

A New Decoupling Controller for Multi-Time Delayed TITO Processes

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Abstract

Multiple-input–multiple-output systems (MIMO) are very popular in the industry, such as distillation column plant, liquid dispensers, infant incubators, quadruple tank... One of the difficulties to handle them is how to eliminate interactions between variables. The decoupling control is a research field related to solving this problem, which has been studied by many authors in the past. This paper addresses to propose a new decoupling control method applied for two input two output processes (TITO) with multi-time delays. Simulation results show that the performance of the system is improved in the sense, that no longer interfering between the loops is occurred as well as the process output disturbances had been completely eliminated.

Keywords: MIMO, TITO, Decoupling control, Plant with multi-time delays

1. Introduction

Time delays are often met in industry. They cause the reducing stability and limiting achievable response time of systems. Moreover, each input-output loop in a multivariable process has usually a different time delay. And therefore, in a particular loop its output could be affected by all the inputs through possibly different time delays [1].

For handling these effects of system delays on its performance there are a huge number of methods presented in the past such as the Smith predictor (SP) for SISO processes presented in [2] and some extensions of it for MIMO systems introduced in [2, 3]. Nevertheless, all these approaches require additionally a few assumptions of controlled MIMO processes, which unfortunately could not be always satisfied.

Therefore a trend of replacing MIMO time delayed processes by many unconnectedly parallel SISO systems and then applying SP for these SISO systems is preferred and desirable [4-7]. The main advantage of this trend is that it allows to deal with single loop control instead of multivariable control. However, since the controller contains also process model in it, the obtained control property of closed loop process is very sensible with model errors.

Decoupling control of multi-time delayed MIMO processes is an approach, which seems to be very close to this trend, but more robust with model errors. The

main purpose of decoupling control is to simplify the controller design afterward in the sense that firstly the time delayed MIMO process is decoupled and then secondly, single PID controllers are tuned for each output/input pair in it [8-13].

As an addition to solving the decoupling control problem, this paper will present a new decoupling controller for multi-time delayed TITO processes.

The organization of this paper is as follows. First the main content of proposed decoupling controller will be given in Section 2. Then Session 3 presents an application example of it to decouple the infant incubator. Finally, some concluding remarks are given in Section 4.

2. Decoupling controller design

Consider an interacted multi-time delayed TITO process described by the following transfer matrix:

$$\mathbf{S} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \quad (1)$$

where

$$S_{ij} = \frac{k_{ij}}{1 + T_{ij}s} e^{-\tau_{ij}s}, \quad i, j = 1, 2 \quad (2)$$

and $k_{ij}, T_{ij} > 0, \tau_{ij} > 0$ denote the steady gains, the time constants and the delayed times of the process respectively.

The aim of decoupling problem here is an open loop controller:

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$$\mathbf{R} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \quad (3)$$

for the interacting process (1) to design, so that the whole system becomes unconnectedly:

$$\begin{aligned} \mathbf{G} = \mathbf{R}\mathbf{S} &= \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \\ &= \begin{pmatrix} R_{11}S_{11} + R_{12}S_{21} & R_{11}S_{12} + R_{12}S_{22} \\ R_{21}S_{11} + R_{22}S_{21} & R_{21}S_{12} + R_{22}S_{22} \end{pmatrix} \\ &= \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix}. \end{aligned} \quad (4)$$

It means that with this open loop decoupling controller (3) all interacting behaviors of the process will be eliminated. Fig.1 illustrates visually this purpose of the decoupling control problem.

Based on Eq. (4) given above the purpose of decoupling control could be exposed that all elements R_{ij} of decoupling controller \mathbf{R} are to be determined, so that they satisfy:

$$\begin{cases} R_{11}S_{12} + R_{12}S_{22} = 0 \\ R_{21}S_{11} + R_{22}S_{21} = 0. \end{cases} \quad (5)$$

