Robust Radio Direction Finding System Using Nested Antenna Array Based on Total Forward – Backward Matrix Pencil Algorithm

Han Trong Thanh**, Nguyen Duc Moi, Vu Van Yem
Hanoi University of Science and Technology – No. 1, Dai Co Viet Str., Hai Ba Trung, Ha Noi, Viet Nam
Received: March 27, 2018; Accepted: May 25, 2018

Abstract

The performance of radio direction finding systems mainly depends on kind of antenna array and signal processing algorithms. In this paper, a Robust radio direction finding system using Nested Antenna Array (NAA) based on Total Forward – Backward Matrix Pencil (TFBMP) method is proposed. By inheriting advantages of both NAA and TFPMP. Therefore, the proposed system can estimate more number of incoming signals than the number of antenna element with only one snapshot. This mean that system size and the sampling frequency in real time receivers can be considerably reduced. The simulation results for DOA estimation using proposed system will be assessed and analyzed to verify its performance.

Keywords: Direction of Arrival (DOA), Nested Antenna Array (NAA), TFBMP.

1. Introduction

Radio Direction Finding (DF) systems have many applications in Radio Navigation, Emergency Aid and intelligent operations, etc... The most important information that estimated by the Radio Direction Finding (DF) system is the Direction of Arrival (DOA) of the incoming signals. *

Thanks to technology development, the electronics and telecommunications devices are usually designed with the smaller size to improve flexible ability and mobility characteristic, especially in military area. In case of DF systems, the system's size mainly belongs to kind of antenna which is used. The most common antenna arrays are Uniform Linear Antenna Array (ULA), Uniform Circular Antenna Array (UCA), Rectangular Linear Antenna Array… They are often employed in DF systems because their simplicity and convenient mathematical model for array processing. However, with those antenna arrays, the number of incoming signals which can be estimated is always less than the number of antenna element. In order to determine the DOA of many more incoming signals, the number of antenna element will be increased. Therefore, the system's size is also significantly increased. To overcome this restriction, in [1-3], the authors proposed a Robust array structure called Nested Antenna Array (NAA). This is a variant of an ULA model which help the DF system can estimate more number of DOA than in case of using ULA model. This fact is due to vectorizing the covariance matrix of the received signals at each antenna element.

In [4-6], Matrix Pencil (MP) algorithm was applied to calculate the DOA information. The achieved results proved that it is can be considered as a high – resolution technique for DOA estimation. This algorithm directly processed the independent data samples. Therefore, it consumes less processing power and is faster executed than the other super – resolution methods for DOA estimation such as MUSIC [7], ESPRIT [8] which generally must calculate the signal covariance matrix. Furthermore, by using this algorithm, the DOA information can be extracted with only one snapshot. It is a remarkable advantage of this technique in comparison with other methods.

In [9-10], an extension of the Matrix Pencil Method named Total Forward – Backward Matrix Pencil (TFBMP) was proposed to accurately calculate the DOA information of the coherent incoming signals. The Total Forward – Backward is the pre – processing technique to break the correlative property of the received signals. This fact helps the Matrix Pencil method to estimate the DOA information of coherent incoming signals. Although TFBMP deals with a larger database, however it is more efficient than the original method, especially for a multipath environment. In [9], TFBMP was used for the high – resolution frequency estimator with the better estimation results than the other methods such as Fourier technique.

In this paper, a robust system using an M – element Nested Antenna Array based on the TFBMP technique to estimate the DOA information is

* Corresponding author: Tel.: (+84) 91.864.1368
Email: thanh.hantrong@hust.edu.vn

proposed. This system will take full advantage of both TFBMP and Nested Antenna Array. The performance of this method will be assessed in many cases that depend on the characteristics of incoming signals as well as antenna array properties.

The paper is organized as follows. Section 2 describes the structure of the NAA and the signal model. In section 3, TFBMP technique for DOAs of those signals is presented in detail. The simulation results are shown in the section 4. The conclusion is given in the section 5.

