

# A Combination of Interpolation and Spatial Correlation Technique to Estimate the Channel in Wideband MIMO-OFDM System

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## Abstract

*In this paper, we combine the interpolation and the spatial correlation techniques to estimate the channel coefficient in wideband multi-input multi-output orthogonal frequency division multiplexing (MIMO-OFDM) system. The simulation is based on the measurement channel modelling method - the Spatial Channel Model (SCM) in the suburban macrocell and microcell environment, under the LTE Advanced standard for 4G. The obtained results show symbol error rate (SER) value when using both different interpolation methods (Linear, Sinc Interpolation-SI and Wiener) and spatial correlation coefficients. The SER results in the SCM channel model are shown that the more of the window step of the interpolation, the worse of the performance system is, as well as, the effect of estimating channel is improved by increasing the distance of antenna element in BS side. Moreover, we can also conclude that of the three interpolation methods, the Wiener interpolation has the best system's performance.*

Keywords: MIMO-OFDM, SFBC, Wiener-Hop interpolation, Sinc interpolation, Linear interpolation, spatial correlation

## 1. Introduction

Nowadays, improving channel capacity for the wireless communication systems based on the limited band is more and more urgent while applications require high throughput. The orthogonal multiplexing OFDM and MIMO diversity technology have been proposed in order to help using the radio resources more efficient.

The Third Generation Partnership 3GPP has proposed the Spatial Channel Model (SCM) [1]. The SCM has been studied for NLOS model for suburban macro, urban macro and urban micro cell. For simulating in NLOS case, authors in [2-3] investigate the spatial correlation properties of the channel model. The LOS model in SCM [4] is defined only for urban micro cell and the spatial correlation properties must be taken into account the K Ricean factor, which is defined by the ratio of power in the direct LOS component to the total power in the diffused non line-of-sight (NLOS) component.

Coding method SFBC [5] which takes advantages of diversity in frequency selective channel transmission scheme has been proposed to apply to the MIMO-OFDM system.

The minimum mean square error (MMSE) detection technique in [6] is applied the inverse of the

channel frequency response to the received signal for restoring the signal and cancelling the interference.

The channel estimation method in MIMO-OFDM receiver using the interpolation algorithms are researched in [7]–[16] for reducing the pilot overhead requirements. The interpolation techniques, especially based on the training sequence estimation or the pilot, are extensively adopted in OFDM channel estimation.

In this paper, we study the performance of the system by the symbol error rate (SER) when using different interpolation methods (Linear, SI and Wiener) and different spatial channel coefficients on the SCM channel model in 2×2 MIMO-OFDM system. The channel model is simulated by using the SCM model under the LTE-A standard in both NLOS and LOS case. We also apply the combination of the SFBC and the MMSE detection to improve the effectiveness of the channel estimation.

The structure of this paper is as follows: Section 2 studies the spatial cross-correlation functions of the SCM channel modelling method. In section 3 and 4, we introduce interpolation techniques for 2×2 MIMO-OFDM system. Section 5 shows simulation results and discussions. Conclusions are given in Section 6.

## 2. The wideband frequency selective SCM channel modelling method

The SCM channel model has the scatterers as can be seen in Fig.1

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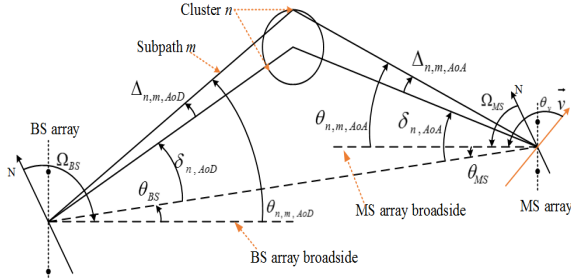


Fig.1. SCM with one cluster of scatterers [1]

