOPF with losses
$R_i^{(2)}$	Revenue of generating i from ACOPF algorithm
$LMP_i^{(1)}$	LMP from DCOPF without losses or FND-based DCOPF with losses
$LMP_i^{(2)}$	LMP form ACOPF algorithm

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1. Introduction

Nowadays, the electric power systems in many countries have been gradually transforming from the vertically integrated model to fully deregulated markets. The different models of the electric markets include: generation competition, wholesale competition and retail competition. The two important market participants are generating companies (Gencos) and distribution companies (Discos). To ensure the reliability of supply and the power system stability, essential ancillary services, such as the one for frequency regulation, must be included [6]. The active power reserve for frequency regulation can be divided into three categories: Regulation Reserve (RR), Spinning Reserve (SR), and Supplemental Reserve (XR) [9]. SR and XR are the two components of the Contingency Reserve (CR). The operating reserve consists of CR and the regulation reserve [9]. The System operator receives offers for energy and reserves from Gencos and bids from Discos. The generating schedule of generation units are determined such that the total social welfare is maximized [1]. The scheduled active power of Gencos, the scheduled purchase of Discos, and the active reserve of generating units are determined on the basis of an optimization model. The energy and reserve markets can be cleared sequentially, or simultaneously [5]. The payment in the electricity market is thus determined on the basis of two elements: the Locational Marginal Price (LMP), and the Reserve Market Clearing Price (RMCP). The LMP consist of three components: marginal energy price, marginal loss price and marginal congestion price. LMP can be calculated as a whole, or from its components [2, 3, 4]. The transmission costs can also be determined from the LMP.

The optimal generation schedule, as well as the allocated active reserve at each Genco can be solved using an optimization problem, based on the full ACOPF model. However, this approach might have some convergence issues, depending on the initial estimates of the solutions. The DCOPF model is simple and always guarantees convergence. However, it does not account for the active losses in the system.

This paper presents a comparative study on different models for the calculation of LMP in a wholesale electricty market with price-sensitive loads and with a reserve market. The remainder of the paper is presented as follows: section 2 presents the LMP calculation method based on the lossless DCOPF model, section 3 presents the iterative DCOPF model with adjusted branch flow limits. The ACOPF model for LMP calculation is presented in section 4. Section 5 presents the method for the calculation of LMP and its components. The calculation examples and comparions of different LMP models are presented in section 6. The conclusion is given in section 7.

2. DCOPF without losses

2.1 Objective function

The objective of the co-optimization model of energy and reserve is to maximize the total social welfare, as shown in Eq. (1) below:

$$
\begin{aligned}
&\sum_{i=1}^{N_G}\sum_{b=1}^{N_{Gi}}\lambda_{Gib}.P_{Gib} \\
&+\sum_{i=1}^{N_G}\left(\lambda_{Gi}^{RR+}.P_{Gi}^{RR+}+\lambda_{Gi}^{RR-}.P_{Gi}^{RR-}+\lambda_{Gi}^{SR}.P_{Gi}^{SR}+\lambda_{Gi}^{XR}.P_{Gi}^{XR}\right) \\
&-\sum_{j=1}^{N_D}\sum_{k=1}^{N_{Dj}}\lambda_{Djk}.P_{Djk}-\sum_{b=1}^{N_{RR+}}\lambda_b^{RR+}.A_b^{RR+}-\sum_{b=1}^{N_{RR-}}\lambda_b^{RR-}.A_b^{RR-} \\
&-\sum_{b=1}^{N_{CR}}\lambda_b^{CR}.A_b^{CR}-\sum_{b=1}^{N_{OR}}\lambda_b^{OR}.A_b^{OR}
\end{aligned}
\tag{1}
$$

2.2 Constraints

2.2.1 Power balance

The active power injected into bus i is subjected to the following constraint:

$$P_i = P_{Gi} - P_{Di}^E - P_{Di}^F = \sum_{j=1}^{N} B_{ij}\left(\delta_i - \delta_j\right) \tag{2}$$

2.2.2 Active power reserve balance

The active power reserve is determined for each area or zone. Within each area, the active power reserve is subjected to the following constraints:

$$\sum_{i=1}^{N_G} P_{Gi}^{RR+} = A^{RR+} \tag{3}$$

$$\sum_{i=1}^{N_G} P_{Gi}^{RR-} = A^{RR-} \tag{4}$$

$$\sum_{i=1}^{N_G}\left(P_{Gi}^{SR}+P_{Gi}^{XR}\right) = A^{CR} \tag{5}$$

$$\sum_{i=1}^{N_G}\left(P_{Gi}^{RR+}+P_{Gi}^{SR}+P_{Gi}^{XR}\right) = A^{OR} \tag{6}$$