Suppose that all elements R_{ij} of decoupling controller (3) are time delayed lead/lags as follows:

$$\begin{aligned} R_{11} &= \frac{c_{11}(1+T_{12}s)}{1+T_{11}'s} e^{-\sigma_{11}s}, & R_{12} &= \frac{c_{12}(1+T_{22}s)}{1+T_{12}'s} e^{-\sigma_{12}s}, \\ R_{21} &= \frac{c_{21}(1+T_{11}s)}{1+T_{21}'s} e^{-\sigma_{21}s}, & R_{22} &= \frac{c_{22}(1+T_{21}s)}{1+T_{22}'s} e^{-\sigma_{22}s} \end{aligned}$$

then the purpose (5) above of decoupling control problem will be converted accordingly into the determination of their coefficients:

$$c_{ij}, T_{ij}', \sigma_{ij} \text{ for all } i, j = 1, 2$$

so that they satisfy:

$$\begin{cases} \frac{c_{11}k_{12}(1+T_{12}s)}{(1+T_{11}'s)(1+T_{12}s)} e^{-(\sigma_{11}+\tau_{12})s} + \\ \quad + \frac{c_{12}k_{22}(1+T_{22}s)}{(1+T_{12}'s)(1+T_{22}s)} e^{-(\sigma_{12}+\tau_{22})s} = 0 \\ \frac{c_{21}k_{11}(1+T_{11}s)}{(1+T_{21}'s)(1+T_{11}s)} e^{-(\sigma_{21}+\tau_{11})s} + \\ \quad + \frac{c_{22}k_{21}(1+T_{21}s)}{(1+T_{22}'s)(1+T_{21}s)} e^{-(\sigma_{22}+\tau_{21})s} = 0. \end{cases} \quad (6)$$

By signifying:

$$\begin{cases} \sigma_{11} + \tau_{12} = \sigma_{12} + \tau_{22} = \tau_1 \\ \sigma_{21} + \tau_{11} = \sigma_{22} + \tau_{21} = \tau_2 \end{cases}$$

where τ_1, τ_2 are chosen so that σ_{ij} become positive for all $i, j = 1, 2$, for example by choosing arbitrarily:

$$\tau_2 \geq \max\{\tau_{11}, \tau_{21}\} \text{ and } \tau_1 \geq \max\{\tau_{12}, \tau_{22}\}, \quad (7)$$

the first coefficients $\sigma_{ij}, i, j = 1, 2$ of controller \mathbf{R} to be designed will be obtained instantly as follows:

$$\begin{cases} \sigma_{11} = \tau_1 - \tau_{12} \text{ and } \sigma_{12} = \tau_1 - \tau_{22} \\ \sigma_{21} = \tau_2 - \tau_{11} \text{ and } \sigma_{22} = \tau_2 - \tau_{21}. \end{cases} \quad (8)$$

Subsequently, together with Eqs. (7) and (8), the required decoupling conditions (6) are changed to:

$$\frac{c_{11}k_{12}}{1+T_{11}'s} + \frac{c_{12}k_{22}}{1+T_{12}'s} = 0 \text{ and } \frac{c_{21}k_{11}}{1+T_{21}'s} + \frac{c_{22}k_{21}}{1+T_{22}'s} = 0. \quad (9)$$

These relevant conditions will be used hereafter to determine the remaining coefficients c_{ij}, T_{ij}' . Obviously the conditions (9) are equivalent to:

$$\begin{cases} c_{12}k_{22}(1+T_{11}'s) + c_{11}k_{12}(1+T_{12}'s) = 0, \forall s \\ c_{22}k_{21}(1+T_{21}'s) + c_{21}k_{11}(1+T_{22}'s) = 0, \forall s \end{cases}$$

which is coincidental with:

$$\begin{cases} c_{12}k_{22} + c_{11}k_{12} = 0 \\ c_{12}k_{22}T_{11}' + c_{11}k_{12}T_{12}' = 0 \\ c_{22}k_{21} + c_{21}k_{11} = 0 \\ c_{22}k_{21}T_{21}' + c_{21}k_{11}T_{22}' = 0. \end{cases} \quad (10)$$

Therefore, by choosing arbitrarily two positive time constants T_1', T_2' and then designating:

$$T_{11}' = T_{12}' = T_1' \text{ and } T_{21}' = T_{22}' = T_2' \quad (11)$$

the conditions (10) will be rewritten shortly in:

$$\begin{cases} c_{12}k_{22} + c_{11}k_{12} = 0 \text{ and} \\ c_{22}k_{21} + c_{21}k_{11} = 0. \end{cases} \quad (12)$$

