Dynamic Model of Flexible Link Manipulators with Translational and Rotational Joints

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Abstract

In this paper, two nonlinear dynamic models of flexible manipulator with translational and rotational joints are formulated based on Finite Element Method (FEM). Dynamic equations of manipulators are derived by using Lagrange equations. The new model TR (Translational-Rotational) is developed based on single flexible link manipulator with only rotational joint. The difference between flexible manipulators which have only rotational joints and others with translational joint is presented in RT model. Dynamic behaviors of flexible manipulators are analyzed through values of joints variable and elastic displacements at the end-effector point. The numerical simulation results are calculated by using MATLAB/SIMULINK toolboxes. The results of this study play an important role in modeling generalized planar flexible two-link robot, in designing the control system and basing on select the suitable structure robot with the same request.

Keywords: Dynamic model, flexible link, translational joint, elastic displacements

1. Introduction

Years ago, a number of researches focused on the flexible manipulator. Most of the investigations on the dynamic modeling of robot manipulators with elastic arms have been confined to manipulators with only revolute joints. Combining such systems with translational joints enables these robots to perform manipulation tasks in a much larger workspace, more flexibility and more applications. The flexible robot arm constructed with couples of rotational–translational joints (R-T/T-R joints) challenge the modeling and analysis for the robot. The two main methods for dynamic modeling of flexible link manipulators are the FEM and assumed mode method (AMM). Although the AMM has been used widely to model and analyze flexible links manipulators but most of studied have only considered the rotational joints. The FEM is a general method and it can be applied to the manipulators with complexly shaped links. This technique is used of this paper.

Few authors have studied the manipulator with only translational joint. Wang and Duo Wei [1] presented a single flexible robot arm with translational joint. Dynamic model analysis is based on a Galerkin approximation with time dependent basis functions. They also proposed a feedback control law in [2]. Kwon and Book [3] present a single link robot which is described and modeled by using assumed modes method (AMM). Other authors have focused on the flexible manipulator with a link slides through a translational joint with a simultaneous rotational motion (R-T robot). Pan et al [4] presented a model R-T with FEM method. The result is differential algebraic equations which are solved by using Newmark method. Yuh and Young [5] proposed the partial differential equations with R-T system by using AMM. Al-Bedoor and Khulief [6] presented a general dynamic model for R-T robot based on FEM and Lagrange approach. They defined a concept which is translational element. The stiffness of translational element is changed. The translational joint variable is distance from origin coordinate system to translational element. The number of element is small because it is hard challenge to build and solve differential equations. Khadem [7] studied a three-dimensional flexible n-degree of freedom manipulator having both revolute and translational joint. A novel approach is presented using the perturbation method. The dynamic equations are derived using the Jourdain's principle and the Gibbs-Appell notation. Korayem [8] also presented a systematic algorithm capable of deriving equations of motion of N-flexible link manipulators with revolute-translational joints by using recursive Gibbs-Appell formulation and AMM.

In this work, two nonlinear dynamic models of flexible manipulator with translational and rotational joints are presented based on using FEM and Lagrange equations. The flexible link is assumed Euler-Bernoulli beam. The first model is shown in

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fig. 1 (called T-R model). The second model is presented in fig. 2 (called R-T model). Flexible link slides through the base of translational joint which rotates at the fixed point. Both motion on horizontal plane and ignore the effect of gravity and structure damping. The boundary conditions of both are differently. The differences between flexible manipulators which have only rotational joints and others with translational joint are mentioned. Furthermore, T-R model has not been mentioned yet.

2. Dynamic modeling

2.1. General equations of motion

In both model, the coordinate system XOY is the fixed frame. The translational joint variable $d\left(t\right)$ is driven by force $F_T\left(t\right)$. The rotational joint variable $q\left(t\right)$ is driven by torque $\tau\left(t\right)$. Both joints are assumed rigid. Flexible link is divided n elements. Elements are assumed interconnected at certain points which are known as nodes. Each element has two nodes. Each node of element j has 2 elastic displacement variables which are flexural (u_{2j-1}) and slope displacements (u_{2j}). Fundamentally, the dynamic equations motion relies on the Lagrange equations with Lagrange function $L = T - P$ are given by [9]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{Q}}\left(t\right)}\right)^T - \left(\frac{\partial L}{\partial \mathbf{Q}\left(t\right)}\right)^T = \mathbf{F}\left(t\right) \tag{1}$$