2. Nested Antenna Array Architecture

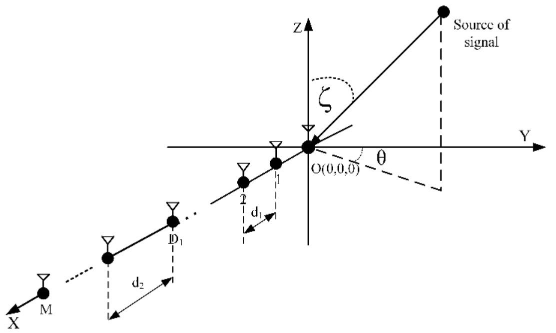

Fig. 1. Nested Antenna array in the coordinate system

In this research, an M – element Nested Antenna Array (NAA) which is a variant of ULA is utilized. Basically, NAA is composed by two ULAs that are hooked together. Two ULAs are called inner and outer array, respectively, in which the inner ULA includes N_1 antenna elements with spacing d_1 and outer ULA has N_2 elements with spacing $d_2 = (N_1 + 1)d_1$. The reference point is defined as the origin of the three-dimensional Cartesian coordinate system shown in Fig.1. Therefore, the position of antenna elements are $p = \{n_1 d_1,\ n_1 = 0,1,\dots,N_1 - 1\} \cup \{n_2 d_2 - d_1,\ n_2 = 1,2,\dots N_2\}$, respectively.

Assume that the incoming signal at the far field of the array impinging on the ULA has DOA information in both elevation (ξ) and azimuth (θ) as shown in Fig.1. However, in this work, only the signal in the same plane with antenna array is concerned. This means that the DOA of signal of interest is estimated in azimuth and $(\xi) = 90^o$.

The phase difference between the m^{th} antenna element and the reference point is:

$$\phi_m = \frac{2\pi}{\lambda} p_m d_1 sin(\theta), \quad (m = 0,1 \dots M-1) \tag{1}$$

where λ is the wavelength of incoming signal, p_m is the position of m^{th} antenna element in the coordinate system.

The phase response of incoming signal at each antenna element is:

$$a_m = g_m e^{j\phi_m} \tag{2}$$

where g_m is the gain of the m^{th} antenna element.

The baseband output at the m^{th} antenna can be modeled as:

$$x_m(t) = s(t)a_m = S(t)e^{j\frac{2\pi}{\lambda}p_m d_1 sin(\theta)} \tag{3}$$

where $s(t)$ is the incoming signal and $S(t) = s(t)g_m$.

In practice, the antenna array can receive several radio signals simultaneously. The received signal at each antenna element will be the sum of all arriving radio signals. In case of K signals from K directions $\theta_1, \theta_2 \dots \theta_K$, respectively, the received signal in AWGN channel at the m^{th} antenna is:

$$\begin{aligned} x_m(t) &= \textstyle\sum_{i=1}^{K} S_i(t) e^{j\beta p_m d_1 sin(\theta_i)} + \eta(t) \\ &= \textstyle\sum_{i=1}^{K} S_i(t)\alpha_i^m + \eta(t), \end{aligned} \tag{4}$$

where $\beta = \frac{2\pi}{\lambda}$ is the propagation factor, $\alpha_i = e^{j\beta p_m d_1 sin(\theta_i)}$ and η_m is Gaussian noise at each antenna element.

3. Total forward – backward matrix pencil method for doa estimation

According to Eq.2, the steering vector or manifold vector in each DOA – θ is defined as:

$$\begin{aligned} \boldsymbol{a}(\theta) &= \left[e^{j\phi_0}\ e^{j\phi_1} \dots\ e^{j\phi_{M-1}}\right]^T \\ &= \left[1\ \ e^{j\frac{2\pi}{\lambda}p_1 d_1 sin(\theta)} \dots\ e^{j\frac{2\pi}{\lambda}p_{M-1} d_1 sin(\theta)}\right]^T \end{aligned} \tag{5}$$

in which T denotes transpose matrix. It can be seen that the vector manifold of two level NAA does not have Vandemonde form. Therefore, the DOA information cannot be directly estimated using any investigated DOA estimation algorithms. To overcome this obstacle, Khatri-Rao Product [11] is used to convert the manifold of the two-level nested array into a form that is similar to the Vandermonde form of the ULA manifold. Firstly, the definitions of Matrices Product will be briefly discussed as follow

Definitions: Given two matrices $A_{m\times n}$ and $B_{p\times q}$

- The Kronecker product [11] of A and B is a mp rows and nq columns matrix.