2.2.3 The active power limit of each generation block

$$0 \le P_{Gib} \le P_{Gib}^{\max} \quad \left(\forall i, b\right) \tag{7}$$

2.2.4 Active power limit of the generating units

For a generating unit that takes part in all reserve markets, its active power is subjected to constraint (8), as follows:

$$0 \le P_{Gi} + P_{Gi}^{RR+} + P_{Gi}^{SR} + P_{Gi}^{XR} \le P_{Gi}^{\max} \quad (\forall i) \\ P_{Gi} - P_{Gi}^{RR-} \ge P_{Gi}^{\min} \tag{8}$$

2.2.5 Constraints on the active power reserve

This constraint is described as follows:

$$0 \le P_{Gi}^{RR+} \le P_{Gi\max}^{RR+} \tag{9}$$

$$0 \le P_{Gi}^{RR-} \le P_{Gi\max}^{RR-} \tag{10}$$

$$0 \le P_{Gi}^{SR} \le P_{Gi\max}^{SR} \tag{11}$$

$$0 \le P_{Gi}^{XR} \le P_{Gi\max}^{XR} \tag{12}$$

2.2.6 Limits on the price-sensitive loads

In a wholesale power market, the loads are considered to consist of two components: fixed load and price-sensitive load. The demand curve of price-sensitive loads can consist of several blocks, each with a lower and an upper limit, as shown in (13)-(14).

$$P_{Dj}^{\mathrm{E}\min} \le P_{Dj}^{E} \le P_{Dj}^{\mathrm{E}\max} \quad (\forall j) \tag{13}$$

$$0 \le P_{Djk}^{E} \le P_{Djk}^{\mathrm{E}\max} \quad (\forall j, \mathrm{k}) \tag{14}$$

2.2.7 Constraints on active power reserve block for each area

The demand curve for active power reserve for each area may consist of several blocks, each has a lower and an upper limit, as in (15)-(18):

$$0 \le A_{b}^{RR+} \le A_{b\max}^{RR+} \tag{15}$$

$$0 \le A_{b}^{RR-} \le A_{b\max}^{RR-} \tag{16}$$

$$0 \le A_{b}^{CR} \le A_{b\max}^{CR} \tag{17}$$

$$0 \le A_{b}^{OR} \le A_{b\max}^{OR} \tag{18}$$

2.2.8 Constraints on the spinning reserve

For each area, the SR should account for at least SR% the CR. The reason is that the spinning reserve can only be provided from units that are actually in operation. Whereas, the XR may be provided, either by online generating units, or by offline fast-start generating units. The constraint on SR is written as follow:

$$\sum_{i=1}^{N_G} P_{Gi}^{SR} \ge SR\%.\sum_{i=1}^{N_G}\left(P_{Gi}^{SR} + P_{Gi}^{XR}\right) \tag{19}$$

2.2.9 Branch flow limits

The branch flow can be expressed by a function of bus voltage angles. Alternatively, they can be expressed by a function of injected active power, via the power distribution factors [2].

$$P_{ij}^{\min} \le P_{ij} = \sum_{m=1}^{N} SF_{ij-m}\left(P_{Gm} - P_{Dm}\right) \le P_{ij}^{\max} \tag{20}$$

3. FND-based iterative DCOPF with losses and branch limits adjusted

The DCOPF model shown in the section 2 does not account for active power losses in the network. It also does not account for reactive power flow in the branch flow limits. These limitations can be overcome using the FND model and adjusting the branch flow limits.

3.1 Fictious Nodal Demand (FND)

The active power losses in the network can be written as follows:

$$P_L = \sum_{l} P_l^2.R_l \tag{21}$$

To account for the active power losses, [2] introduced a concept of FND, where the active power losses in the network is introduced by adjusting the demand at load buses. The load demand at each bus is written as follows:

$$P_i = P_{Gi} - P_{Di} - FND_i = P_{Gi} - P_{Di} - C_i.P_L \tag{22}$$

where C_i is loss distribution factor. In the literature, various approaches are applied to calculate C_i. One common methodology is to use the real-time or historical load ratios as Eq. (23).

$$C_i = \frac{P_{Di}}{\sum_{i} P_{Di}} \tag{23}$$

Consequently, the branch flow can be determined from the injected power at all buses, using the power distribution factors:

$$P_l = \sum_{i=1}^{N} SF_{l-i}\left(P_{Gi} - P_{Di} - FND_i\right) \tag{24}$$

It is relevant to note that this model is similar to those employed by PJM [11].