Where, T and P are kinetic and potential energy of system. External generalized forces vector is defined as

$$\mathbf{F}\left(t\right) = \left[F_1\left(t\right) \quad F_2\left(t\right) \quad 0 \quad . \quad . \quad 0 \quad 0\right]^T \tag{2}$$

Specific generalized coordinate vector is given by

$$\mathbf{Q}\left(t\right) = \left[q_1\left(t\right) \quad q_2\left(t\right) \quad u_1 \quad . \quad . \quad u_{2n+1} \quad u_{2n+2}\right]^T \tag{3}$$

Elastic displacements vector of element j is

$$\mathbf{Q}_j\left(t\right) = \left[u_{2j-1} \quad u_{2j} \quad u_{2j+1} \quad u_{2j+2}\right]^T \tag{4}$$

In T-R case, driving forces and joints variables are $F_1\left(t\right) = F_T\left(t\right); F_2\left(t\right) = \tau\left(t\right); q_1\left(t\right) = d\left(t\right); q_2\left(t\right) = q\left(t\right)$ Kinetic and potential are $T = T^{T-R}$ and $P = P^{T-R}$.

In R-T case, respectively, $F_1(t) = \tau(t); F_2(t) = F_T(t)$, $q_1(t) = q(t); q_2(t) = d(t)$ and $T = T^{R-T}, P = P^{R-T}$. When kinetic and potential energy are known, it is possible to express Lagrange equations as shown

$$\mathbf{M}\left(\mathbf{Q}\right)\ddot{\mathbf{Q}} + \mathbf{C}\left(\mathbf{Q},\dot{\mathbf{Q}}\right)\dot{\mathbf{Q}} + \mathbf{KQ} = \mathbf{F}\left(t\right) \tag{5}$$

Coriolis force are calculated as

$$\mathbf{C}\left(\mathbf{Q},\dot{\mathbf{Q}}\right).\dot{\mathbf{Q}} = \dot{\mathbf{M}}\left(\mathbf{Q}\right)\dot{\mathbf{Q}} - \frac{1}{2}\left(\frac{\partial}{\partial \mathbf{Q}}\left(\dot{\mathbf{Q}}^T.\mathbf{M}.\dot{\mathbf{Q}}\right)\right) \tag{6}$$

Structural damping is ignored in this paper. Mass matrix $\mathbf{M}$ and stiffness matrix $\mathbf{K}$ are calculated by using FEM theory in each detail case, respectively. Size of matrices $\mathbf{M},\mathbf{K}$ and $\mathbf{C}$ is $(2n+4)\times(2n+4)$ and size of $\mathbf{F}\left(t\right)$ and $\mathbf{Q}(t)$ is $(2n+4)\times 1$.

2.2. The T-R configurations

The T-R model is shown as fig. 1. The translational joint moves along its axis. The translational joint variable $d\left(t\right)$ is distance from origin point of fixed coordinate to position of translational joint. Coordinate system $X_1O_1Y_1$ is attached to end point of link 1. Coordinate system $X_2O_2Y_2$ is attached to first point of link 2. Link 1 with length L_1 is assumed rigid and link 2 with length L_2 is assumed flexibility. Symbol m_t is mass of payload on the end-effector point. Coordinate $\mathbf{r}_{01}$ of end point of link 1 on XOY is computed as $\mathbf{r}_{01} = \left[L_1 \quad d\left(t\right)\right]^T$. Coordinate $\mathbf{r}_{2j}$ of element j on $X_2O_2Y_2$ can be found as $\mathbf{r}_{2j} = \left[\left(j-1\right)l_e + x_j \quad w_j\left(x_j,t\right)\right]^T; 0 \le x_j \le l_e$. Where, length of each element is $l_e = L_2/n$ and $w_j\left(x_j,t\right) = \mathbf{N}_j\left(x_j\right)\mathbf{Q}_j\left(t\right)$ is total elastic displacement of element. The vector of shape function $\mathbf{N}_j\left(x_j\right)$ is presented in [9]. Coordinate of element j on $X_1O_1Y_1$ is $\mathbf{r}_{21j} = \mathbf{T}_2^1.\mathbf{r}_{2j}$. Where, $\mathbf{T}_2^1 = \begin{bmatrix} \cos q\left(t\right) & -\sin q\left(t\right) \\ \sin q\left(t\right) & \cos q\left(t\right) \end{bmatrix}$ is transformation matrix from $X_2O_2Y_2$ to $X_1O_1Y_1$.