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \dots & a_{1n}B \\ a_{21}B & a_{22}B & \dots & a_{2n}B \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1}B & a_{m2}B & \dots & a_{mn}B \end{bmatrix} \quad (6)$$

- Khatri – Rao Product of A and B is a mn rows and p columns matrix which is rewritten by the Kronecker product as the following

$$A \odot B = \left[a_1 \otimes b_1 | a_2 \otimes b_2 \dots a_p \otimes b_p\right]_{mn\times p} \quad (7)$$

where "$\otimes$" and "$\odot$" denote Kronecker and Khatri – Rao product and $a_1, a_2 \dots a_p$ and $b_1, b_2 \dots b_p$ are the columns of matrixes A and B, respectively.

Let us consider an array of M elements, with $\vec{d}_i$ denoting the position vector of the i^{th} element. Define the set

$$D = \{\vec{d}_i - \vec{d}_j\}, with\ i, j = 1 \div M \quad (8)$$

In the definition of the set D, the repetition of its elements is allowed. The set D_u which consists of some separate elements of the set D is also defined. Then, the difference co-array of the given array is defined as the array which has elements located at positions given by the set D_u. The number of elements this array directly decides the distinct values of the cross-correlation terms in the covariance matrix of the signal received by an antenna array.

The difference co-array of a two-level nested antenna array is a filled ULA array with $2N_2(N_1 + 1) - 1$ elements whose positions are given by the set P_{ca} defined as

$$P_{ca} = \{md_1, m = -M_{ca}, \dots, M_{ca}\ ; \\ M_{ca} = N_2(N_1 + 1) - 1\} \quad (9)$$

In case of the two-level nested array with $N1 + N2$ elements, the dimension of the virtual array manifold $A^* \odot A$ is $(N_1 + N_2)^2 \times K$, where $(*)$ denotes the complex conjugate matrix and K is the number of incoming signals. A new matrix $\tilde{A}$ of size $(2N_2(N_1 + 1) - 1) \times K$ is constructed by removing the repeat rows from $A^* \odot A$ (after their first occurrence) and also sorting them so that the i^{th} row corresponds to the element location $\{-M_{ca} + i\}$.

It can be seen that $\tilde{A}$ behaves like the manifold of a virtual ULA array (longer than original array) with $2N_2(N_1 + 1) - 1$ elements. The elements of this array has position given by the distinct values of set P_{ca}. This array is precisely the difference co-array of the original array.

As above analysis, instead of working with the original antenna array, the DOA information can be calculated by using the new virtual ULA array (ULA) with $\widetilde{M}$ elements

$$\widetilde{M} = 2M_{ca} + 1 = 2N_2(N_1 + 1) - 1 \quad (10)$$

The manifold vector as Eq.5 can be rewriten as

$$\widetilde{\boldsymbol{a}}(\theta) = \left[e^{j\phi_0}\ \ e^{j\phi_1} \dots\ \ e^{j\phi_{\widetilde{M}-1}}\right]^T \quad (11)$$

where $\phi_{\widetilde{m}} = \frac{2\pi}{\lambda}\widetilde{m}d_1 \sin(\theta)$ and $\widetilde{m} = -M_{ca} \div M_{ca}$

The discrete time output signal at m^{th} element now is

$$x_m = \sum_{i=1}^{K} A_i . e^{j\beta \widetilde{m}_m d_1 sin(\theta_i)} + \eta_m \\ = \sum_{i=1}^{K} A_i . \alpha_i^{\widetilde{m}_m} + \eta_m \quad (12)$$

where $m = 0, 1, 2 \dots \widetilde{M}$.

Base on TFBMPM, two matrices Y_{0fb} and Y_{1fb} are defined as:

$$Y_{0fb_{2(\widetilde{M}-L)\times L}} = \begin{bmatrix} z_0 & z_1 & \cdots & z_{L-2} & z_{L-1} \\ z_L^* & z_{L-1}^* & \cdots & z_2^* & z_1^* \end{bmatrix} \quad (13)$$

$$Y_{1fb_{2(\widetilde{M}-L)\times L}} = \begin{bmatrix} z_1 & z_2 & \cdots & z_{L-1} & z_L \\ z_{L-1}^* & z_{L-2}^* & \cdots & z_1^* & z_0^* \end{bmatrix} \quad (14)$$

where $z_\tau(\tau = 0, \dots, L)$ is defined as

$$z_j^T = [x_j\ \ x_{j+1}\ \dots\ x_{\widetilde{M}-L+j-1}]\ ;\ j = 0, \dots, L \quad (15)$$

and L is chosen as pencil parameter with the condition:

$$K \le L \le \widetilde{M} - K \quad \text{if } \widetilde{M} \text{ is even,} \\ K \le L \le \widetilde{M} - K + 1 \quad \text{if } \widetilde{M} \text{ is odd} \quad (16)$$