### 2.1. The SCM channel in NLOS environment

Authors in [1] assume that with  $S$  element linear BS array and  $U$  element linear MS array, the channel impulse response function of the channel for the  $n^{th}$  multipath component ( $n = 1, \dots, N$ ), the  $(u, s)^{th}$  component ( $s = 1, \dots, S$ ;  $u = 1, \dots, U$ ) is given for the wideband frequency channel as:

$$h_{u,s,n}^{SCM}(t) = \sqrt{\frac{P_n \sigma_{SF}}{M}} \sum_{m=1}^M \left\{ \begin{array}{l} \sqrt{(\theta_{n,m,AoD})} \exp(j[kd_s \sin(\theta_{n,m,AoD}) + \Phi_{n,m}]) \\ \times \sqrt{(\theta_{n,m,AoA})} \exp(jkd_u \sin(\theta_{n,m,AoA})) \\ \times \exp[jk\|v\| \cos(\theta_{n,m,AoA} - \theta_v) t] \end{array} \right\} \quad (1)$$

$$h_{u,s,n}^{SCM}(\tau, t) = h_{u,s,n}^{SCM}(t) \delta(\tau - \tau_n)$$

where  $\tau$  is the time delay of the channel;  $\sigma_{SF}$  is the lognormal shadow fading random variable;  $d_s$  and  $d_u$  are the distance of BS and MS antenna elements, respectively.

We assumed the lognormal shadow fading is zero and antenna gain of each array element of both BS and MS are equal to one. The transfer function  $H_{usn}^{NLOS}$  in the frequency domain denotes the Fourier transform of the channel impulse response  $h_{u,s,n}(t)$  is given as [3]:

$$H_{usn}^{NLOS}(f, t) = \sum_{n=1}^N h_{u,s,n}(t) \times \exp(-j2\pi\tau_n f) \quad (2)$$

$$= \sum_{n=1}^N \sqrt{\frac{P_n}{M}} \sum_{m=1}^M \left\{ \begin{array}{l} \exp(j[kd_s \sin(\theta_{n,m,AoD}) + \Phi_{n,m}]) \times \\ \exp(jkd_u \sin(\theta_{n,m,AoA})) \times \\ \exp[jk\|v\| \cos(\theta_{n,m,AoA} - \theta_v) \tau] \end{array} \right\} \times \exp(-j2\pi\tau_n f)$$

The spatial cross correlation function (CCF) of the NLOS MIMO channel becomes:

$$\rho_{s_2 u_2}^{s_1 u_1} NLOS(\Delta d_s, \Delta d_u, \Delta t = 0, \Delta f = 0) = \langle H_{u_1 s_1}^{SCM}(f, t) \times H_{u_2 s_2}^{*SCM}(f, t + \Delta t) \rangle$$

$$= \sum_{n=1}^N \frac{P_n}{M} \sum_{m=1}^M \left\{ \begin{array}{l} \exp(jk\Delta d_s \sin(\theta_{n,m,AoD})) \\ \exp(jk\Delta d_u \sin(\theta_{n,m,AoA})) \end{array} \right\} \quad (3)$$

### 2.2. The SCM channel in LOS environment

Authors in [1] describe the LOS case for urban micro environment, the channel impulse response function of the channel based on the  $K$  Ricean factor of the direct (the earliest) arriving path ( $n = 1$ ) is as:

$$h_{us,n=1}^{LOS} = \sqrt{\frac{1}{K+1}} h_{us,n=1}(t) + \sigma_{SF} \sqrt{\frac{K}{K+1}} \left( \begin{array}{l} \sqrt{G_{BS}(\theta_{BS})} \exp(jkd_s \sin(\theta_{BS})) \times \\ \sqrt{G_{MS}(\theta_{MS})} \exp(jkd_u \sin(\theta_{MS}) + \phi_{LOS}) \\ \times \exp(jk\|v\| \cos(\theta_{MS} - \theta_v) t) \end{array} \right) \quad (4)$$

The channel coefficients for others paths ( $n \neq 1$ ) are written as:

$$h_{us,n}^{LOS} = \sqrt{\frac{1}{K+1}} h_{us,n} \quad \text{where } n = 2, \dots, N$$

Therefore, we have  $H_{usn}^{LOS}(\cdot)$  the Fourier transform of the impulse response  $h_{us,n}^{LOS}$  as follows [4]:

$$H_{usn}^{LOS}(f, t) = \sum_{n=1}^N h_{us,n}^{LOS}(t) \times \exp(-j2\pi\tau_n f)$$

$$= \sum_{n=1}^N \sqrt{\frac{1}{K+1}} h_{us,n}(t) \times \exp(-j2\pi\tau_n f) + \sqrt{\frac{K}{K+1}} \left( \begin{array}{l} \exp(jkd_s \sin(\theta_{BS})) \\ \times \exp(jkd_u \sin(\theta_{MS})) \\ \times \exp(jk\|v\| \cos(\theta_{MS} - \theta_v) t) \end{array} \right) \times \exp(-j2\pi\tau_n f) \quad (5)$$

