Active Disturbance Rejection Based Approach for Velocity Control of a Three-Mass System

Do Trong Hieu*, Nguyen Duy Vinh*, Nguyen Tung Lam

Hanoi University of Science and Technology, No. 1, Dai Co Viet, Hai Ba Trung, Hanoi, Viet Nam

Received: March 01, 2019; Accepted: June 24, 2019

Abstract

The paper deals with a velocity control problem of a three-mass system. The equations of motion of the system with limited shaft stiffness and damping is derived via d'Alembert principle. Based on the system dynamics, an active disturbance rejection control is developed for the system via a support of an extended state observer. The designed process with systematic and simple approach shows better performances compared to PID control. Several numerical simulation scenarios are carried out to verify the robustness of the control.

Keywords: Three-mass system, active disturbance rejection, extended state observer, vibration suppression.

1. Introduction

Guaranteeing motion performance in drive systems is always a challenging task for design and control engineers. Normally treated as a lumped-mass system, finite stiffness, viscous and damping effects of the transmission shaft greatly affect the system motion quality [1]. Research works on multi-mass system can be extensively found in the literature. In order to tackle resonances and varying rotational inertia, a filter and an adaptive speed control is developed in [2]. In driving systems, a three-mass system which represents driving, coupling, and load inertia can be considered as a fundamental problem and can be expanded to multi-mass system without loss of generality. An extensive control design comparison can be found in [3], where control performances of an electrical drive system with elastic coupling when using PI, predictive speed, and cascade control backed by force dynamics controls. It is shown that PI control give worst performance when facing system parameter changing. Similar idea of using force dynamics control in position control of two-mass system with speed sensor located at the motor side is found in [4]. In multi-mass system, it is challenging to gather all system parameter information, due to the limitation, a system identification is presented in [5], however, the accuracy of the proposed method is heavily depended on system identification setup process and excitation signal. Other control approaches to multi-mass systems such as model predictive control, backstepping control and fuzzy control are examined in [6, 7] and [8], respectively. Several researches dedicate to attenuate backlash affects in multi-mass system [3]. Although many strategies have been proposed, robustness and other practical concerns continue to pose challenges.

In recent years, Active Disturbance Rejection Control (ADRC) is interested in to replace the traditional PID controller. This concept was originally proposed by J. Han [9, 10] but only become transparent to application engineers since a new parameter tuning method is proposed in [11]. This control method shows several advantages for disturbance rejection and for process with inaccurate parameters. ADRC is a powerful control method where system models are expanded with a new state variable, including all unknown kinetic and disturbance, that commonly happens in system formulation. The new state is estimated by using the Extended State Observer (ESO). An application of ADRC for rigid coupling motion control system can be found in [12]. In [13, 14], the authors referred to decoupling control for multivariable system using ADRC. An ADRC based solution for resonance suppression in motion control of two-inertia systems is proposed in [15]. These studies show the advantages and potential of ADRC approach in system control.

In general, the control of the three-mass system requires speed feedback at the input of the third inertia, practically, speed sensor is deployed to measure driving inertia speed. Based on this configuration, the paper develops a velocity tracking performance based on active disturbance rejection control backed with extended state observer. The

* Corresponding author: Tel.: (+84) 949.910.429
Email: hieu.dotrong@hust.edu.vn
nguyenduyvinh1993@hotmail.com

proposed control structure is simple and practical. The tracking result shows highly improved performances compared to classical PID control. This paper is structured as follow. The three-mass system modeling is presented in section 2. In section 3, we present the velocity control design of three-mass system based on ADRC approach as well as the parameters tuning procedure of the ADRC. Subsequently, some simulation results are given, followed by several concluding remarks in section 4.

2. System Modeling

Consider a mechanical system as shown in Fig 1 with an assumption of ignoring backlash, friction and elascity of the system. This three-mass system consists of three rigid bodies with moment of inertia J_1, J_2, J_3 and two flexible connected shafts with coefficients of elasticity k_1, k_2 and the damping coefficients b_1, b_2. m_e is the input torque of the electric motor and m_L is the load torque of the working machine. θ_i (i=1,2,3) is angular position.

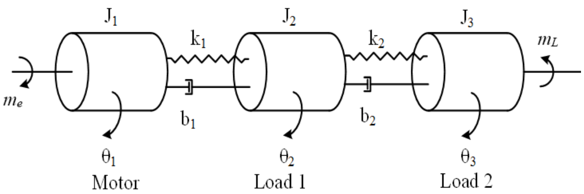

Fig. 1. Three- mass system with flexible connection.