3.2 Adjustment of the branch flow limits

According to [9], when taking into account the reactive power flow, the branch flow limits are determined by Eq. (25) – (32).

$$P_{ij}^{\max *} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{25}$$

$$a = P_{ij\odot}^2 + Q_{ij\odot}^2 \tag{26}$$

$$b = -P_{ij\odot}\left[\left(S_{ij}^{\max}\right)^2 - M^2\right] \tag{27}$$

$$c = \frac{1}{4}\left[\left(S_{ij}^{\max}\right)^2 - M^2\right] - Q_{ij\odot}^2\left(S_{ij}^{\max}\right)^2 \tag{28}$$

$$M^2 = S_{ij\odot}^2 - P_{ij\odot}^2 - Q_{ij\odot}^2 \tag{29}$$

$$P_{ij\odot} = \left|\dot{U}_i\right|^2 G_{ij} \tag{30}$$

$$Q_{ij\odot} = -\left|\dot{U}_i\right|^2\left(B_{ii} + B_{ij}\right) \tag{31}$$

$$S_{ij\odot} = \left|\dot{U}_i\right|\left|\dot{U}_j\right|\left|Y_{ij}\right| \tag{32}$$

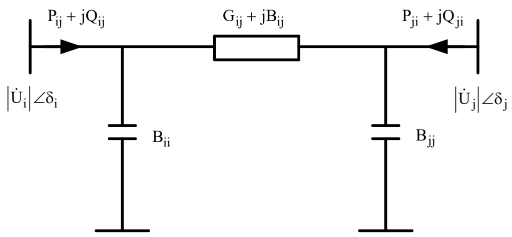

Fig. 1. The PI model of the transmission lines

3.3 Iterative DCOPF Algorithm

With the FND introduced at load buses, the active power demand at each bus is now subjected to the following constraint:

$$P_i = P_{Gi} - P_{Di}^E - P_{Di}^F - FND_i = \sum_{j=1}^{N} B_{ij}\left(\delta_i - \delta_j\right) \tag{33}$$

$$P_{ij}^{\min *} \le P_{ij} = \sum_{m=1}^{N} SF_{ij-m}\left(P_{Gm} - P_{Dm} - FND_m\right) \le P_{ij}^{\max *} \tag{34}$$

The iterative DCOPF that takes into account active power losses consist of the following steps:

1) Temporarily, ignore the active power losses in the network, i.e., $P_L = 0$, $FND_i = 0$.

2) Solve the DCOPF model to obtain the scheduled active power of the generators and the scheduled demand of loads.

3) Determine the new estimates of P_L, FND_i

4) Solve the DCOPF model with newly updated load demand.

5) Check for the convergence criteria:

$$\max\left|P_{Gi}^{(k)} - P_{Gi}^{(k+1)}\right| \le \varepsilon \ \left(\forall i\right) \tag{35}$$

$$\max\left|P_{Dj}^{(k)} - P_{Dj}^{(k+1)}\right| \le \varepsilon \ \left(\forall j\right) \tag{36}$$

If the convergence criteria is not satisfied, go to step 3.

4. ACOPF-based LMP Algorithm

The mathematical model of the ACOPF has the same objective as that of the model presented in section 2. In the ACOPF model, the bus power balance constraint and the branch flow constraint are modified. In addition, the reactive power limits of generators and the voltage limits constraints are added. These constraints are presented as follows:

$$\begin{aligned} P_i &= P_{Gi} - P_{Di} = \left|\dot{U}_i\right|\sum_{k=1}^{n}\left|\dot{U}_j\right|\left(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}\right) \\ Q_i &= Q_{Gi} - Q_{Di} = \left|\dot{U}_i\right|\sum_{k=1}^{n}\left|\dot{U}_j\right|\left(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}\right) \end{aligned} \tag{37}$$

$$Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max} \tag{38}$$

$$\left|U_i\right|^{\min} \le \left|U_i\right| \le \left|U_i\right|^{\max} \tag{39}$$

$$0 \le S_{ij} = \sqrt{P_{ij}^2 + Q_{ij}^2} \le S_{ij}^{\max} \tag{40}$$

The ACOPF model can be solved by using iterative linear programming method (iterative LP) [8].