The spatial CCF of the LOS MIMO channel is as:

$$\rho_{u_2 s_2}^{u_1 s_1} LOS(\Delta d_s, \Delta d_u) = \sum_{n=1}^N \frac{1}{K+1} \cdot \frac{P_n}{M} \sum_{m=1}^M \left\{ \begin{array}{l} \exp(jk\Delta d_s \sin(\theta_{n,m,AoD})) \\ \times \exp(jk\Delta d_u \sin(\theta_{n,m,AoA})) \end{array} \right\} + \frac{K}{K+1} \left\{ \begin{array}{l} \exp(jk\Delta d_s \sin(\theta_{BS})) \\ \times \exp(jk\Delta d_u \sin(\theta_{MS})) \end{array} \right\} \quad (6)$$

where the angles  $\theta_{BS}$  and  $\theta_{MS}$  are the AoD and the AoA of the LOS component;  $\phi_{LOS}$  is the phase shift.

### 3. Interpolation Methods for 2x2 MIMO-OFDM system

We investigate three popular interpolation methods: Linear, Sinc and Wiener that have been introduced in [7] to [16].

#### 3.1. The Linear Interpolation

This method [7]-[13] relies on the channel coefficient of two consecutive pilot positions in both time and frequency domain. The assumption is that the interpolation approach is in shift invariant.

If the frequency interval of the neighboring pilot subcarrier is  $L$ , the index of the non-pilot subcarrier between two adjacent pilots is  $l$ , the index of pilot subcarriers is  $p$ . The transfer function for non-pilot subcarriers between  $k^{th}$  and  $(k+1)^{th}$  pilots is described as:

$$\tilde{H}(kL + l) = \left(1 - \frac{l}{L}\right) \tilde{H}_p(k) + \left(\frac{l}{L}\right) \tilde{H}_p(k+1) \quad 7)$$

where  $H_p(k)$  is the transfer function of the pilot.

#### 3.2. The Sinc Interpolation (SI)

This method has been introduced in [14]-[15]. We assume that  $h(n); n = 1, 2 \dots N$  is the channel coefficient in the all of OFDM symbols and

$h(k); k = 1, 2 \dots N_{pilot}$  is the channel coefficient in the pilot symbols in the time domain. The closed form expression calculates the channel coefficient in the data symbols bases on pilot positions as following:

$$h(n) = \sum_{k=1}^{N_{pilot}} h(k) \times \frac{\sin\left(\frac{\pi(n-kL)}{L}\right)}{\pi(n-kL)} \quad (8)$$

The correctness and effectiveness of this method depends on the step value  $L$ , that is similar to the Linear method.

### 3.3. The Wiener Interpolation

This method has been introduced in [7] and [16]. We assumed that  $\hat{H}_{i,l}$  is the channel coefficient at  $i^{th}$  OFDM symbol at the  $l^{th}$  sub-carrier.  $\hat{H}_{i,p}$  is the channel coefficient at the  $p^{th}$  sub-carrier and the  $i^{th}$  OFDM symbol on which contain the pilot data. The input of Wiener filter is described as, where  $w_{i',p,i,l}$  is the filter coefficients.

$$\hat{H}_{i,l} = \sum_{i',p} w_{i',p,i,l} \hat{H}_{i',p} \quad (9)$$

Set the matrix coefficient of the filter as:

$$W_{i,l}^T = (w_{1,1,i,l}, \dots, w_{i',p,i,l}, \dots, w_{(\ell_t-1)D_t+1,(\ell_f-1)D_f+1,i,l}) \quad (10)$$

Therefore, we have :

$$\hat{H}_{i,l} = W_{i,l}^T \hat{H}_{i,p} \quad (11)$$

where  $\ell_t, \ell_f$  are the number of OFDM symbols which contain pilots in the time and frequency axis, respectively.

### 4. Description the $2 \times 2$ MIMO-OFDM system

We consider a  $2 \times 2$  MIMO system as in Fig.2. In the transmitter side, the signal is modulated by the constellation of QAM64, then feed to the SFBC encoder and finally goes to the OFDM before going to antennas. The SCM channel modelling is used as the medium physic between transmitter and receiver. The receiver basically do the visa versa of the transmitter but channel estimator is added to increase the system performance by using different interpolation methods.