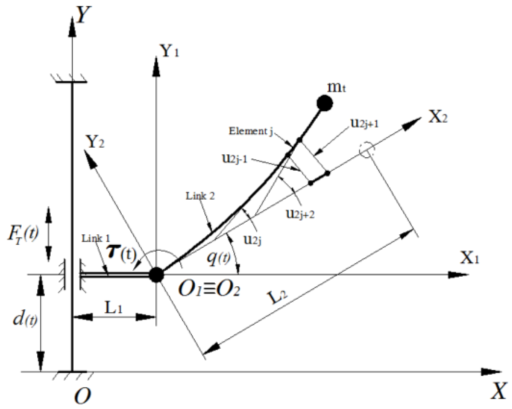

Fig. 1. T-R model

Coordinate $\mathbf{r}_{02j}$ of element j on XOY is $\mathbf{r}_{02j} = \mathbf{r}_{01} + \mathbf{r}_{21j}$. Coordinate $\mathbf{r}_{0E}$ of end point of flexible link 2 on $\mathbf{XOY}$ can be computed as

$$\mathbf{r}_{0E}^{T-R} = \begin{bmatrix} L_1 + L_2\cos q(t) - u_{2n+1}\sin q(t) \\ d(t) + L_2 \sin q(t) + u_{2n+1}\cos q(t) \end{bmatrix} \qquad (7)$$

If assumed that robot with all of links are rigid, coordinate $\mathbf{r}_{0E\_rigid}^{T-R}$ on $\mathbf{XOY}$ is given as

$$\mathbf{r}_{0E\_rigid}^{T-R} = \begin{bmatrix} L_1 + L_2\cos q(t) & d(t) + L_2\sin q(t) \end{bmatrix}^T \qquad (8)$$

The kinematic energy of rigid link 1 can be computed as $T_1^{T-R} = \frac{1}{2} m_1 .\dot{\mathbf{r}}_{01}^T .\dot{\mathbf{r}}_{01}$. Where, symbol m_1 is the mass of first link. Kinetic energy of element j is determined as

$$T_{2j}^{T-R} = \frac{1}{2}\int_0^{l_e} m_2 \left(\frac{\partial \mathbf{r}_{02j}}{\partial t}\right)^T \left(\frac{\partial \mathbf{r}_{02j}}{\partial t}\right) dx_j$$
$$T_{2j}^{T-R} = \frac{1}{2}\left(\dot{\mathbf{Q}}_{jg}^{T-R}\right)^T (t)\mathbf{M}_j^{T-R}\dot{\mathbf{Q}}_{jg}^{T-R}(t) \qquad (9)$$

Where, m_2 is mass per meter of link 2. Generalized elastic displacement vector of element j is given as

$$\mathbf{Q}_{jg}^{T-R}(t) = \begin{bmatrix} d(t) & q(t) & u_{2j-1} & u_{2j} & u_{2j+1} & u_{2j+2} \end{bmatrix}^T \qquad (1$$

Each element of inertial mass matrix $\mathbf{M}_j^{T-R}$ can be computed as

$$\mathbf{M}_j^{T-R}(s,e) = \int_0^{l_e} m_2 \left[\frac{\partial \mathbf{r}_{02j}}{\partial Q_{js}}\right]^T \left[\frac{\partial \mathbf{r}_{02j}}{\partial Q_{je}}\right] dx_j; \qquad (11)$$
$$s,e = 1,2,..,6$$

Where, Q_{js} and Q_{je} are the s^{th}, e^{th} element of $\mathbf{Q}_{jg}^{T-R}(t)$ vector. It can be shown that $\mathbf{M}_j^{T-R}$ is of the form

$$\mathbf{M}_j^{T-R} = \begin{bmatrix} \mathbf{M}_{rr} & \mathbf{M}_{rf} \\ \mathbf{M}_{fr} & \mathbf{M}_{base} \end{bmatrix} \qquad (12)$$