Afterward, these reduced conditions (12) will be used to guide for determination of the last four coefficients $c_{ij}, i, j = 1, 2$. Simply it could be realized through two following steps. Firstly choose arbitrary two positive constants c_1, c_2 and designate them accordingly with:

$$c_{11} = c_1, c_{22} = c_2, \quad (13)$$

then, secondly, the last two remaining coefficient c_{12}, c_{21} are obtained immediately from Eq. (12):

$$\begin{cases} c_{12} = -c_1 k_{12} / k_{22} \text{ and} \\ c_{21} = -c_2 k_{21} / k_{11}. \end{cases} \quad (14)$$

The following procedure exhibits all design steps of decoupling controller \mathbf{R} , which is obtained by summarizing completely all calculations given above.

Design procedure

1. Choose two parameters τ_1, τ_2 accordingly to Eq. (7) and determine $\sigma_{ij}, i, j = 1, 2$ with (8).
2. Choose arbitrarily $T_1' > 0, T_2' > 0$ and determine $T_{ij}', i, j = 1, 2$ with (11).
3. Choose arbitrarily $c_1 > 0, c_2 > 0$ and determine $c_{ij}, i, j = 1, 2$ accordingly to Eqs. (13), (14).

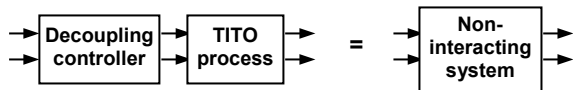


Fig.1. Purpose of decoupling control

The controller \mathbf{R} obtained by using this design procedure will decouple the interacted process given in (1) in the sense of (4) with:

$$\begin{aligned} G_1 &= R_{11}S_{11} + R_{12}S_{21} \\ &= \frac{c_1 k_{11}(1 + T_{12}s)}{(1 + T_1's)(1 + T_{11}s)} e^{-(\tau_1 - \tau_{12} + \tau_{11})s} \\ &\quad + \frac{-c_1 k_{12} k_{21}(1 + T_{22}s)}{k_{22}(1 + T_1's)(1 + T_{21}s)} e^{-(\tau_1 - \tau_{22} + \tau_{21})s} \end{aligned} \quad (15)$$

and

$$\begin{aligned} G_2 &= R_{21}S_{12} + R_{22}S_{22} \\ &= \frac{-c_2 k_{21} k_{12}(1 + T_{11}s)}{k_{11}(1 + T_2's)(1 + T_{12}s)} e^{-(\tau_2 - \tau_{11} + \tau_{12})s} \\ &\quad + \frac{c_2 k_{22}(1 + T_{21}s)}{(1 + T_2's)(1 + T_{22}s)} e^{-(\tau_2 - \tau_{21} + \tau_{22})s} \end{aligned} \quad (16)$$

Furthermore, since the decoupling controller \mathbf{R} contains arbitrarily chosen parameters $\tau_1, \tau_2, T_1', T_2'$ and c_1, c_2 , it arises here some opportunities to select them for improving additionally the performance of decoupled system \mathbf{G} . For example, based on both Eqs. (15) and (16) it is easy to recognize that to avoid the overshoot occurred in separate loops G_1, G_2 , two

parameters T_1', T_2' should be so selected that they satisfy additionally:

$$T_1' > \max\{T_{12}, T_{22}\} \text{ and } T_2' > \max\{T_{11}, T_{21}\}. \quad (17)$$

Finally, because the choice of τ_1, τ_2 is based on the inequality (7), this design algorithm could be also applicable to processes (1) with all bounded uncertain time delays $\tau_{ij}, i, j = 1, 2$ contained in their elements, which are exhibited in (2).

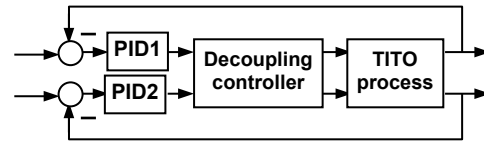


Fig.2. Control interacting process with two separate PIDs.

After decoupling, it is clearly that all available SISO control methods could be now applied separately for G_1 and G_2 in \mathbf{G} . For example, two separate PID controllers could be used discretely to control the whole system \mathbf{G} as illustrated in Fig.2.