Fig.3 shows the simulated channel interpolation methods in the MIMO-OFDM system with the arrangement of user data, reference signal and zero data in frequency domain. It means that on the same  $i^{th}$  symbol and the same the  $k^{th}$  sub-carrier, the existing reference signal (RS-pilot) in this antenna can be gotten by setting the other to zero and vice versa.

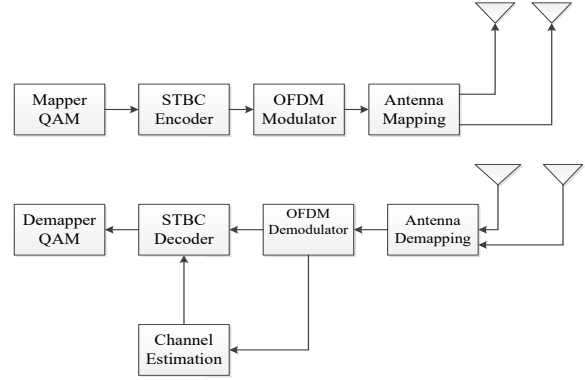


Fig. 2. The  $2 \times 2$  MIMO-OFDM system

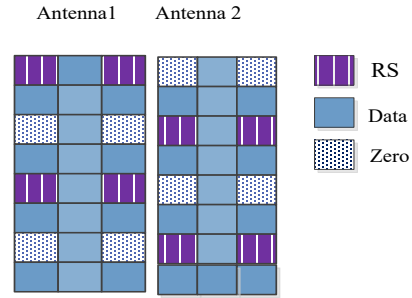


Fig.3. Arrange user data, reference signal and zero data in frequency grid

We denote the square matrix  $F_L$  with  $N_{FFT} \times N_{FFT}$  as the following format:

$$F_L = \begin{bmatrix} F_{0,0} & \dots & F_{0,N_{FFT}-1} \\ \vdots & \ddots & \vdots \\ F_{N_{FFT}-1,0} & \dots & F_{N_{FFT}-1,N_{FFT}-1} \end{bmatrix} \quad (12)$$

where  $F_{pq} = e^{-j2\pi(\frac{pq}{N})}$  and the RS can be generated in antenna 1 and 2, respectively as below:

$$\begin{aligned} X_{p,1}(k) &= e^{-jD_f \pi k^2 / N_{FFT}} \\ X_{p,2}(k) &= e^{-jD_f \pi (k+M)^2 / N_{FFT}} \\ M &= N_{FFT} / D_f \end{aligned} \quad (13)$$

Therefore the channel coefficients at the pilot positions is as:

$$H_p(k) = (Q^H Q)^{-1} Q^H Y_r \quad (14)$$

$$Q = \left[ \text{diag} \left( X_{p,1}(k) \right) \times F_L, \text{diag} \left( X_{p,2}(k) \right) \times F_L \right]$$

### 5. Simulation results and discussions

Under the simulation of the Vehicle A model C with the speed of  $30 \text{ km/h}$ , the channel profile delay is described in [1], the bandwidth of LTE-A standard is  $5 \text{ MHz}$ . The parameters for simulating the channel modelling as well as the MIMO-OFDM system can be given as in Table 1.

In the time domain, the carrier wave at  $2 \text{ GHz}$ , the maximum Doppler frequency can be gotten as:  $f_{Dmax} = \frac{v}{c} f_o = 55.556 \text{ Hz}$ . The coherence time is

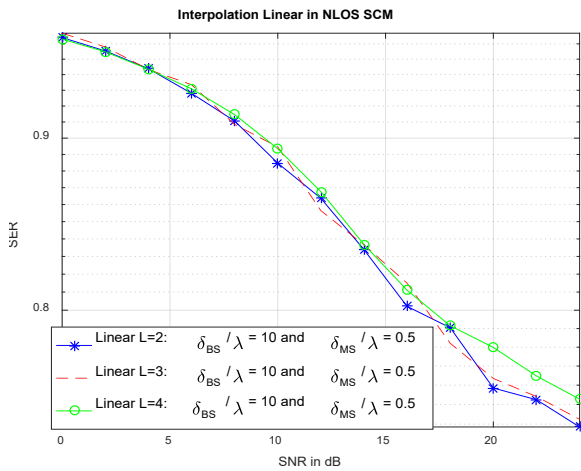
$(\Delta t)_c = \frac{1}{2f_{Dmax}} = 900\mu s$ . The  $T_s = ta \times NFFT = 66.662 \mu s$ , so  $(\Delta t)_c \gg T_s$  therefore the channel is independent in time domain.