5. LMP Calculation & Components

The LMP consists of the following components [8]:

$$LMP_i = LMP_E - LF_i.LMP_E + \sum_l SF_{l-i}.\mu_l \tag{41}$$

In the DCOPF model, the loss factors are determined as follows:

$$LF_i = \frac{\partial P_L}{\partial P_i} = \sum_l 2.R_l.SF_{l-i}.\left(\sum_{j=1}^{N} SF_{l-j}.P_j\right) \tag{42}$$

In the ACOPF model, the loss factors are calculated with the following expression:

$$\begin{aligned} &\left[\frac{\partial P_L}{\partial P_1}\frac{\partial P_L}{\partial P_2}\cdots\frac{\partial P_L}{\partial P_N}\right] = \\ &\left[\frac{\partial P_L}{\partial \delta_1}\frac{\partial P_L}{\partial U_1}\frac{\partial P_L}{\partial \delta_2}\frac{\partial P_L}{\partial U_2}\cdots\frac{\partial P_L}{\partial \delta_N}\frac{\partial P_L}{\partial U_N}\right].\begin{bmatrix} \frac{\partial \delta_1}{\partial P_1}\ \frac{\partial \delta_2}{\partial P_2}\cdots\frac{\partial \delta_N}{\partial P_N} \\ \frac{\partial U_1}{\partial P_1}\ \frac{\partial U_2}{\partial P_2}\cdots\frac{\partial U_N}{\partial P_N} \end{bmatrix} \end{aligned} \tag{43}$$

6. Results obtained with 3-bus system

In this section, we analyze the results obtained with a simple 3-bus system, using the optimization models described above.

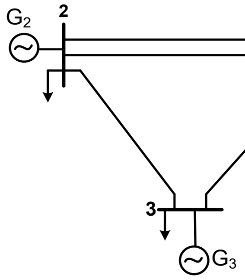

Fig. 2. A three-bus system

In the power system of Fig. 2, there are 3 power plants which all take part in the reserve market. The offers for energy by the power plants and bids of price-sensitive loads (which consist of 3 blocks) are shown in Table 1.

Table 1. Energy Offers of generators

	Generator 1			Generator 2		
Block	1	2	3	1	2	3
Power (MW)	200	130	170	150	100	150
Price ($/MWh)	5	7	9	4.5	8	10

The FND-based iterative DCOPF model is solved using the iterative LP method. After 4 steps, the solution converges to an error tolerance of 0,0001 MW. The LMPs and their components are shown in Table 2.

Table 2. LMP at bus 2

	DCOPF without losses	DCOPF with losses	ACOPF
LMP ($/MWh)	8	8.322	8.299
LMP_E ($/MWh)	7	7	7
LMP_L ($/MWh)	0.3	0.322	0.295
LMP_C ($/MWh)	0.7	1	1.004

The optimal values of generating power, reserve power and price-sensitive loads are shown in Table 3. The results in Table 3 show that the system has enough active power reserve. The RMCP results are shown in Table 4. The income of the power plants, which consists of the energy and the reserve components are shown in Table 5 and Fig. 3.

The differences in LMP and revenue, obtained with DCOPF without losses and FND-based model with losses, are determined according to (44)-(45), and are shown in Fig. 4 and Fig. 5.

$$D_{LMP}(\%) = \frac{LMP_i^{(1)} - LMP_i^{(2)}}{LMP_i^{(2)}}.100 \quad (i = 1, 2, 3) \qquad (44)$$

$$D_R(\%) = \frac{R_i^{(1)} - R_i^{(2)}}{R_i^{(2)}}.100 \qquad (i = G_1, G_2, G_3) \qquad (45)$$

Table 3. Active power generation, reserve power and price-sensitive load demands

	DCOPF without losses	DCOPF with losses	ACOPF
P_{G1} (MW)	293.3	280.9	279.2
P_{G2} (MW)	226.7	246.5	248.6
P_{G3} (MW)	300	300	300
P_{D2} (MW)	300	300	300
P_{D3} (MW)	120	120	120
$P_{G1}^{RR+}(MW)$	60	60	60
$P_{G1}^{RR-}(MW)$	60	60	60
$P_{G1}^{SR}(MW)$	0	0	0
$P_{G2}^{RR+}(MW)$	0	0	0
$P_{G2}^{RR-}(MW)$	0	0	0
$P_{G2}^{SR}(MW)$	36	36	36
$P_{G2}^{XR}(MW)$	0	0	0
$P_{G3}^{SR}(MW)$	0	0	0
$P_{G3}^{XR}(MW)$	54	54	54

