## Research on Optimality of Beamforming in MIMO Model to Improve SER in Multipath Mobile Transmission Environment

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## Abstract

Some papers are researching on how to optimize the beam weighs in generally. They discover beam patterns are related with upper bound of SER and can allocate power to these beams. The environment is used to illustrate these beams are Ricean and Rayleigh distributed. However, in multipath mobile environments, how they are applied in the transmit beams needs to be made clear. This paper concentrates on use of the multipath mobile channel matrix of MIMO to form the beams along with the physical paths at the transmitter. The paper also uses power allocation for these beams on principle of " water filling", the gain of path is better, more transmit power is assigned to the path. The simulation can show the SER is improved if using more beams for more paths and also the optimal power allocation is giving the lower SER compared with the case using equal power allocation to all paths.

Keywords: MIMO, SER, beamforming.

## 1. Introduction

The true channel matrix, that the transmitter does not fully know, can be modeled as a Gaussian random matrix (or vector) whose mean and covariance is given in the feedback. Two point by point type of criticism are channel mean (CM) and channel covariance (CC) [1],[2],[7],[8]. The author concentrates on the CC, incorporating be explored for processing rapidly changing MIMO channels.

In the current 4G communication, downlink technology uses Orthogonal Frequency Division Multiple (OFDM) and MIMO to speed up to 100 Mbps (expecting a 2x2 MIMO configuration with 20MHz bandwidth). The good capacity of MIMO relies on the exact estimation of Channel State Information (CSI) [2]. It uses a training sequence to be known at the receiver and the transmitter. The disadvantage is time needs to be spent for exchanging the trainning sequence between the transmitter and the receiver. In the FDD (Full Duplex Devision) mode, both the pilot-aided training overhead and the feedback overhead for channel side information (CSI) acquisition are increased proportionally with the BS antenna size. However, the proportion of radio resources allocated to CSI acquisition is severely restricted by the channel coherence period. The situation is made worse in an environment with high user equipment (UE) mobility [3]. The author

considers reducing CSI by using the subspace estimation instead the other information of channel.

In massive MIMO systems, normally for 5G, the pilot sequence is used to estimate the CSI in both directions. These are based on picking up the strongest channel impulse responses. CSI can be estimated at the receiver side only, or at both at the transmitter and the receiver. Estimation at both sides has some advantages: the CSI does not have to be transmitted, which yields low latency and high capacity. In addition, more power can be allocated to the OFDM subchannels with higher channel gain. They state that schemes with estimation at the receiver side only has higher outage probability with fast fading channels but have lower complexity. They conclude new techniques should be introduced to reduce the training time will improve the performance of FDD systems in massive MIMO to get better channel gain, capacity, received power, and reduce latency [4]. The author considers the CSI estimation at the receiver only where some good transmit dimensions and corresponding power allocation are applied at the transmitter. The time of transmitting these dimensions to the transmitter is suitable because the spatial features of the channel changes little.

Recent MIMO system investigations have considered more realistic channel conditions and taken into account the imperfect CSI at both transmit and receive sides. It is said that solutions to enhance MIMO system robustness against imperfect CSI come from two methods: using space-time coding or channel coding and proposing improved sub-

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optimum detectors [5]. Moreover, an another paper argues that statistical CSI acquisition in Massive MIMO should be formulated as a problem of covariance estimation with missing data. This point of view has been adopted in the context of subspace estimaton. This paper can handle the case of scheduling and dynamic pilot sequence allocation, and provides asymptotically contamination-free covariance estimates without requiring dedicated pilot sequences [6]. Based on these research directions, the author gives subspace estimation (use channel covariance matrix) at the receiver (suboptimum detector) in multipath environment (considered realistic) that helps increasing the channel capacity. This is because the spatial feature of the channel changes little. The proposed method does not need the dedicated pilot sequences in some circumstances.

There exists a probability of a symbol error during the transmission through the encoder and transmit antenna and then receive antenna and decoder. The formula for this can be expressed as [7], [8]. In realistic MIMO model, the *SER* is limited due to coding and constellation size, transmission environment. In this section, the *SER* should be seen as the upper *SER* values. This value can be denoted  $P_{s,bound}$ . For this study, two cases the covariance feedback and mean feedback where the *SER* is different are considered.

In the case of the covariance matrix feedback [8]:  $P_{s,bound} = \alpha \left| \mathbf{I}_{N} + \mathbf{A} \frac{gE_{s}}{N_{0}} \right|^{-1}$ (1)

where  $\alpha$  is a factor that can be determined by the number of the transmit elements M:  $\alpha = (M-1)/M$ .

g is a constellation factor due to the type of modulation at the transmitter.