From the parameters mean excess delay  $\bar{\tau}$  and the mean square excess delay spread  $\bar{\tau}^2$ , we can calculate  $(\bar{\tau})^2 = 2474 ns$  and  $\bar{\tau}^2 = 6120500 ns^2$ . Therefore the delay spread (RMS)  $\sigma_\tau = \sqrt{\bar{\tau}^2 - (\bar{\tau})^2} = 2.4735 \mu s$ . The coherence bandwidth of the channel is  $B_c = \frac{1}{5\sigma_\tau} = 80.857$  (KHz). In the frequency domain, the bandwidth  $B_s = 5$  MHz  $\gg B_c$  therefore, it is the wideband and frequency selective channel.

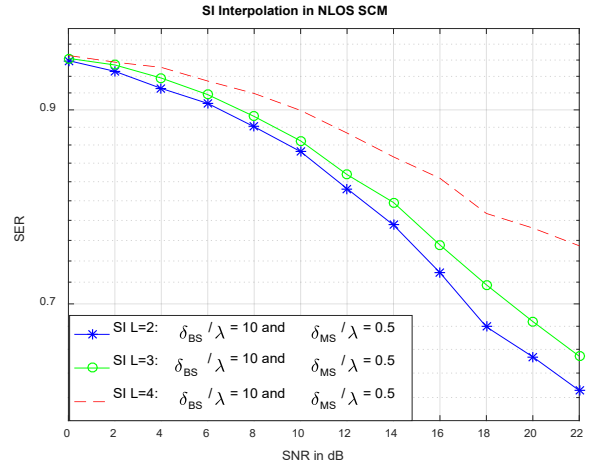
**Table 1.** Simulation parameters for  $2 \times 2$  MIMO-OFDM system

Parameters	Value
No of OFDM symbols	11
Number of subcarrier	300
Length of GI	128
Number of IFFT	512
Modulation	<i>QAM 64</i>
Frequency sampling	$T_s = 130.21 ns$
Sampling frequency	$f_s = 7.68$ MHz
Maximum access delay	$\tau_{max} = 2473.96 ns$

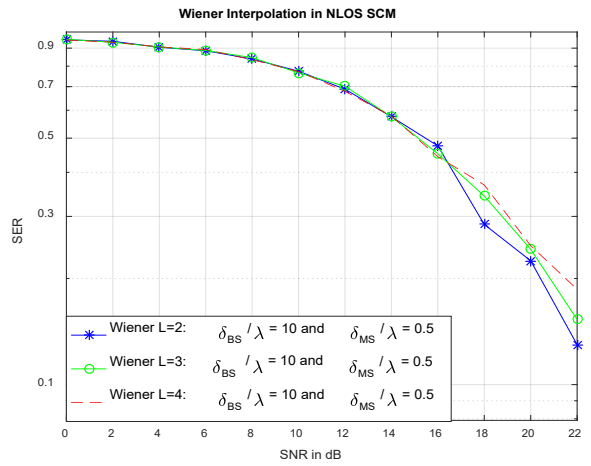
Fig.4 - Fig.6 are the SER of the NLOS SCM channel model when using Linear, SI and Wiener interpolation, respectively. The parameters for the distance of the antenna array in BS and MS side are  $\Delta d_s = 10\lambda, \Delta d_u = 0.5\lambda$ , respectively. We estimate the channel coefficient only in time domain with the window step  $L$  from 2 to 4. From these graphs, we can see that if the step  $L$  is increased the system's performance is decreased.



**Fig.4.** Linear Interpolations of SCM NLOS



**Fig.5.** SI interpolation of SCM NLOS



**Fig.6.** Wiener interpolation of SCM NLOS

Figure 7-9 are the results of estimating channel of Linear, SI and Wiener interpolation in case with step window  $L = 2$  with different spatial correlation coefficients which obtained from the spacing of the antenna array in both sides. We can see that, the SER of the system can be reduced by increasing the distance of the BS antenna elements. However, the differential from these graphs are small as can be seen in Table 2.