Using the principle of d'Alembert, the equations of motion for the three-mass model are as follow [16]:

$$\begin{cases} J_1.\ddot{\theta}_1 + b_1(\dot{\theta}_1 - \dot{\theta}_2) + k_1(\theta_1 - \theta_2) = m_e \\ J_2\ddot{\theta}_2 + b_2(\dot{\theta}_2 - \dot{\theta}_3) + k_2(\theta_2 - \theta_3) - b_1(\dot{\theta}_1 - \dot{\theta}_2) - k_1(\theta_1 - \theta_2) = 0 \\ J_3\ddot{\theta}_3 - b_2(\dot{\theta}_2 - \dot{\theta}_3) - k_2(\theta_2 - \theta_3) = -m_L \end{cases} \tag{1}$$

With

$$m_{si} = k_i(\theta_i - \theta_{i+1}) \tag{2}$$

the equations of motion in (1) become:

$$\begin{cases} J_1.\ddot{\theta}_1 + b_1(\omega_1 - \omega_2) + m_{s1} = m_e \\ J_2\ddot{\theta}_2 + b_2(\omega_2 - \omega_3) + m_{s2} - b_1(\omega_1 - \omega_2) - m_{s1} = 0 \\ J_3\ddot{\theta}_3 - b_2(\dot{\theta}_2 - \dot{\theta}_3) - m_{s2} = -m_L \\ \dot{m}_{s1} = k_1(\omega_1 - \omega_2) \\ \dot{m}_{s2} = k_2(\omega_2 - \omega_3) \end{cases} \tag{3}$$

A simple calculation shows that

$$\Rightarrow \begin{cases} \dot{\omega}_1 = \dfrac{m_e - b_1(\omega_1 - \omega_2) - m_{s1}}{J_1} \\ \dot{\omega}_2 = \dfrac{-b_2(\omega_2 - \omega_3) - m_{s2} + b_1(\omega_1 - \omega_2) + m_{s1}}{J_2} \\ \dot{\omega}_3 = \dfrac{-m_L + b_2(\dot{\theta}_2 - \dot{\theta}_3) + m_{s2}}{J_3} \\ \dot{m}_{s1} = k_1(\omega_1 - \omega_2) \\ \dot{m}_{s2} = k_2(\omega_2 - \omega_3) \end{cases}$$

where ω_1 is angular speed of motor, ω_2 and ω_3 are angular speed of load 1 and load 2.

It should be noted that when $b_1 = b_2 = 0$, the equations in (3) will be the model of three-inertia system studied in [17, 18]. In this paper we will consider the general case where $b_i \neq 0$.

The three-mass system can be also described in the state space form as:

$$\begin{cases} \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \\ \dot{m}_{s1} \\ \dot{m}_{s2} \end{bmatrix} = \begin{bmatrix} -\frac{b_1}{J_1} & \frac{b_1}{J_1} & 0 & -\frac{1}{J_1} & 0 \\ \frac{b_1}{J_2} & \left(-\frac{b_2}{J_2} - \frac{b_1}{J_2}\right) & \frac{b_2}{J_2} & \frac{1}{J_2} & -\frac{1}{J_2} \\ 0 & \frac{b_2}{J_3} & -\frac{b_2}{J_3} & 0 & \frac{1}{J_3} \\ k_1 & -k_1 & 0 & 0 & 0 \\ 0 & k_2 & -k_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ m_{s1} \\ m_{s2} \end{bmatrix} + \begin{bmatrix} \frac{1}{J_1} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{J_3} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} m_e \\ m_L \end{bmatrix} \\ \underline{y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ m_{s1} \\ m_{s2} \end{bmatrix} \end{cases} \tag{4}$$

From the state space equation, we have the following transfer functions that characterize the relationship between input torque and speed:

$$\frac{\omega_1(s)}{m_e(s)} = \frac{c_o s^4 + c_1 s^3 + c_2 s^2 + c_3 s + c_4}{s^5 + d_1 s^4 + d_2 s^3 + d_3 s^2 + d_4 s}$$

$$\frac{\omega_3(s)}{m_e(s)} = \frac{c_0^{'}.s^2 + c_1^{'}.s + c_2^{'}}{s^5 + d_1.s^4 + d_2.s^3 + d_3.s^2 + d_4.s}$$