Based on Eq.13 and Eq.14, all data matrix is constructed as:

$$Y_{fb_{2(\widetilde{M}-L)\times(L+1)}} = \begin{bmatrix} z_0 & z_1 & \cdots & z_{L-1} & z_L \\ z_L^* & z_{L-1}^* & \cdots & z_1^* & z_0^* \end{bmatrix} \quad (17)$$

In order to estimate the DOA information, the Singular Value Decomposition (SVD) of this matrix will be performed:

$$Y_{fb_{2(\widetilde{M}-L)\times(L+1)}} = \\ U_{2(\widetilde{M}-L)\times 2(\widetilde{M}-L)} \Sigma_{2(\widetilde{M}-L)\times(L+1)} V^H_{(L+1)\times(L+1)} \quad (18)$$

where H denotes complex conjugate transpose of a matrix, U, Σ, and V are given by

$$\Sigma = diag\{\sigma_1, \sigma_2, \dots, \sigma_p\} \tag{19}$$

$$p = min\{2(\widetilde{M} - L), L+1\} \tag{20}$$

$$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_p \geq 0 \tag{21}$$

$$U = [u_1, u_2, \dots, u_{2(\widetilde{M}-L)}] \tag{22}$$

$$Y_{fb}^H u_i = \sigma_i v_i, i = 1, \dots, p \tag{23}$$

$$V = [v_1, v_2, \dots, v_{(L+1)}] \tag{24}$$

$$Y_{fb}^H v_i = \sigma_i u_i, i = 1, \dots, p \tag{25}$$

$$U^H U = I, \; V^H V = I \tag{26}$$

σ_i are the singular values of Y_{fb} and the vector u_i and v_i are the i^{th} left and right singular vector, respectively. In the next step, the K largest singular values of Y_{fb} can be achieved by using the singular value filtering.

$$\bar{Y}_{fb_{2(\widetilde{M}-L)\times(L+1)}} = \bar{U}_{2(\widetilde{M}-L)\times K}\bar{\Sigma}_{K\times K}\bar{V}_{K\times(L+1)}^H \tag{27}$$

where

$$\bar{\Sigma} = diag\{\sigma_1, \sigma_2, \dots, \sigma_K\} \tag{28}$$

has K largest singular values of Σ, and the matrices $\bar{U}$ and $\bar{V}$ is K-truncation of V:

$$\bar{V} = [\bar{V}_0, v_{L+1}], \; \bar{V} = [v_1, \bar{V}_1] \tag{29}$$

Similar to Eq.27, $\bar{Y}_{0fb}$ and $\bar{Y}_{1fb}$ are obtained as

$$\bar{Y}_{0fb} = \bar{U}\bar{\Sigma}\bar{V}_0^H, \; \bar{Y}_{1fb} = \bar{U}\bar{\Sigma}\bar{V}_1^H \tag{30}$$

Base on above equations, the matrix pencil can be established as

$$MP = \bar{Y}_{1fb} - z\bar{Y}_{0fb} \tag{31}$$

Left multiplying MP by $\bar{Y}_{0fb}^+$ yields

$$q^H(\bar{Y}_{1fb}\bar{Y}_{0fb}^+ - zI) = 0^H \tag{32}$$

where $\bar{Y}_{0fb}^+$ is the Moore-Penrose pseudo inverse of Y_{0fb}

$$\bar{Y}_{0fb}^+ = (\bar{V}_0^H)^+\bar{\Sigma}^{-1}\bar{U}^+ \tag{33}$$

Substituting Eq.30 and Eq.33 into Eq.32 the equivalent generalized Eigen-problem becomes

$$q^H(\bar{V}_1^H - z\bar{V}_0^H) = 0^H \tag{34}$$

By left multiplying by $\bar{V}_0$, Eq.34 becomes

$$q^H(\bar{V}_1^H\bar{V}_0 - z\bar{V}_0^H\bar{V}_0) = 0^H \tag{35}$$

Using the values of the generalized eigenvalues, z, of Eq.35, DOA information of incoming signal can be numerical calculated as

$$\theta_i = sin^{-1}[\frac{\Im[\ln(z_i)]}{\beta d_1}] \tag{36}$$

where $\Im\,[ln(z_i)]$ is the imaginary part $ln(z_i)$.