**Table 2.** SERs of Interpolation methods at SNR = 22 dB

$\{\Delta d_s, \Delta d_u\}$	$\{0.5\lambda, 0.5\lambda\}$	$\{10\lambda, 0.5\lambda\}$	$\{30\lambda, 0.5\lambda\}$
SER of Linear	.7351	.7306	.7269
SER of SI	.7573	.6734	.6629
SER of Wiener	.2439	.1665	.1204

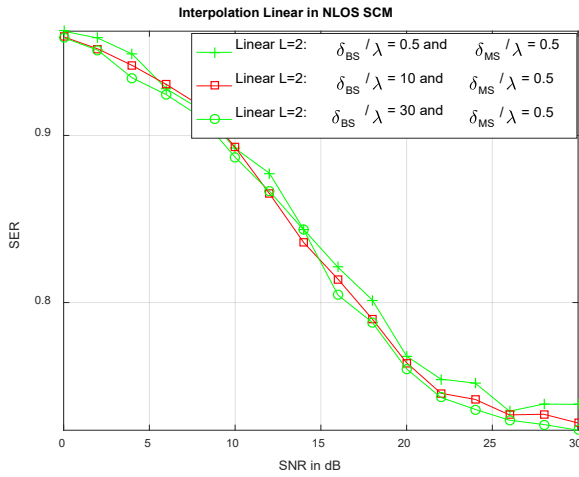


Fig.7. Spatial correlation and Linear Interpolation of SCM NLOS

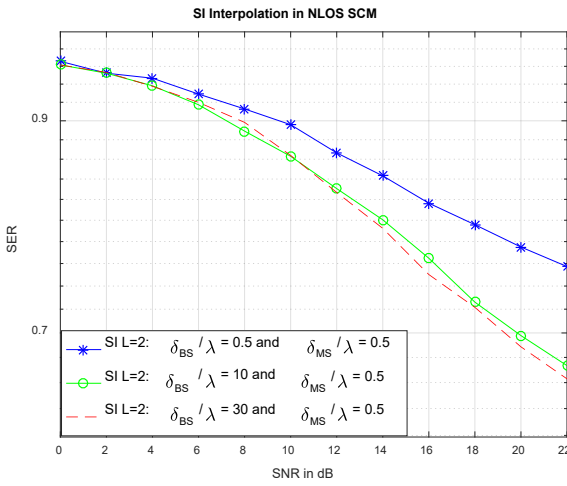


Fig.8. Spatial correlation and SI Interpolation of SCM NLOS

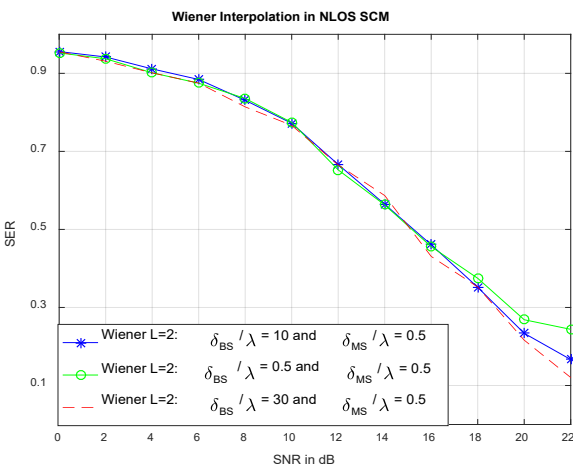


Fig.9. Spatial correlation and Wiener Interpolation of SCM NLOS

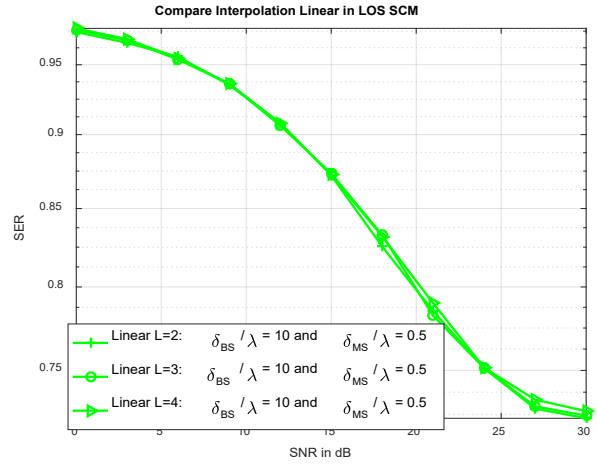