With:

$$c_o = \frac{1}{J_1} \qquad c_1 = \frac{J_2 b_2 + J_3 b_1 + J_3 b_2}{J_1 J_2 J_3}$$

$$c_2 = \frac{J_2 k_2 + J_3 k_1 + J_3 k_2 + b_1 b_2}{J_1 J_2 J_3} \qquad c_3 = \frac{b_1 k_2 + b_2 k_1}{J_1 J_2 J_3}$$

$$c_4 = \frac{k_1 k_2}{J_1 J_2 J_3}$$

$$d_1 = \frac{J_2J_3b_1 + J_1J_2b_2 + J_1J_3b_1 + J_1J_3b_2}{J_1J_2J_3}$$

$$d_2 = \frac{J_1b_1b_2 + J_2b_1b_2 + J_3b_1b_2 + J_2J_3k_1 + J_1J_2k_2 + J_1J_3k_1 + J_1J_3k_2}{J_1J_2J_3}$$

$$d_3 = \frac{J_1b_2k_1 + J_3b_2k_1 + J_2b_1k_2 + J_2b_2k_1 + J_3b_1k_2 + J_1b_1k_2}{J_1J_2J_3}$$

$$d_4 = \frac{J_1k_1k_2 + J_2k_1k_2 + J_3k_1k_2}{J_1J_2J_3}$$

$$c'_o = \frac{b_1b_2}{J_1J_2J_3} \qquad c'_1 = \frac{k_1b_2 + k_2b_1}{J_1J_2J_3}$$

$$c'_2 = \frac{k_1k_2}{J_1J_2J_3}$$

3. Velocity Control Design

3.1 Controller design

In common practice, the sensor is mounted at the motor end, where only the motion of the motor is measured and fed back even though the objective is also to control the motion

of the load. This setup is called motor feedback configuration. In this paper, we aim to control the motor velocity and load velocity using this configuration.

First equation in (3) can be rewritten as:

$$\begin{aligned}\frac{d\omega_1}{dt} &= \left(-\frac{b_1}{J_1}\omega_1 + \frac{b_1}{J_1}\omega_2 - \frac{1}{J_1}m_{s1}\right) + \frac{1}{J_1}m_e \\ &= f(\omega_1, \omega_2, m_{s1}) + b.u\end{aligned} \tag{5}$$

where

$u = m_e$ is the control input, $b=1/J_1$,

$$f(\omega_1, \omega_2, m_{s1}) = \left(-\frac{b_1}{J_1}\omega_1 + \frac{b_1}{J_1}\omega_2 - \frac{1}{J_1}m_{s1}\right)$$

According to [9], the generalized term $f(\omega_1, \omega_2, m_{s1})$ is insignificant while only its real time estimate $\hat{f}$ is important. An extended state observer (ESO) is constructed to provide $\hat{f}$ such that we can compensate the impact of $f(\omega_1, \omega_2, m_{s1})$ on our model by means of disturbance rejection. This allows the control law:

$$u = \frac{u_0 - \hat{f}}{b} \tag{6}$$

to reduces the plant in (5) to a form of:

$$\frac{d\omega_1}{dt} = f + b.u \simeq u_0 \tag{7}$$

The ESO was originally proposed by J. Han [9] and made practical by the tuning method proposed by Gao [11], which simplified its implementation and made the design transparent to engineers. The main idea is to use an augmented state space model of equation (5) that includes f, short for $f(\omega_1, \omega_2, m_{s1})$) as an additional state. In particular, let $x_1 = \omega_1$, $x_2 = f$. The augmented state space form of equation (5) is

$$\begin{cases} \begin{pmatrix}\dot{x}_1 \\ \dot{x}_2\end{pmatrix} = \underbrace{\begin{pmatrix}0 & 1 \\ 0 & 0\end{pmatrix}}_{A} . \begin{pmatrix}x_1 \\ x_2\end{pmatrix} + \underbrace{\begin{pmatrix}b \\ 0\end{pmatrix}}_{B} .u + \underbrace{\begin{pmatrix}0 \\ 1\end{pmatrix}}_{E} .\dot{f} \\ y = \underbrace{\begin{pmatrix}1 & 0\end{pmatrix}}_{C} \begin{pmatrix}x_1 \\ x_2\end{pmatrix} \end{cases} \tag{8}$$

The state observer

$$\begin{cases} \dot{z}_1 = z_2 + b.u + l_1.(y - z_1) \\ \dot{z}_2 = l_2.(y - z_1) \end{cases} \tag{9}$$

with l_1, l_2 are observer parameters to be determined, provides an estimate of the state of equation (8). z_1, z_2 will track y (ω_1) and f respectively. The convergence of ESO is extensively discussed in [19].