 $\frac{E_s}{N_0}$  is the average energy per noise density of a

symbol.

$$\mathbf{A} = \mathbf{D}_{h}^{1/2} \mathbf{U}_{h}^{H} \mathbf{C}^{H} \mathbf{C} \mathbf{U}_{h} \mathbf{D}_{h}^{1/2}$$
(2)

that is determined from the pre-coder  ${\bf C}$  and the covariance matrix

$$\mathbf{R}_{hh} = \mathbf{U}_h \mathbf{D}_h \mathbf{U}_h^H \tag{3}$$

where  $\mathbf{U}_h$ ,  $\mathbf{D}_h$  are the unitary and diagonal matrices, respectively?

### 2. Transmit dimensions and power allocation

The transmitted beam algorithms can be expressed in terms of beam dimensions and power allocation.

Using the probability of codeword error, the *SER*, the channel capacity obtained from [7], [8], [9], [10], [11] and [12], the optimal dimensions and power allocation depend on the chosen criteria and on types of feedback. These findings are given according to three criteria: the codeword error probability, the *SER*, the Shannon capacity of channel. For the *SER*, there are two cases of feedback: the covariance feedback and the mean feedback.

In the mean feedback, when mean of the channel vector is known at the transmitter, the upper bound of the *SER* is [1]:

$$P_{s,bound} = \alpha \prod_{\mu=1}^{M} \frac{1}{1 + \delta_{\mu}\beta} \exp\left(\frac{-K_{\mu}\delta_{\mu}\beta}{1 + \delta_{\mu}\beta}\right)$$
(4)

where  $\alpha = (M-1)/M$ 

 $\beta = g \sigma_{e}^{2} \frac{E_{s}}{N_{0}}$  where  $\sigma_{e}^{2}$  is the variance of the channel vector **h** at the transmitter and g is the constellation-specific constant [9].

$$K_{\mu} = \frac{\left[ \mathbf{U}_{c}^{H} \, \bar{\mathbf{h}} \right]^{2}}{\sigma_{\varepsilon}^{2}} \tag{5}$$

where **h** is mean of the channel vector **h**, the vector  $\bar{\mathbf{h}}$  which is unbiased at the transmitter and matrix  $\mathbf{U}_c$  consists of eigenvectors of the pre-coder

**C**. This relationship is defined as:

$$\mathbf{C}^{H}\mathbf{C} = \mathbf{U}_{c}\mathbf{D}_{c}\mathbf{U}_{c}^{H} \tag{6}$$

(using the SVD of  $\mathbf{C}^{H}\mathbf{C}$ ).

 $\delta_{\mu}$  is the  $\mu$  th eigenvalue of  $\mathbf{D}_{c}$ 

When considering optimal beamforming offered by the mean feedback for the SER, the matrix representing the optimal dimensions is defined as [1]:

where 
$$\mathbf{\hat{h}}\mathbf{h} = \mathbf{U}_{h}\mathbf{D}_{h}\mathbf{U}_{h}^{H}, \mathbf{D}_{h} = diag(\lambda, 0, ..., 0)$$
 (7)

## (if $\mathbf{h}$ is a vector, $\mathbf{D}_{h}$ has only one eigenvalue $\lambda$ )

For the optimal allocation, depending on the distribution of channel, two power allocations for the Ricean distribution and the Nagakami- m distribution are considered as in [13].

For the case of the Ricean distribution, the optimal power allocation is expressed as:

$$\mathbf{D}_{c} = diag(\delta_{1}, \delta_{2}, ..., \delta_{M})$$
(8)
where
$$\delta_{2} = .. = \delta_{M} = \left[\frac{2a}{b + \sqrt{b^{2} - 4ac}} - \frac{1}{\beta}\right]_{+}$$
(9)
$$\delta_{1} = 1 - (M - 1)\delta_{2}$$

with

$$a = \left(1 + \frac{M}{\beta}\right)^{2},$$
  

$$b = \left[\frac{\lambda}{\beta\sigma_{\epsilon}^{2}} + \left(1 + \frac{M}{\beta}\right)(2M - 1)\right],$$
  

$$c = M(M - 1)$$

For the case of the Nagakami-m distribution, the optimal power allocation is defined as:

$$\delta_{2} = ... = \delta_{M} = \begin{cases} \delta_{2}^{0}, \frac{E_{s}}{N_{0}} > \gamma_{th} \\ 0, \frac{E_{s}}{N_{0}} < \gamma_{th} \end{cases}$$
(10)  
$$\delta_{1} = 1 - \delta_{2} (M - 1) \\ \delta_{2}^{0} = \frac{\sigma_{e}^{2} (\sigma_{e}^{2} + 2\lambda)}{M \sigma_{e}^{2} (\sigma_{e}^{2} + 2\lambda) + \lambda^{2}} \\ \text{Where} \\ \cdot \left[ 1 + \frac{1}{\beta} \left( M - \frac{\lambda}{\sigma_{e}^{2} + 2\lambda} \right) \right] - \frac{1}{\beta} \\ \text{and} \ \gamma_{th} = \frac{\lambda}{g \sigma_{e}^{4}} \left( \frac{\sigma_{e}^{2} + \lambda}{\sigma_{e}^{2} + 2\lambda} \right) \end{cases}$$

because  $\mathbf{\tilde{h}}\mathbf{h}^{-H}$  is one rank matrix and the constraint is:  $\sum_{\mu=1}^{M} \delta_{\mu} = 1 \text{ and } \delta_{\mu} \ge 0.$ 