Where, matrix $\mathbf{M}_{base}$ is the base mass matrix which is given as

$$\mathbf{M}_{base} = \begin{bmatrix} \frac{13}{35}m_2 l_e & \frac{11}{210}m_2 l_e^2 & \frac{9}{70}m_2 l_e & -\frac{13}{420}m_2 l_e^2 \\ \frac{11}{210}m_2 l_e^2 & \frac{1}{105}m_2 l_e^3 & \frac{13}{420}m_2 l_e^2 & -\frac{1}{140}m_2 l_e^3 \\ \frac{9}{70}m_2 l_e & \frac{13}{420}m_2 l_e^2 & \frac{13}{35}m_2 l_e & -\frac{11}{210}m_2 l_e^2 \\ -\frac{13}{420}m_2 l_e^2 & -\frac{1}{140}m_2 l_e^3 & -\frac{11}{210}m_2 l_e^2 & \frac{1}{105}m_2 l_e^3 \end{bmatrix} \qquad (13)$$

Matrix $\mathbf{M}_{rr} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$ has elements which are computed as

$$m_{11} = m_2 . l_e ; m_{12} = m_{21};$$
$$m_{12} = -\frac{1}{12} m_2 l_e \begin{pmatrix} \sin(q) l_e u_{2j} - \sin(q) l_e u_{2j+2} \\ -12\cos(q) j l_e + 6\sin(q) u_{2j-1} \\ +6\sin(q) u_{2j+1} + 6\cos(q) l_e \end{pmatrix}$$
$$m_{22} = \frac{1}{210} m_2 l_e \begin{pmatrix} 210 l_e^2 j(j-1) + l_e^2 (2u_{2j}^2 - 3u_{2j}u_{2j+2} \\ +2u_{2j+2}^2) + 22 l_e (u_{2j-1}u_{2j} - u_{2j+1}u_{2j+2}) \\ +13 l_e (u_{2j}u_{2j+1} - u_{2j-1}u_{2j+2}) + 70 l_e^2 \\ +78(u_{2j-1}^2 + u_{2j+1}^2) + 54 u_{2j-1}u_{2j+1} \end{pmatrix} \qquad (14)$$

Matrix $\mathbf{M}_{rf} = \mathbf{M}_{fr}^T = \begin{bmatrix} m_{13} & m_{14} & m_{15} & m_{16} \\ m_{23} & m_{24} & m_{25} & m_{26} \end{bmatrix}$ has elements which are computed as

$$m_{13} = m_{15} = \frac{1}{2} m_2 l_e \cos q ; m_{14} = \frac{1}{12} m_2 l_e^2 \cos q;$$
$$m_{16} = -m_{14}; m_{23} = \frac{1}{20} m_2 l_e^2 (10j - 7);$$
$$m_{24} = \frac{1}{60} m_2 l_e^3 (5j-3); m_{25} = \frac{1}{20} m_2 l_e^2 (10j-3); \qquad (15)$$
$$m_{26} = -\frac{1}{60} m_2 l_e^3 (5j-2);$$

Total elastic kinetic energy of link 2 is yielded as

$$T_e^{T-R} = \sum_{j=1}^{n} T_{2j}^{T-R} = \frac{1}{2}\left(\dot{\mathbf{Q}}^{T-R}\right)^T (t)\mathbf{M}_e^{T-R}\dot{\mathbf{Q}}^{T-R}(t) \qquad (16)$$

The mass matrix $\mathbf{M}_e^{T-R}$ is constituted from matrices of elements follow FEM theory, respectively. The generalized coordinate vector of system is defined as

$$\mathbf{Q}^{T-R}(t) = \begin{bmatrix} d(t) & q(t) & u_1 & . & . & u_{2n+1} & u_{2n+2} \end{bmatrix}^T \qquad (17$$

Kinetic energy of payload is $T_P^{T-R} = \frac{1}{2} m_t . \left(\dot{\mathbf{r}}_{0E}^{T-R}\right)^T .\dot{\mathbf{r}}_{0E}^{T-R}$ and kinetic energy of system is determined as

$$T^{T-R} = T_1^{T-R} + T_e^{T-R} + T_P^{T-R}$$
$$T^{T-R} = \frac{1}{2}\left(\dot{\mathbf{Q}}^{T-R}\right)^T (t)\mathbf{M}^{T-R}\dot{\mathbf{Q}}^{T-R}(t) \qquad (18)$$