Then the control law

$$u_0 = \frac{K_P.(\omega_{ref} - z_1)}{b} \tag{10}$$

reduces equation (5) to:

$$\frac{d\omega_1}{dt} = f + b.\frac{u_0 - z_2}{b} \simeq u_0 = K_P.(\omega_{ref} - z_1) \tag{11}$$

where ω_{ref} is the set point for velocity.

Taking the Laplace Transform of (11), one has:

$$\frac{\omega_1(s)}{\omega_{ref}(s)} \simeq \frac{K_p}{s + K_p} \tag{12}$$

The velocity control based on ADRC for the system is then constructed as depict in Fig. 2.

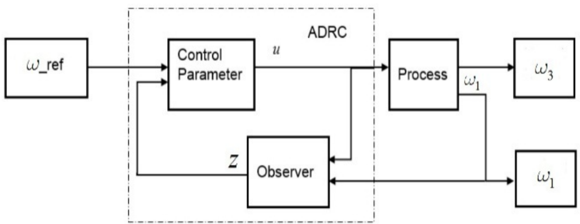

Fig. 2. ADRC structure for 3-mass system.

The parameters of ADRC K_P, l_1 and l_2 can be determined according to [20]:

- Get the desired settling time T_{settle}.
- K_p can be calculated from the desired first-order system with 2%-settling time:

$$K_p = \frac{4}{T_{settle}}$$

- Since the observer dynamics must be fast enough, the observer poles $s_{1/2}^{ESO}$ must be placed left of the close-loop pole s^{CL}, for suggestion:

$$s_{1/2}^{ESO} = s^{ESO} \approx (3...10).s^{CL} \text{ with } s^{CL} = -K_p$$

- The observer parameters can be computed from its characteristic polynomial:

$$\det\left(sI - \left(A - LC\right)\right) = s^2 + l_1 s + l_2 \overset{!}{=} \left(s - s^{ESO}\right)^2$$

Then

$$l_1 = -2.s^{ESO}; l_2 = \left(s^{ESO}\right)^2$$

3.2 Simulation

This section dedicates to numerical verification of the closed-loop performance. The parameters of the system are given as:

Symbol	Value (Unit)
J_1	1.88×10^{-3} kg.m^2
J_2	1.57×10^{-3} kg.m^2
J_3	1.57×10^{-3} kg.m^2
k_1	186 N.m/rad
k_2	186 N.m/rad
b_1	0.008 N.m.s/rad
b_2	0.008 N.m.s/rad

The observer gains and controller gains of ADRC are selected as follow: $K_p = 20$, $l_1 = 600$, $l_2 = 90000$.

In this section, the proposed method is tested in simulation and the results are compared to the responses of PID controller. The transfer function of this PID controller is:

$$G_{PID}(s) = P + I.\frac{1}{s} + D.\frac{N}{1 + N.\frac{1}{s}}$$

The parameters of PID controller are determined by using tuning tool in Matlab/Simulink with:

P = 0.1166, I = 0.0217, D = -0.0025, N = 45.1412

In these tested simulations, the reference command input is 30 rad/s at 0s, and the disturbance input m_L is applied at 1.5s with the value of 0.1 N.m. The simulation results show that the ESO can estimate the value of disturbance almost correctly. Fig. 3 shows that the velocity of motor, load 1 and load 2 reach the desired value with settling time of 0.1s. Compare to PID controller, the designed velocity controller gives smoother response and has no overshoot as shown in Fig. 4. The control signal is shown in Fig. 5 and the estimation $\hat{f}$ of f is presented in Fig. 6 where one can see that the ESO has good performance.

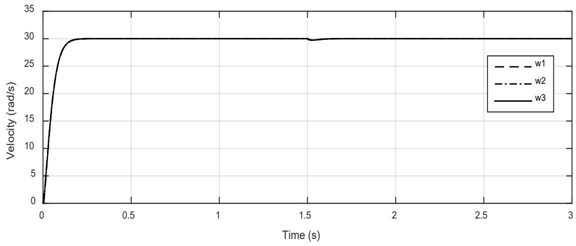

Fig. 3. Velocity responses of the system with designed controller.