From equations from (8) - (10) that it can be seen the transmit power is divided such that the strongest dimension corresponding to the eigenvalue  $\lambda$  receives the most power and the remaining power is equally divided among the other eigenvectors. The optimal power allocation can be chosen by the upper bound of the cost function (1). The power constraint can be defined as:

$$\ln P_{s,bound} = \ln \alpha - \frac{\lambda}{\sigma_{\epsilon}^{2}} -$$

$$\ln \left| \mathbf{I}_{M} + \beta \mathbf{D}_{c} \right| + \frac{\lambda}{\sigma_{\epsilon}^{2} \left( 1 + \beta \delta_{1} \right)}$$
(11)

In case of the covariance matrix, [2] presented the optimal pre-coder **C** as:

$$\mathbf{C} = \mathbf{\Phi} \mathbf{D}_f \mathbf{U}_h^H \tag{12}$$

where  $\Phi$  is the matrix consisting of orthogonal columns and is used to multiply a symbol before this symbol goes to beam-forming matrix:

$$\mathbf{W} = \mathbf{D}_f \mathbf{U}_h^H \tag{13}$$

 $\mathbf{U}_h$  is the matrix representing the optimal dimensions and is explained in (3.28).

 $\mathbf{D}_{f}$  is the matrix representing the optimal power allocation:

$$f_{\mu}^{2} = \left[\frac{1}{\frac{1}{M}} + \frac{N_{0}}{gE_{s}} \left(\frac{1}{\frac{N}{M}} \frac{\bar{M}}{L} - \frac{1}{\lambda_{l}} - \frac{1}{\lambda_{\mu}}\right)\right]_{+}$$
(14)  
with  $\mathbf{D}_{f} = diag\left(f_{1}, f_{2}, ..., f_{M}\right)$ 

where  $\lambda_i$  is the ith eigenvalue of the covariance matrix  $\mathbf{R}_{hh} = \mathbf{U}_h \mathbf{D}_h \mathbf{U}_h^H$ 

 $\overline{M}$  is number of the non-zero eigenvalues of  $\mathbf{R}_{hh}$ . When  $\lambda_1 \leq \lambda_2 \leq ... \leq \lambda_M$ , it is easy to see  $f_1 \geq f_2 \geq ... \geq f_M$ .  $\overline{M}$  can be determined as follows:  $\frac{1}{r} + \frac{N_o}{gE_s} \left( \frac{1}{r} \sum_{l=1}^r \frac{1}{\lambda_l} - \frac{1}{\lambda_\mu} \right)$  is tested from r = 1 to M in the secure

in the sequel.

If finding *r* so that:

$$\frac{1}{r} + \frac{N_o}{gE_s} \left( \frac{1}{r} \sum_{l=1}^r \frac{1}{\lambda_l} - \frac{1}{\lambda_\mu} \right) \ge 0$$
(15)

$$\frac{1}{r+1} + \frac{N_o}{gE_s} \left( \frac{1}{r+1} \sum_{l=1}^{r+1} \frac{1}{\lambda_l} - \frac{1}{\lambda_\mu} \right) < 0$$
(16)

Then 
$$M = r, f_{r+1} = f_{r+2} = \dots = f_M = 0$$
.

# **3.** Forming transmit beamforming in multipath transmission environment

The channel matrix in the MIMO model in the discrete physical model stated as:



Fig.1. MIMO model with moving the receiver

$$\mathbf{H} = \left[ h_{nm} \right]_{N\mathbf{x}M} \tag{17}$$

where  $h_{nm}$  is the connection coefficient between the *m* th element at the transmit antenna and the nth element at the receive antenna where:

$$h_{nm} = \sum_{l=1}^{L} \alpha_l e^{-j\theta_l} e^{-j\kappa \left( (m-1)\sin\phi_l s_T - (n-1)\sin\theta_l s_R \right)}$$

$$e^{ju_l vt}$$
(18)