Matrix $\mathbf{M}^{T-R}$ is total mass matrix. The gravity effects can be ignored as the robot movement is confined to the horizontal plane. Defining E and I are Young's modulus and area moment of inertia link 2. Elastic potential energy of element j with stiffness matrix $\mathbf{K}_{base}$ which is presented as [9] is given as

$$P_j^{T-R} = \frac{1}{2}\int_0^{l_e} EI\left[\frac{\partial^2 w_j\left(x_j,t\right)}{\partial x_j^2}\right]^2 dx_j \tag{19}$$

$$P_j^{T-R} = \frac{1}{2}\mathbf{Q}_j^T(t)\mathbf{K}_j^{T-R}\mathbf{Q}_j(t)$$

Stiffness matrix of element j is defined as

$$\mathbf{K}_j^{T-R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathbf{K}_{base} \end{bmatrix} \tag{20}$$

Where,

$$\mathbf{K}_{base} = \begin{bmatrix} \frac{12EI}{l_e^3} & \frac{6EI}{l_e^2} & -\frac{12EI}{l_e^3} & \frac{6EI}{l_e^2} \\ \frac{6EI}{l_e^2} & \frac{4EI}{l_e} & -\frac{6EI}{l_e^2} & \frac{2EI}{l_e} \\ -\frac{12EI}{l_e^3} & -\frac{6EI}{l_e^2} & \frac{12EI}{l_e^3} & -\frac{6EI}{l_e^2} \\ \frac{6EI}{l_e^2} & \frac{2EI}{l_e} & -\frac{6EI}{l_e^2} & \frac{4EI}{l_e} \end{bmatrix} \tag{21}$$

Total elastic potential energy of system is yielded as

$$P^{T-R} = \sum_{j=1}^{n} P_j = \frac{1}{2}\left(\mathbf{Q}^{T-R}\right)^T(t)\mathbf{K}^{T-R}\mathbf{Q}^{T-R}(t) \tag{22}$$

Total stiffness matrix $\mathbf{K}^{T-R}$ is constituted from matrices of elements follow FEM theory similar $\mathbf{M}^{T-R}$ matrix, respectively.

2.3. The R-T configurations

The R-T model is shown as fig. 2. Flexible link 2 is divided n elements. The position of elements k continuously changes through translational joints. The angle β_0 is the angle between O_1X_1 and O_2X_2. The translational joint variable $d(t)$ is the distance from element k at the origin coordinate system to the end point of link 2. The value of k depends on time. The position of other elements is changed, too. Total length of flexible link 2 is L_2. Length of each element is $l_e = L_2/n$. Element k is inside the hub of the translational joint and we have contact formulas $k.l_e = L_2 - d(t)$. Effect of the length of hub is ignored.

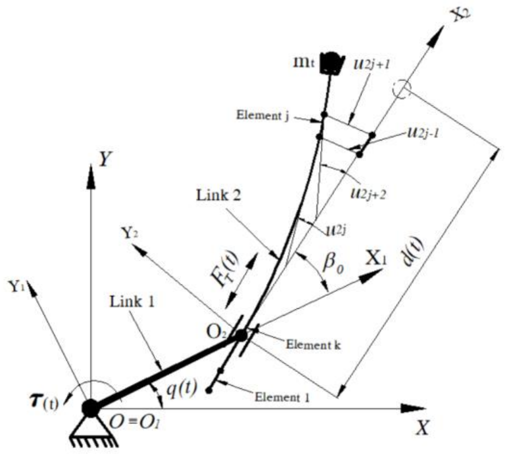

Fig. 2. R-T model

It is noteworthy to mention that value of k is the number of position element k. So, it must be take raw value of k in computing process. Coordinate $\mathbf{r}_j$ on $X_2O_2Y_2$ of element j is $\mathbf{r}_j = \left[(j-1)l_e - L_2 + d(t) + x_j \quad w_j\left(x_j,t\right)\right]^T$ and on XOY is $\mathbf{r}_{0j} = \mathbf{r}_{01} + \mathbf{T}_1^0.\mathbf{T}_1^2.\mathbf{r}_j$. Transformation matrix $\mathbf{T}_0^1$ between XOY and $X_1O_1Y_1$ is $\mathbf{T}_1^0 = \begin{bmatrix} \cos q(t) & -\sin q(t) \\ \sin q(t) & \cos q(t) \end{bmatrix}$. Transformation matrix $\mathbf{T}_1^2$ between $X_1O_1Y_1$ and $X_2O_2Y_2$ is $\mathbf{T}_1^2 = \begin{bmatrix} \cos\beta_0 & -\sin\beta_0 \\ \sin\beta_0 & \cos\beta_0 \end{bmatrix}$. Coordinate of end point of link 1 on XOY is $\mathbf{r}_{01} = \left[L_1.\cos q(t) \quad L_1.\sin(t)\right]^T$. Coordinate of end-effector is given as

$$\mathbf{r}_{0E}^{R-T} = \begin{bmatrix} \left[d(t)+L_1\right].\cos q(t) - u_{2n+1}.\sin q(t) \\ \left[d(t)+L_1\right].\sin q(t) + u_{2n+1}.\cos q(t) \end{bmatrix} \tag{23}$$