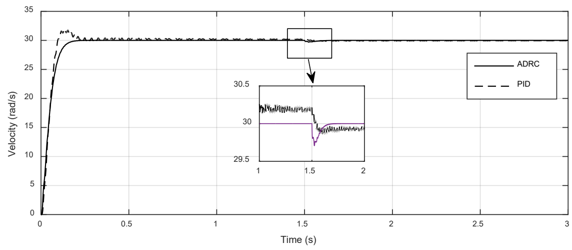

Fig. 4. Tracking and disturbance rejection performance of the system (load 2 velocity response)

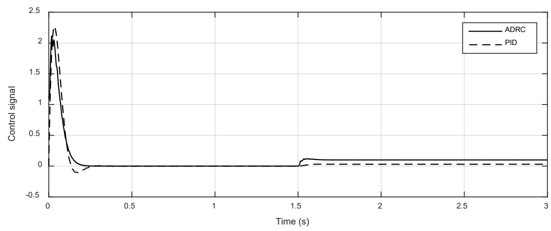

Fig. 5. Control signal

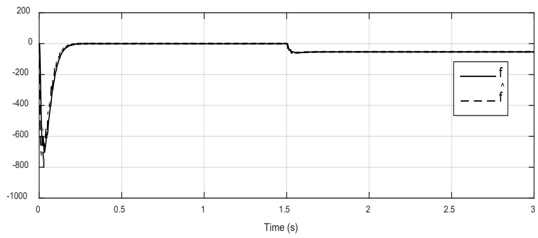

Fig. 6. Estimation of f

The ADRC shows better performance in term of lower overshoot and shorter settling time while bearing a simple design approach.

3.3 Robustness

In order to test the robustness of the designed controller, some situations are considered. In the first case (Fig. 7), only the values of b_1 and b_2 are changed b_1=0.008 N.m.s/rad, b_2=0.016 N.m.s/rad. Other parameters are kept as in section 3.2. The second case (Fig. 8) is considered when $b_1 = b_2 = 0$.

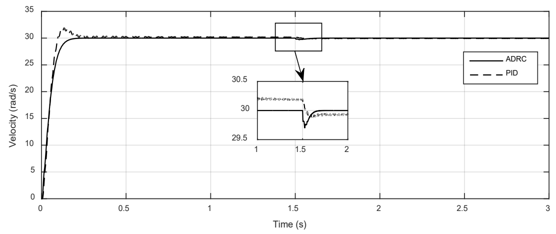

Fig. 7. Load 2 velocity response

(b_1 = 0.008 and b_2 = 0.016).

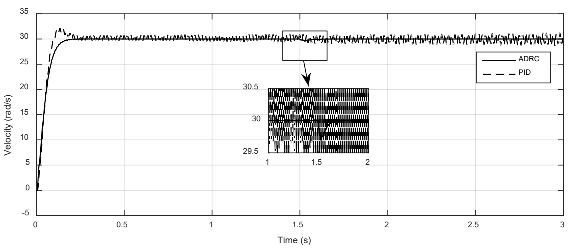

Fig. 8. Load 2 velocity response ($b_1 = b_2 = 0$).

And in the last case (Fig. 9), we supposed that the parameters of the system are changes with J_1=1.5x10^{-3} kg.m^2, J_2=1.57x10^{-3} kg.m^2, J_3=1.57 kg.m^2, k_1=175 N.m/rad, k_2=175 N.m/rad, b_1=0.005 N.m.s/rad, b_2=0.005 N.m.s/rad.

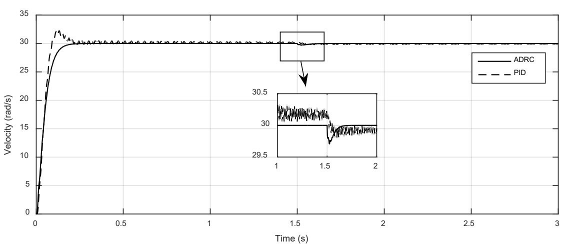

Fig. 9. Tracking and disturbance rejection performance (load 2 velocity response) when the parameters of the system are modified.

As seen in Fig. 7, Fig. 8 and Fig. 9, the PID controller show bad performance when $b_1 = b_2 = 0$ while the designed controller still has good response in all the situations. It can be concluded that ADRC have better robust properties compared to classical PID.

4. Conclusion

This paper has proposed an approach for the velocity control problem of three-mass system based on Active Disturbance Rejection Control. From the positive performances in term of reference tracking and disturbance reduction of the closed-loop system, one can observe that the use of ADRC method has advantages such as less dependence on the modeling and simple implementation. ADRC method requires little knowledge of the plant, is simple in tuning method and promises strong robustness. This approach can be considered as a control tool for practitioners. ADRC can be considered as a promising practical method, not only for robotic engineering, but also for many other systems that share the flexibility nature such as crane systems and liquid transfer process.

