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An Innovative Image Denoising Method Using Curvelet Transform and Histogram Segmentation

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Abstract

A new image denoising method based on Curvelet transform and histogram segmentation is proposed. This paper first explores the concept and the propertites of the Curvelet transform for curved singularities analysis then applies Curvelet transform and histogram segmentation to estimate optimum threshold for image denoising. In the simulations, the Wrap (Wrapping-based transform) algorithm was used to realize the Curvelet transform, which adds a wrap step to the Unequally Spaced Fast Fourier Transform (USFFT) method. The simulation results show the denoising effectiveness of the proposed method, show that Curvelet transform has a better denoising result and a certain increase in PSNR (Peak Signal-to-Noise Ratio), especially for the images those contain curved singularities.

Keywords: Curvelet transform, Image denoise, Histogram segmentation.

1. Introduction

In recent years, Wavelet transform, especially second-generation wavelet transform, has been being used as an effective method for various applications such as astronomy, acoustics, nuclear engineering, voice, magnetic resonance imaging, optics, earthquake prediction, radar, partial differential equations, image processing, etc [1-2].

Among image processing tasks, noise removal is basic step and it plays extremely important role in digital image processing. The purpose of noise removal is to obtain a good estimate of the original image from its noised version meanwhile preserving important structures of images such as edge and curve. Traditional wavelet based denoising algorithm proposed by Donoho and Johnstone basically shrinks the wavelet coefficients on adopting an universal threshold with dimension $N, \lambda = \sigma\sqrt{2\ln N}$ and adopting also hard-soft shrink wavelet (detail) coefficients [3].

Curvelet transforms are recently developed as mathematical tools that overcome the weakness of the separable wavelet transform in representing curves and edges. Curvelet transform shows better performance than wavelet transform in represent multiscale edge [4].

In this paper, we analyse Curvelet properties and propose an innovative image denoising algorithm based on segmentation threshold for Curvelet shrinkage. We know that basic property of the Curvelet transform is piecewise smooth with discontinuities. In order to remove noise while preserving important information of images, we divide an image into different regions by gray level histogram. Each segmentation provides threshold for Curvelet shrinkage. The total shrinkage is mean of all threshold values.

The rest of the paper is organized as follows. In section 2, the necessary background is given about Curvelets for image denoise. In section 3, the proposed method is shown with histogram segmentation. Section 4 provides simulation results of the proposed method. Finally, the conclusions of this paper are for concluding remarks, and suggestions for further researches.

2. Methodology

2.1. Curvelet transform

Curvelet transform is defined in both continuous and digital domain and for higher dimensions. The basic structure of Curvelets is derived from a ridge-like form called Ridgelet [5]. Curvelets are obtained by parabolic dilations rotations and translations of elementary function φ and are indexed by scale parameter a satisfying $0<a<1$, location parameter b and orientation parameter θ. Curvelets have the approximate form as

$$\varphi_{a,b,\theta}(x) = a^{-3/4}\varphi\left(D_a R_\theta (x-b)\right),\ D_a = \begin{bmatrix} 1/a & 0 \\ 0 & 1/\sqrt{a} \end{bmatrix} \quad (1)$$

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D_a is the parabolic scaling matrix, R_θ is the rotation by θ radians and $\varphi(x_1, x_2)$, $x_1, x_2 \in \mathbb{R}^2$ is an admissible profile. Thus, if φ is supported near the unit square, the envelope of $\varphi_{a,b,\theta}$ is supported near an $a \times \sqrt{a}$ rectangle with the minor axis pointing in the direction of θ.

Curvelets obey the principle of harmonic analysis: It is possible to decompose and reconstruct an arbitrary function $f(x_1, x_2)$ as a superposition of Curvelets. If the scale, rotation and location are discretized as:

$$a_j = 2^{-j}\text{, where } j = 0,1,2,\dots \tag{2}$$

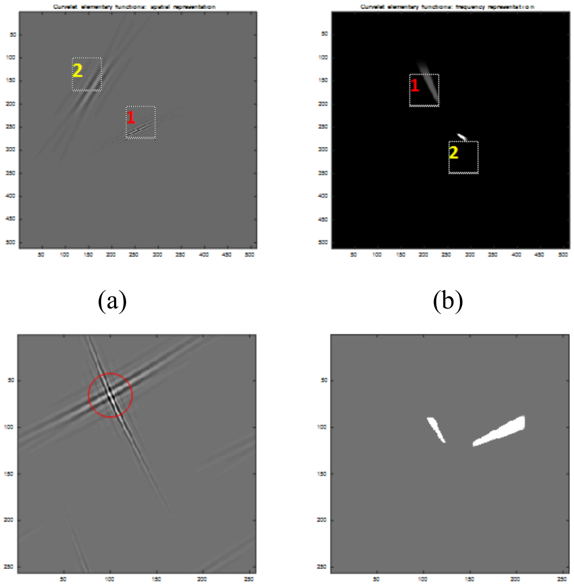
$$\theta_{j,i} = 2\pi l \cdot 2^{-\lfloor j/2 \rfloor}, \quad j = 0,1,\dots,2^{\lfloor j/2 \rfloor} - 1 \tag{3}$$

$$b_k^{(j,l)} = R_{\theta_{j,l}} \begin{bmatrix} k_1 2^{-j} \\ k_2 2^{-j/2} \end{bmatrix}, k_1 k_2 \in \mathbb{Z} \tag{4}$$