Fig.10. SER of Linear interpolation of SCM LOS

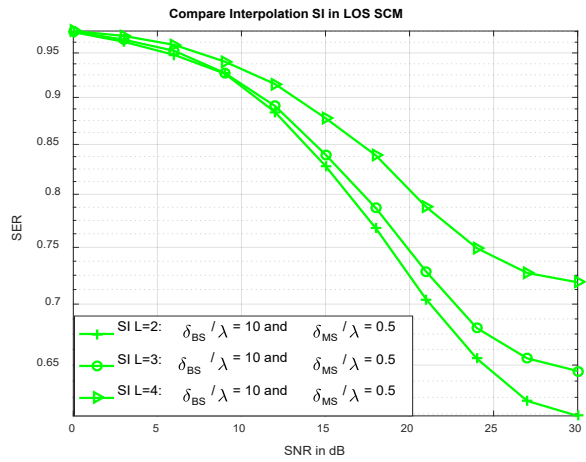


Fig.11. SER of SI interpolation of SCM LOS

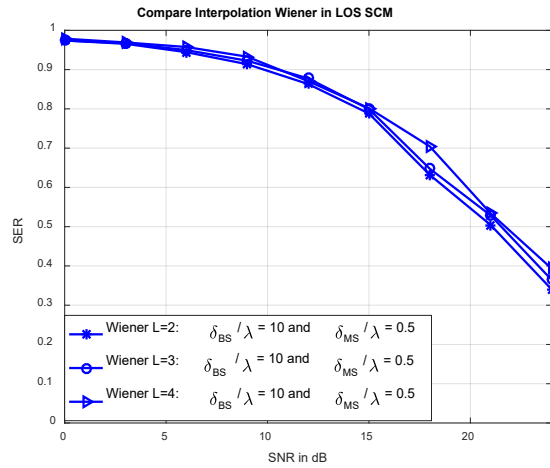
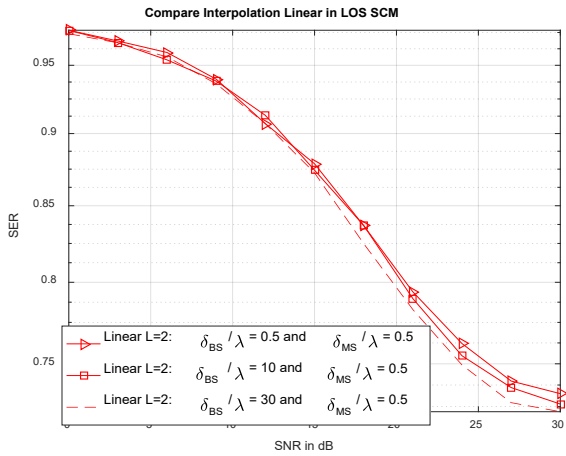


Fig.12. SER of Wiener interpolation of SCM LOS

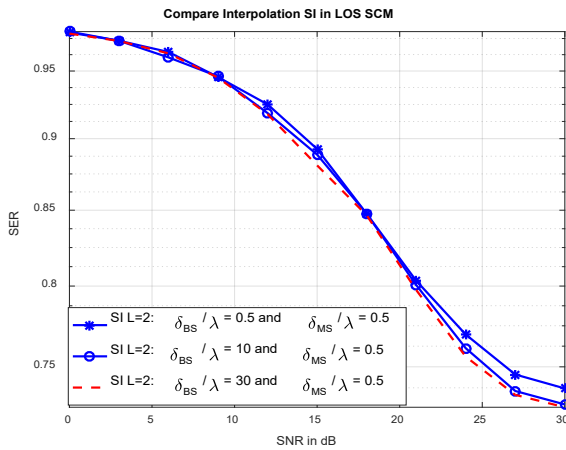
As mention above, in the case of SCM LOS in the urban microcell, Fig. 10 - Fig.12 are the results of estimating channel by using interpolating methods. In this case, we have the same conclusion that the more increasing of the step  $L$ , the worse of the performance of the system is.