If assumed that link 2 is rigid, coordinate $\mathbf{r}_{0E\_rigid}^{R-T}$ on XOY is given as

$$\mathbf{r}_{0E\_rigid}^{R-T} = \begin{bmatrix} \left[d(t)+L_1\right].\cos q(t) \\ \left[d(t)+L_1\right].\sin q(t) \end{bmatrix} \tag{24}$$

Kinetic and potential energy of R-T system are calculated by using the same method in T-R case

$$T^{R-T} = T_e^{R-T} + T_P^{R-T}$$

$$T^{R-T} = \frac{1}{2}\left(\dot{\mathbf{Q}}^{R-T}\right)^T(t)\mathbf{M}_e^{R-T}\dot{\mathbf{Q}}^{R-T}(t) + \frac{1}{2}m_t.\left(\dot{\mathbf{r}}_{0E}^{R-T}\right)^T.\dot{\mathbf{r}}_{0E}^{R-T} \tag{25}$$

$$P^{R-T}=\sum_{j=1}^{n}P_j^{R-T}=\frac{1}{2}\left(\mathbf{Q}^{R-T}\right)^T(t)\mathbf{K}^{R-T}\mathbf{Q}^{R-T}(t) \qquad (26)$$

Where, T_e^{R-T} is the elastic kinetic and T_P^{R-T} is kinetic energy of payload. Mass matrix $\mathbf{M}_e^{R-T}$ and stiffness matrix $\mathbf{K}^{R-T}$ can be calculated by using FEM theory following mass and stiffness matrices of all elements. Mass matrix of element j with mass per meter m_2 is given as $\mathbf{M}_j^{T-R}$ at recipe (12). It noted that $\mathbf{M}_{base}$ is the same but others elements are difference. They are computed as

$$\begin{aligned}
m_{11} &= f(m,le,j,L,d(t),Q_j(t));\\
m_{12} &= -\frac{1}{12}m_2l_e\left(l_eu_{2j}-l_eu_{2j+2}+6u_{2j-1}+6u_{2j+1}\right)\\
m_{13} &= \frac{1}{20}m_2l_e\left(10jl_e+10d(t)+10L_1-10L_2-7l_e\right)\\
m_{14} &= \frac{1}{60}m_2l_e^2\left(5jl_e+5d(t)+5L_1-5L_2-3l_e\right)\\
m_{15} &= \frac{1}{20}m_2l_e\left(10jl_e+10d(t)+10L_1-10L_2-3l_e\right)\\
m_{16} &= -\frac{1}{60}m_2l_e^2\left(5jl_e+5d(t)+5L_1-5L_2-2l_e\right)\\
m_{22} &= m.l_e;\, m_{23}=m_{24}=m_{25}=m_{26}=0;
\end{aligned} \qquad (27)$$

Matrices $\mathbf{M}_{base}^{R-T}$ and $\mathbf{K}_j^{R-T}$ have format as $\mathbf{M}_{base}^{T-R},\mathbf{K}_j^{T-R}$.

3. Boundary conditions

In T-R case, rotational joint of link 2 is constrained so that elastic displacements of first node of element 1 on link 2 can be zero. Thus variables u_1,u_2 are zero. The rows and columns 3th, 4th of matrices $\mathbf{M},\mathbf{K},\mathbf{C}$ and $\mathbf{F}(t),\mathbf{Q}(t)$ vectors are eliminated as presented in FEM theory.