References

[1] R. Seifried; Dynamics of Underactuated Multibody Systems – Modeling; Control and Optimal Design, vol. 205. Springer 2014.

[2] S. Brock, D. Luczak, K. Nowopolski, T. Pajchrowski, and K. Zawirski; Two Approaches to Speed Control for Multi-Mass System with Variable Mechanical Parameters; IEEE Transactions on Industrial Electronics, vol. 64, no. 4, pp. 3338–3347, 2017.

[3] C. Ma and H. Yoichi; Backlash Vibration Suppression Control of Torsional System by Novel Fractional Order PIDk Controller, vol. 124, no. 3, pp. 312–317, 2004.

[4] J. Vittek, V. Vavrúsˇ, P. Brisˇ, and L. Gorel; Forced Dynamics Control of the Elastic Joint Drive with Single Rotor Position Sensor, Automatika; Journal for Control, Measurement, Electronics, Computing and Communications, vol. 54, no. 3, pp. 337–347, 2013.

[5] Ł. Dominik and K. Nowopolski; Identification of multi-mass mechanical systems in electrical drives; Proceedings of the 16th International Conference on Mechatronics – Mechatronika 2014.

[6] P. J. Serkies and K. Szaba; Model predictive control of the two-mass with mechanical backlash; Computer Applications in Electrical Engineering, pp. 170–180, 2011.

[7] M. Mola, A. Khayatian, and M. Dehghani; Backstepping position control of two-mass systems with unknown backlash; 2013 9th Asian Control Conference, ASCC 2013.

[8] H. Ikeda, T. Hanamoto; Fuzzy Controller of Three-Inertia Resonance System designed by Differential Evolution; Journal of International Conference on Electrical Machines and Systems Vol. 3, No. 2, pp. 184~189, 2014.

[9] J. Han; From PID to active disturbance rejection control; IEEE Trans. Ind. Electronics., Vol 56, No.3, pp. 900-906, 2009.

[10] Z.Gao, Y.Huang, J.Han; An alternative paradigm for control system design; Proceedings of 40th IEEE Conference on Decision and Control, Orlando, Florida, December 4-7, pp. 4578-4585, 2001.

[11] Z.Gao (2003); Scaling and Parameterization Based Controller Tuning; Proceedings of the 2003 American Control Conference, pp. 4989–4996, 2003.

[12] Y. X. Su, C. H. Zheng, B. Y. Duan; Automatic disturbances rejection controller for precise motion control of permanent-magnet synchronous motors; IEEE Trans. Ind. Electron. 52, 814–823, 2005.

[13] Q. Zheng, Z. Chen, Z. Gao; A Dynamic Decoupling Control and Its Applications to Chemical Processes; Proceeding of American Control Conference, New York, USA, 2007.

[14] T.H. Do; Application of First-order Active Disturbance Rejection Control for Multivariable Process, Special Issue on Measurement, Control and Automation, Vol 17, pp 30-35, 2016.

[15] S. Zhao and Z. Gao; An Active Disturbance Rejection based Approach to Vibration Suppression in Two-Inertia Systems; Asian Journal of Control, Vol 15, No. 3, pp. 146-155, 2013.

[16] D. Luczak; Mathematical Model of Multi-mass Electric Drive System with Flexible Connection; 9th International Conference on Methods and Models in Automation and Robotics, pp.590-595, 2014.

[17] H. Ikeda, T. Hanamoto, T. Tsuji and M. Tomizuka; Design of Vibration Suppression Controller for 3-Inertia System Using Taguchi Method. International Symposium on Power Electronics; Electrical Drives, Automation and Motion, pp.19-23, 2006

[18] H. Ikeda, T. Hanamoto and T. Tsuji; Vibration Suppression Controller for 3-Mass System Designed by Particle Swarm Optimization; International Conference on Electrical Machines and Systems, 2009.

[19] D. Yoo, S. S. T. Yau, Z.Gao (2006); On convergence of the linear extended observer; Proceedings of the IEEE International Symposium on Intelligent Control, Munich, Germany. pp. 1645–1650, 2006.

[20] G. Herbst; A Simulative Study on Active Disturbance Rejection Control as a Control Tool for Practitioners; In Siemens AG, Clemens-Winkler-Strabe 3, Germany, 2013.