4. Simulation results

The performance of the proposed approach is examined by simulation using Matlab. This work is divided into many cases depending on antenna's structure and characteristic of incoming signal. In all simulation, the number of antenna element can be varied. However, the distance between two elements in succession of inner antenna array $d_1 = 0.3\lambda$ is constant. This supposition is to guarantee the acceptable mutual coupling factor between antenna elements. Moreover, in order to evaluate the accuracy of the simulation, the Root Mean Square Error (RMSE) is used. This parameter is defined as

$$RMSE = \sqrt{\frac{\sum_{i=1}^{K}(x_i - x_i')^2}{K}} \tag{37}$$

where x_i is the expected value and x_i' is the estimated value of measurement object i^{th} and K is the number of measurement objects.

In the first simulation, assuming that there are 8 signals imping on a 6 – element antenna array ($M = 16$). The DOAs are $-80, -30, -10, 0, 10, 45, 60$ and 85 in degrees. The simulation result is plotted in Fig.2. It has to be noticed that the estimated DOAs in the simulation are the numerical values calculated by Eq.36 in Section 3. However, in order to demonstrate visually the result, it is illustrated in 2 – dimension Cartesian coordinate system, in which the X – Axis is the DOA of incoming signals and the Y – Axis is indicating factor. This factor is set to 1 corresponding to the estimated DOA. Obviously, the proposed system has accurately estimated the DOA information of 8 incoming signals while there are 6 antenna elements. This fact cannot be done by using 6 element ULA arrays with the same algorithm. Moreover, by using TFBMP, the DOA information can be calculated with only one snapshot. This is a significant advantage of TFPMP in comparison with other high resolution algorithms such as MUSIC. This issue helps to reduce considerably the sampling frequency as well as the amount of processing data

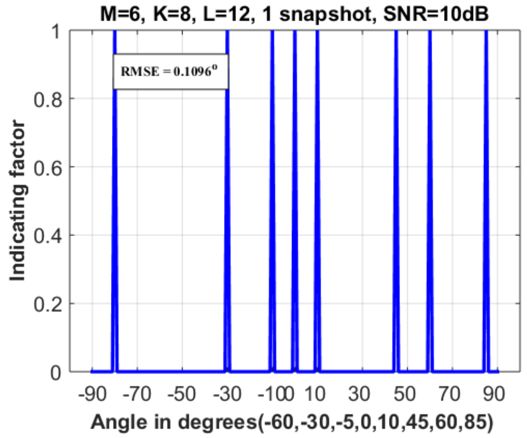

Fig.2. DOA estimation with NAA with one snapshot

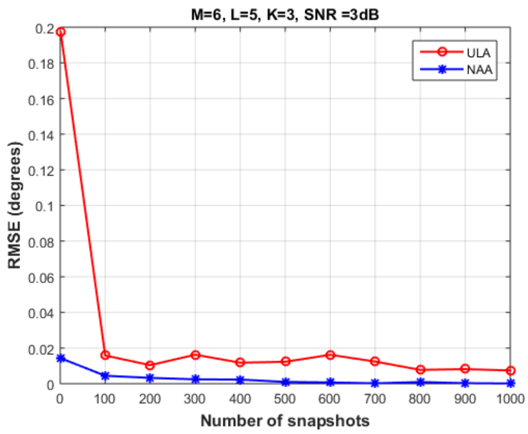

Fig.3. Accuracy comparison between ULA and NAA

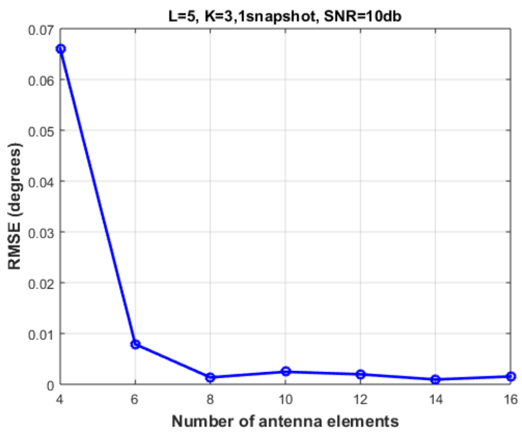

Fig.4. Impact of number of antenna element on accuracy of DOA estimation

The second simulation is executed to compare the performance of ULA and NAA using TFPMP with the same number of antenna element ($M = 6$) and 3 incoming signals in AWGN with $SNR = 3dB$, while the number of snapshots is varied. The result presented in Fig.3 shows that NAA works better than ULA in the same situation. It can be explained that although the number of antenna element in practical is $M = 6$, but after applying $Kr - product$, the virtual antenna is generated with 23 elements. However, with more antenna elements, the NAA needs more time to estimate DOA than ULA. Therefore, the trade-off between the computation time and the accuracy of the algorithm could be taken into account