In R-T case, the displacements of element k are zero because assumed that the translational joint hub is treated as rigid. The rows and columns $(2k-1)^{th};(2k)^{th}$ of matrices $\mathbf{M},\mathbf{K},\mathbf{C}$ and $\mathbf{F}(t),\mathbf{Q}(t)$ vectors are eliminated and values of these are changed, too. It is noteworthy to mention that value of k depends on time. This boundary condition is clearly different point between flexible link manipulator with only rotational joints and flexible link manipulator with combine translational joint and rotational joint. So now, size of matrices $\mathbf{M},\mathbf{K},\mathbf{C}$ is $(2n+2)\times(2n+2)$ and $(2n+2)\times 1$ is size of $\mathbf{F}(t)$ and $\mathbf{Q}(t)$ vector. The k variable is continuously updated for each time step in solving process. Vector generalized coordinate $\mathbf{Q}(t)$ is rebuilt after each loop too because of changing value k variable.

4. Simulation results

Simulation specifications of two flexible models are given by Table. 1.

Table 1. Parameters of dynamic models

Property	T-R model	R-T model
Length of link 1 (m)	L_1=0.1	L_1=0.2
Mass of link 1 (kg)	m_1=1.4	m_1=0.9
Flexible link parameters		
Length of flexible link (m)	L_2=0.3	L_2=0.8
Width (m)	b=0.03	b=0.01
Thickness (m)	h=0.003	h=0.002
Number of element	n=5	
Cross section area (m^2) (A=b.h)	A=9.10^{-5}	A=2.10^{-5}
Mass density (kg/m^3)	ρ=7850	
Mass per meter (kg/m) (m_2=ρ.A)	0.7	0.157
Young's modulus (N/m^2)	E=2.10^{10}	
Inertial moment of cross section (m^4)	I=$b.h^3/12$=$1.67.10^{-12}$	
Mass of payload (g)	m_t=100	m_t=20
Initial value at t=0 (m)	0	d_0=0.4
β_0		0
Simulation time (s)	T=10	

Dynamic equations (5) for both models are solved by using MATLAB/SIMULINK toolboxes with boundary conditions. The simulation results of T-R model are shown in fig. 3, fig. 4 and fig. 5. Driving force and torque of joints are built using Bang-Bang rule which are shown fig. 3. Driving time is 0.5(s). Dynamic behaviors of system are described through values of joint variables and elastic displacements in fig. 4 and fig. 5. Maximum values of translational and rotational joint are 0.075(m) and 0.24(rad). These values are shown in fig. 4. Elastic displacements at the end-effector point are presented in fig. 5. Maximum values of these displacements are 0.14(m) and 0.25(rad) at 0.6(s). These displacements are fast reduced after that with 0.02(m) and 0.035 (rad). The simulation results of R-T model are shown from fig. 6 to fig. 8. The Bang-Bang rule in fig. 6 is used for driving torque and force of joints. Fig. 7 shows values of joint variables. It note that value of rotational joint continuously increases while maximum value of translational joint increases from initial value 0.4(m) to 0.51(m) at 0.5(s). Values different of rotational joint between both models can be explained by effect of higher nonlinearity in R-T model, difference of boundary condition and direction of motion of flexible link when value of translational joint increase over time. Elastic displacements at the end-effector point are shown in fig. 8. Values of these displacements are smaller than T-R case.