 $\alpha_l$  is the magnitude of path l,  $\kappa = \frac{2\pi}{\lambda}$  where  $\lambda$  is wavelength of signal, vt = z where v is the velocity of the receiver, t is the time of moving the receiver and z is the distance the receiver moves. The important relationship of matrix  $\mathbf{H}(t)$  is:

$$\mathbf{R}_{hh}(\tau) = \begin{bmatrix} E \left\langle \mathbf{H}^{H}(t_{1} - \tau) \mathbf{H}(t_{1}) \right\rangle & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & E \left\langle \mathbf{H}^{H}(t_{K} - \tau) \mathbf{H}(t_{K}) \right\rangle \end{bmatrix}$$
(19)

Applying SVD at the receiver to decompose the covariance matrix  $\mathbf{R}_{hh}$ , i.e.  $\mathbf{R}_{hh} = \mathbf{U}\boldsymbol{\Sigma}V^{H}$  leads to the vectors  $\mathbf{u}_{l}, l = 1 \rightarrow L$  of matrix:

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \dots & \mathbf{u}_L \end{bmatrix}$$
(20)

The productive transmit vector at the *p* th observation  $\mathbf{w}_{lp}, l = 1 \rightarrow L$  are then  $\mathbf{u}_{lp}^{H}, l = 1 \rightarrow L$ ,

where  $\mathbf{u}_{lp}, l = 1 \rightarrow L$  consists of the M(p-1)+1 th to the Mp th entries of vector  $u_l, l = 1 \rightarrow L$ .

In terms of the vectors offered by the covariance matrix at the receiver, the array factor (beam patterns) of the vector as defined:

$$4F_{lp}(\phi) = \frac{1}{M} \sum_{m=1}^{M} \mathbf{w}_{lp}(m) e^{-j(\kappa(m-1)s_T \sin \phi)}$$
(21)

## 4. Results and discussion

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It is assumed that multipath transmision environment has 4 paths with gains  $\alpha_1 = 0.6, \alpha_2 = 0.4 \alpha_3 = 0.3 \alpha_4 = 0.3$ .

Wavelength of the signal is 0.1 m. Distance between transmit and receive antennas is  $s_T = s_R = 0.5$  m. Velocity of the receiver is v = 40 km. Transmit and receive angles:  $15^\circ, 45^\circ, 75^\circ, 105^\circ; 135^\circ, 165^\circ, 195^\circ, 225^\circ$ .

Number of observations at the receiver is 10 times. Number of transmit and receive antennas is 6. Type of modulation in the transmitter is QPSK. Based on the formula in paragraph 1, 2, we can simulate the SER along with signal to noise power ratio per one symbol, from 10 dB to 15 dB. Figure 2 compares the SER between forming beam patterns with equal and optimum power allocation. It is clear the SER is higher in the case of optimum power allocation.

If we just use information from 3 paths for forming beam patterns, the SER in this case is lower than 4 path's use. However, 4 paths need more one beam to transmit, that leads complex transmit beam strucrure, illustrated in figure 3. Figure 4 combines 4 cases of using 1 path meant the strongest beam [7], 2 paths, 3 paths and 4 paths. It is if using 1 path is give higher SER comparing other cases. Figure 5 summarises one case for 4 paths but equal power allocation, 4 other cases of using 1 path meant the strongest beam, 2 paths, 3 paths and 4 paths. Using 4 paths for beam patterning is effective compared remaining cases, however, we need more complex struture in transmitter, also in receiver, to create the beam patterns.

Contribution of the paper is firstly changing the mathematical MIMO model using CSI to a realistic mobile environment for MIMO. Secondly, concentration on spatial characteristics of the mobile channel to forming the beams and coresponding adaptive power allocation. Thirdly, this proves using more beams tracking on more paths is better in improving SER. The simulations also state if only using one path (the strongest beam [7]), the SER is higher much comparing the other case. Last but not least, adaptive power allocation gives lower SER than equal power.



**Fig. 2.** Comparison SER for 4 paths in case of optimum and equal power allocation



Fig. 3. Comparison of SER in using 3 paths and 4 paths



**Fig.4.** Comparison of SER between using 1, 2, 3, 4 paths forming beams

1,2,3,4 paths (optimum) and 4 paths (equal)

The SER in case of partial CSI

**Fig.5.** Comparison of SER between using 4 paths (equal power) and 1(the strongest beam), 2, 3, 4 paths forming beams (optimum power)

14 14.5 15

12 12.5 13 13.5

The average energy per noise power of one symbol-db

#### 5. Conclusion

10 10.5 11 11.5

The paper has applied the configuration of beam patterns in the multipath mobile environment, and allocate power to these beam patterns. This is normally formulated by the mathematical MIMO model in case mean or covariance channel matrix. The paper shows if more the physical transmission paths are used, the SER is more improved, *even using the strongest beam.* The SER is even lower when "water filling" power allocation is implemented at the transmitter comparing with the case of using equal power.

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