Moreover, the result plotted in Fig.3 also proclaim that the RMSE will decrease in proportion to the increasing of number of snapshots. This relationship is suitable for statistical characteristic of data.

The number of antenna element also impacts to the performance of proposed system. In this case, it is assumed that there are 3 incoming signals in AWGN channel with $SNR = 10dB$ imping on the array. The simulation result shown in Fig.4 indicates that if the number of antenna element increases, the accuracy in DOA estimation will be increased. However, it can be seen that when the number of element is more than 8, the accuracy of DOA estimation varies insignificantly. It means that the number of antenna element should be chosen to satisfy both minimizing system size and DOA estimation accuracy.

5. Conclusions

In this paper, a Robust DF system using Total Forward Backward Matrix Pencil method with Nested Antenna Array is proposed. This system has some advatages in comparison with other DF systems which uses other DOA algorithms and popular kind of antenna arrays. By using NAA, the proposed system can estimate DOA information of more sources than the number of antenna elements. Moreover, with TFBMP method, DOA information is extracted with only one snapshot. Therefore, the computational complexity and size of the DF system can be reduced significantly and the proposed system can be implemented in practical.

Acknowledgments

This research is carried out in the framework of the project funded by the Hanoi University of Science and Technology (HUST), Vietnam with the title “Research on Wideband DOA estimation algorithms for advance Radio Direction Finding System” under the grant number **T2017-PC-113**. The authors would like to thank HUST for their financial support.

References

[1] Pal, Piya, and P. P. Vaidyanathan. "Nested arrays: A novel approach to array processing with enhanced degrees of freedom." IEEE Transactions on Signal Processing 58.8 (2010), pp. 4167-4181.

[2] IIZUKA, Yuki; ICHIGE, Koichi. "Extension of two-level nested array with larger aperture and more degrees of freedom", in IEEE International Symposium on Antennas and Propagation (ISAP) (2016). p. 442-443.

[3] QI, Han; JIANG, Hong; ZHOU, Erning. "Multi-target direction finding in MIMO radar exploiting nested array. In: Natural Computation," in 12th IEEE International Conference on Fuzzy Systems and Knowledge Discovery (ICNC-FSKD), (2016). p. 1904-1909.

[4] Y. Hua and T. Sarkar, "Matrix pencil method for estimating parameters of exponentially damped/undamped sinusoids in noise," IEEE Transaction on Acoustics, Speech, and Signal Processing, 38(5)(1990) 814–824.

[5] R. S. Adve, T. K. Sarkar, O. M. Pereira-Filho, and S. M. Rao, "Extrapolation of time-domain responses from three-dimensional conducting objects utilizing the matrix pencil technique," IEEE Transactions on Antennas and Propagation, 45(1)(1997) 147-156.

[6] N. Dharamdial, R. Adve, and R. Farha, "Multipath delay estimations using matrix pencil," in Proc. Wireless Communications and Networking Conference (WCNC), 1(2003) 632-635.

[7] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," IEEE Transactions on Antennas and Propagation, 34(3)(1986) 276-280.

[8] Ottersten B. and Kailath T. "Direction-of-arrival estimation for wide-band signals using the ESPRIT algorithm," IEEE Transactions on Acoustics, Speech and Signal Processing, vol. 38 (1990), pp. 317-327.

[9] J. E. F. del Rio and T. K. Sarkar, "Comparison between the matrix pencil method and the fourier transform technique for high-resolution spectral estimation," in Digital Signal Processing, 6(11)(1996) 108–125.

[10] Sales, Kirk L. Reducing the Number of Ultrasound Array Elements with the Matrix Pencil Method. Michigan State University. Electrical Engineering, 2012.

[11] Liu, Shuangzhe, and Gõtz Trenkler. "Hadamard, Khatri-Rao, Kronecker and other matrix products." Int. J. Inf. Syst. Sci 4.1 (2008), pp. 160-177.