Table 4. Reserve Market Clearing Prices (RMCPs)

	DCOPF without losses	DCOPF with losses	ACOPF
$RMCP^{RR+}$ ($/MWh)	7	7	7
$RMCP^{RR-}$ ($/MWh)	7	7	7
$RMCP^{SR}$ ($/MWh)	4	4	4
$RMCP^{XR}$ ($/MWh)	4	4	4

Table 5. The revenue of power plant 2

	DCOPF without losses	DCOPF with losses	ACOPF
Energy Revenue ($/h)	1813.60	2051.37	2063.13
Reserve Revenue ($/h)	144.00	144.00	144.00
Total Revenue ($/h)	1957.60	2195.37	2207.13

The obtained results show that: the solution of FND-based DCOPF with losses is very close to that of the full ACOPF model (the difference is less than 1%). The DCOPF model without losses is the least accurate.

On the other hand, in terms of computational performance, the FND-based iterative DCOPF model is much better than the ACOPF model. In addition, the DCOPF model always guarantees convergence, whereas the ACOPF model might have some convergence issues, depending on the initial estimates of the solutions.

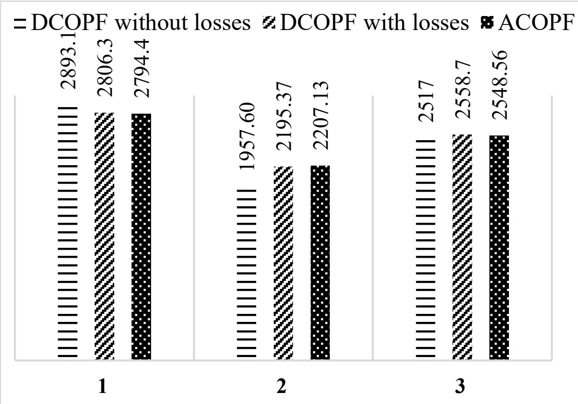

Fig. 3. Total revenue of the power plants ($/h)

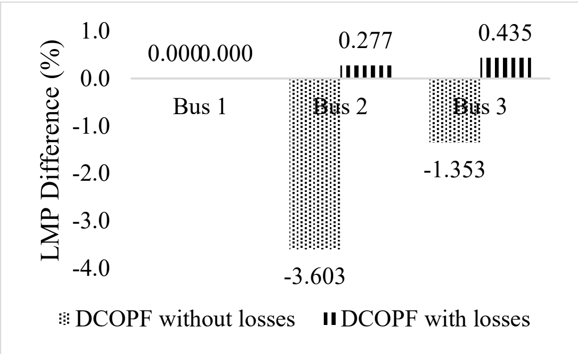

Fig. 4. Difference of LMP in percentage between each DCOPF algorithm and the ACOPF one

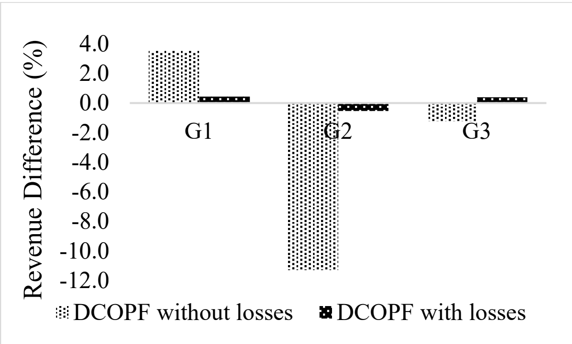

Fig. 5. Difference of revenue in percentage between each DCOPF algorithm and the ACOPF one

7. Conclusion

This paper studies the electric market model which includes a market for active power reserve. Different mathematical models are analyzed, including the lossless DCOPF, DCOPF with losses and the full ACOPF model. The results show that the optimal solution obtained with the DCOPF with losses is accurate, and is very close to the solution obtained with the ACOPF model.

Based on the FND-based iterative DCOPF, the power companies and the purchasing agencies can calculate their revenue and profit. This models also allows market participants to study and to derive strategies for generation expansion planning and transmission planning.

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