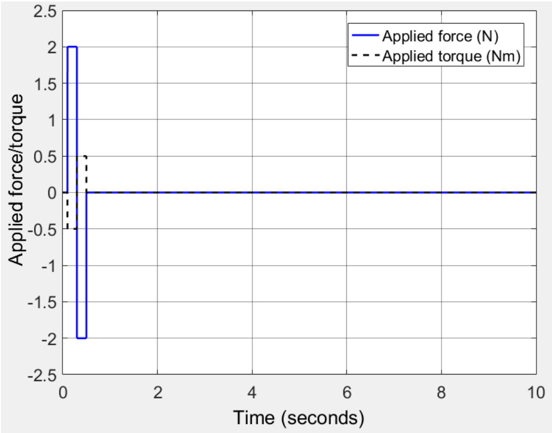

Fig. 3. Driving force and torque at joints of T-R model

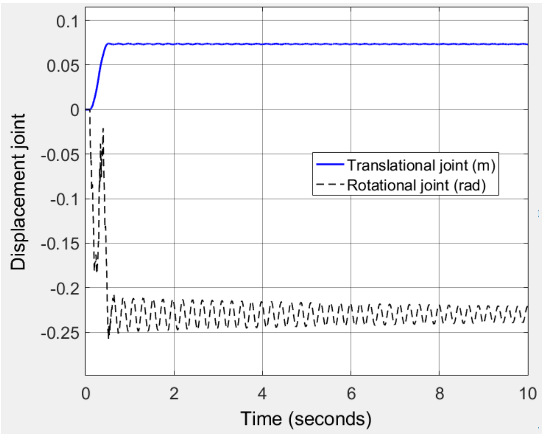

Fig. 4. Values of joint variables in T-R model

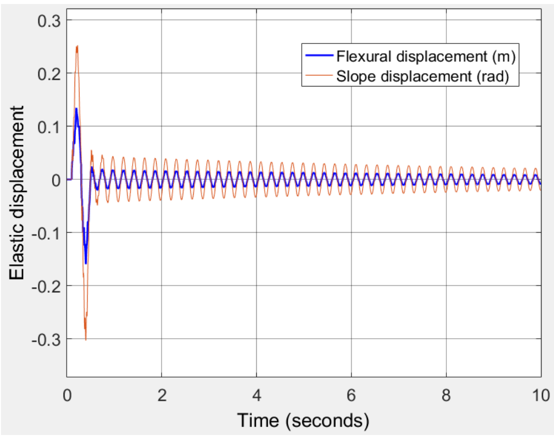

Fig. 5. Value of elastic displacements at the end-effector point in T-R model

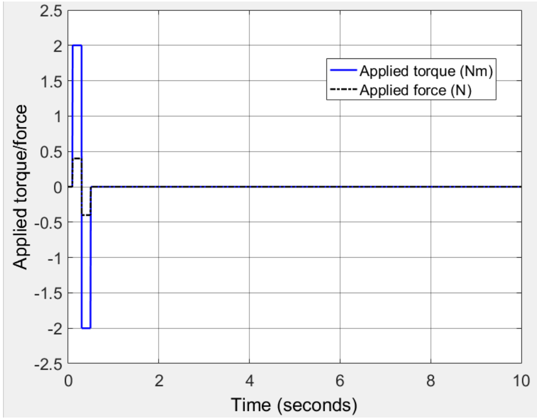

Fig. 6. Driving torque and force at joints in R-T model

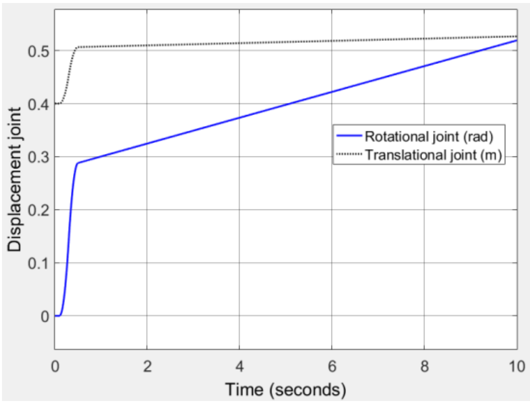

Fig. 7. Values of joint variables in R-T model

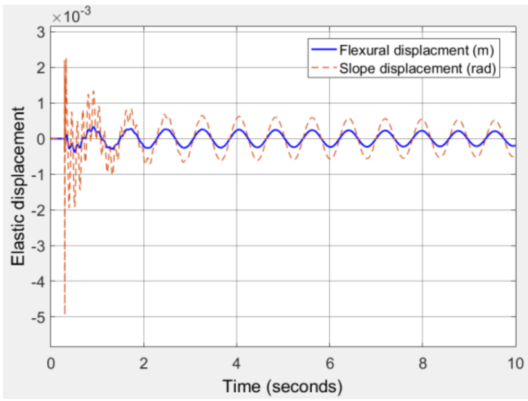

Fig. 8. Value of elastic displacement at the end-effector point in R-T model

5. Conclusion

Mathematical model of two flexible manipulators with translational and rotational joints has been presented based upon finite element method and Lagrange equations. T-R model is the new model which is developed from single flexible link manipulator with only rotational joint. R-T model shows difference points between translational joint and rotational joint. These difference points are boundary conditions. The final models derived for the flexible robot are nonlinear and complex. Simulation results show that the response of flexible robots is unstable especial without effect of structure damping and needs to control. Moreover, the effects of structure damping and varying payload on dynamic characteristic of models have been studied and discussed. The results in this paper are helpful and important in development of modeling generalized planar flexible two-link robot which combines translational and rotational joints and designing control system.

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