3. Illustrated simulation

To investigate experimentally the performance of proposed decoupling controller it will be hereafter demonstrated on the incubator presented in [14] as an interacting TITO process \mathbf{S} . This TITO multi-time delayed process has the transfer matrix given in Eq. (1) with following particular elements:

$$\begin{aligned} S_{11} &= \frac{0.58}{1 + 1796s} e^{-90s}, \quad S_{21} = \frac{-0.4}{1 + 1100s} e^{-92s}, \\ S_{12} &= \frac{5.56 \cdot 10^{-4}}{1 + 255s} e^{-0.18s}, \quad S_{22} = \frac{1.67 \cdot 10^{-3}}{1 + 45s} e^{-0.02s}. \end{aligned} \quad (18)$$

Using the proposed procedure above for designing a decoupling controller described by transfer matrix \mathbf{R} given in Eq. (3), it will be obtained particularly:

$$\begin{aligned} R_{11} &= \frac{0.00167(1 + 255s)}{1 + 300s} e^{-91.82s}, \\ R_{12} &= \frac{-0.000556(1 + 45s)}{1 + 300s} e^{-91.98s}, \\ R_{21} &= \frac{0.4(1 + 1796s)}{1 + 1800s} e^{-2s} \\ \text{and } R_{22} &= \frac{0.58(1 + 1100s)}{1 + 1800s}. \end{aligned} \quad (19)$$

Fig.3 represents the detailed execution of decoupling structure given in Fig.2 on Matlab Simulink, where the multi-time delayed TITO process,

the decoupling controller and two PID controllers are respectively the transfer matrix S fixed in Eq. (18), the transfer matrix R specified in Eq. (19) and:

$$\text{PID1: } 2400 + 1.38/s, \quad \text{PID2: } 2703 + 56.54/s$$

which are achieved by using the PID tuning tool of Matlab for G_1 and G_2 separately, where their initial values have been determined first by using Ziegler-Nichols method.

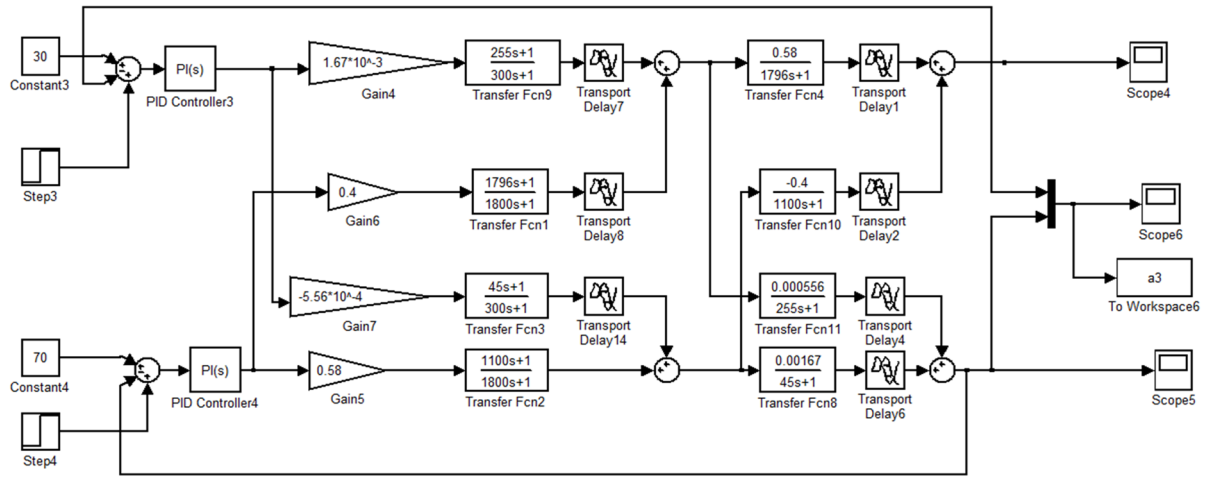


Fig. 3. Matlab demonstrated simulation for decoupling control.