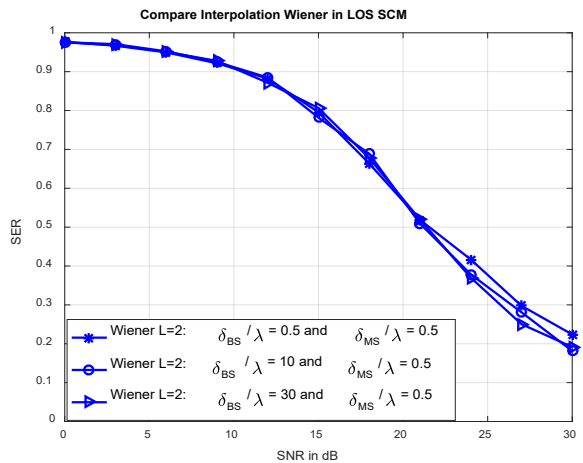
**Fig.13.** Spatial correlation and Linear Interpolation of SCM LOS

Fig. 13 - Fig. 15 are the results of interpolating methods with step window  $L = 2$  combined different spatial correlation coefficients. As one can see, the more of the spacing of the BS antenna, the lesser of the SER is.

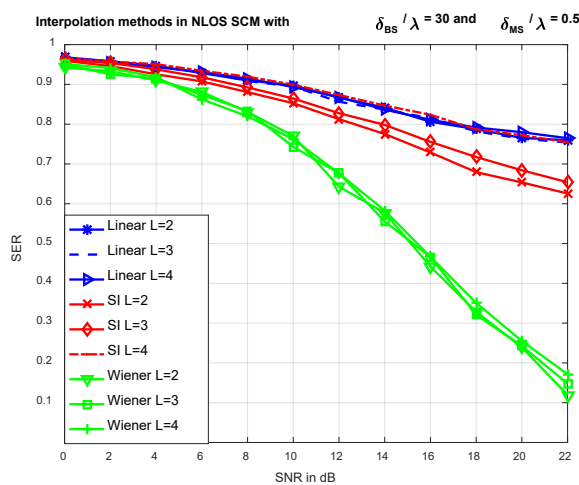
Fig.16 - Fig.17 comparing the three of interpolation scenario of both NLOS and LOS case by changing distance of pilot and the spatial correlation at  $\{\Delta d_s = 30\lambda, \Delta d_u = 10\lambda\}$ . Of the three interpolation techniques, the most channel estimation effective is the Wiener, followed by the SI and the worst case is the linear. As the same analyzed characteristic above, the system's performance is better if the window step  $L$  is decreased.



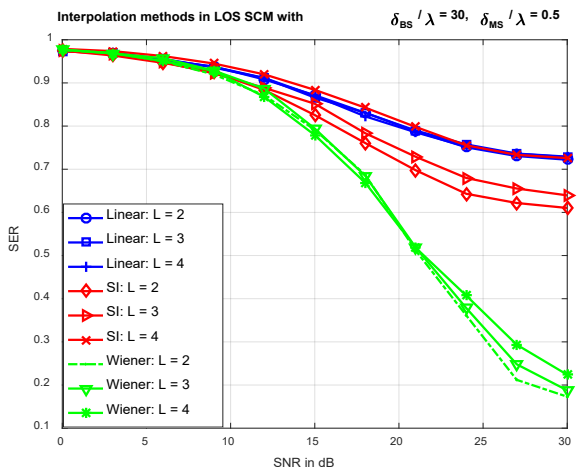
**Fig. 14.** Spatial correlation and SI Interpolation of SCM LOS



**Fig. 15.** Spatial correlation and Wiener Interpolation of SCM LOS



**Fig. 16.** SER of Linear, SI and Wiener Interpolations in SCM NLOS case



**Fig. 17.** SER of Linear, SI and Wiener Interpolations, in SCM LOS case

## 6. Conclusion

In this paper, we studied interpolation methods and spatial correlation techniques applied to MIMO OFDM 2x2 systems to estimate the channel coefficients in both case NLOS and LOS of the SCM. From the SER results, we conclude that the channel coefficients using Wiener interpolation has the best effectiveness of estimating channel, the linear interpolation has the worst result. Moreover, the effectiveness of the estimating channel depends on the spatial correlation, especially by rising the distance of the BS antenna element. Finally, the SER depends on the pilot positions by the step  $L$ , the higher of the  $L$  step, the worse of the system's performance can get.

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