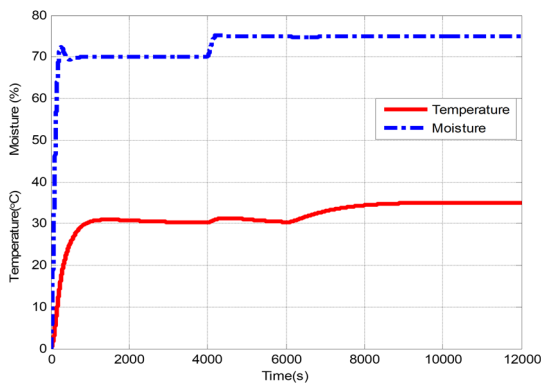


Fig. 4. Step response without decoupling

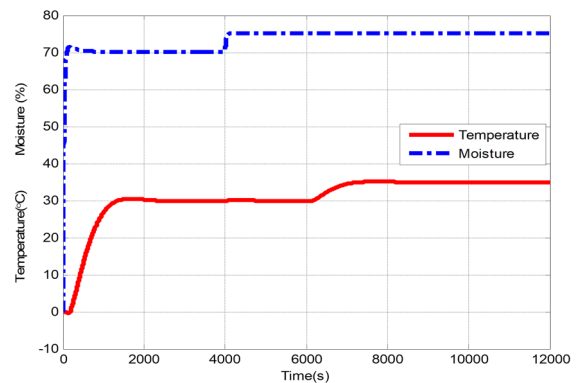


Fig. 5. Step response with decoupling controller

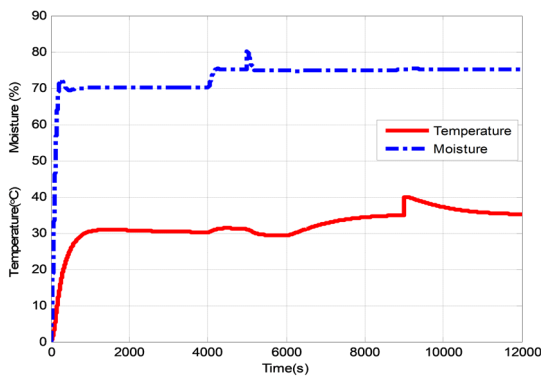


Fig. 6. Influence of the process disturbance in the non-decoupling system

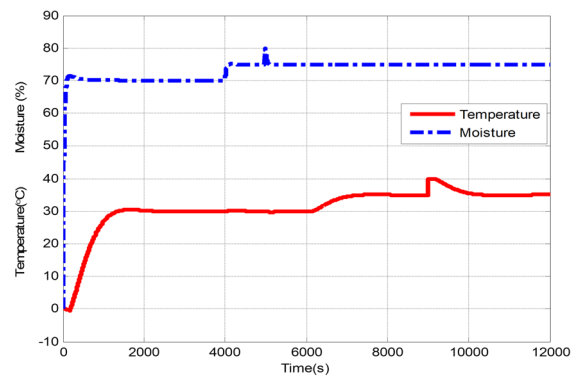


Fig. 7. Influence of the process disturbance in the decoupling system

As shown in obtained simulation results displayed in Fig.4-Fig.7, the closed loop system has a settling time of 1270s and 88s for temperature control loop as well as for moisture loop, respectively.

The simulation results for non-disturbed case are presented in Fig.4 and Fig.5. Whereas Fig.4 shows the result without using decoupling controller \mathbf{R} , the simulation result by using \mathbf{R} is exhibited in Fig.5. It is recognizable there that the proposed decoupling controller had eliminated completely all interacting behaviors inside the process \mathbf{S} .

Fig.6 and Fig.7 show the simulation results again for both circumstances, without and with decoupling controller \mathbf{R} , but now in the presence of input disturbance on the first loop G_1 . Once again the brilliant decoupling performance of proposed controller \mathbf{R} has been seen there, that this disturbance does not affect on the other loop, i.e. on G_2 . Hence the proposed open loop controller \mathbf{R} has met entirely the perfect decoupling performance for interacting controlled systems as expecting.

4. Conclusions

In this work a design method of open loop controller \mathbf{R} for decoupling multi-time delayed TITO processes is proposed, which does not require any assumption about the controlled objects as usual by other methods. Additionally, since the proposed decoupling algorithm contains in it some arbitrarily chosen factors, it provides therefore in principle many decoupling controllers for a certain TITO process.

Finally, the simulation results obtained there have confirmed once more that the decoupling controller attained by using this method has provided an excellent decoupling performance as expecting and therefore it could be now deployed in practice.

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