So that $\varphi_{j,k,l} = \varphi_{a,b_k^{(j,l)},\theta_{j,l}}$, the function f can be expressed in terms of the Curvelet family $(\varphi_{j,k,l})$ as

$$f = \sum_{j,k,l} \langle f, \varphi_{j,k,l} \rangle \varphi_{j,k,l} \tag{5}$$

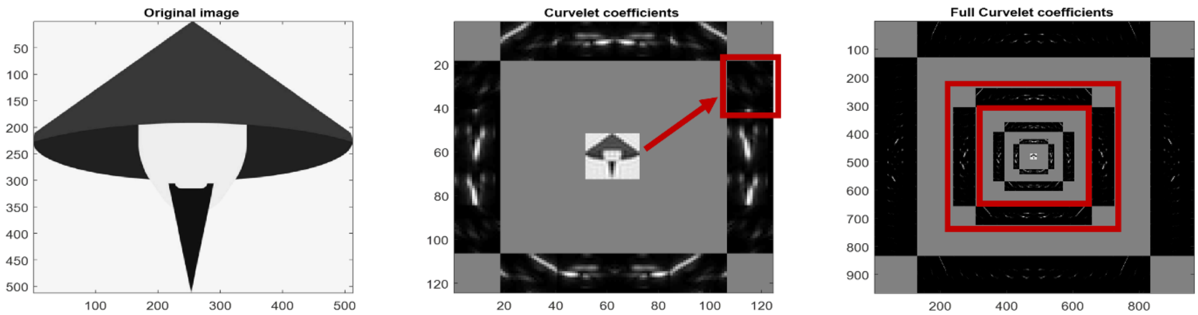
$$\|f\|_2^2 = \sum_{j,k,l} \left| \langle f, \varphi_{j,k,l} \rangle \right|^2, \forall f \in L^2(\mathbb{R}^2) \tag{6}$$



(a) (b) (c) (d)

Fig. 1. Spatial and frequency representation of Curvelet elementary functions; (a) spatial, (b) frequency representation of two Curvelets at different scales, rotations and translations; (c) and (d) illustrates a synthetic image, which comprises two intersecting reflectors, and its representation as a sum of weighted Curvelet elementary functions.

The elementary functions are not isotropic and highly oscillatory in a direction. The oscillations in different elementary functions can occupy different frequency bands which is a multi-resolution property [6].



(a) (b) (c)

Fig. 2. Curvelet coefficients magnitude of an image. (a) Original image, (b) Red rectangle represents one direction at one scale, (c) Inside area of two red rectangles is one scale of all directions.

Curvelet transform provides a strong directional characterization in which elements are highly anisotropic at fine scales. With these properties, Curvelet solve the isotropic and limited directional analysis of classic wavelet transform. Unlike the wavelet transform, it has directional parameters. The decomposition into Curvelet coefficients cannot only be used for image analysis but also for image manipulation [7].

In the Curvelet transform, most of the energy is localized in only a few coefficients as $w_{j,k,l} = \langle f, \varphi_{j,k,l} \rangle$. A Curvelet intersecting a discontinuity parallel to its longitudinal support will have coefficients of significant amplitude and if a Curvelet intersects a discontinuity at an arbitrary angle, it will have small coefficients. A Curvelet not intersecting a discontinuity will have zero coefficients [8].

2.2. Fast digital Curvelet transform (FDCT)

The second generation Curvelet transform is faster and less redundant compared to its first generation version. There are two different digital implementations of FDCT: Curvelets via USFFT (Unequally Spaced Fast Fourier Transform) and Curvelets via Wrapping. FDCT wrapping is the fastest Curvelet transform [12]. The algorithm of the fast digital Curvelet transform by wrapping is as follows:

Algorithm 1:

1. Apply the two dimensional fast Fourier transform (2D-FFT) and obtain Fourier samples $\widetilde{f}[n_1,n_2], -n/2 \le n_1, n_2 \prec n/2$;

2. For each scale j and angle θ, form the product $\widetilde{U}_{j,\theta}[n_1,n_2]\widetilde{f}[n_1,n_2]$;

3. Wrap this product around the origin and obtain $\widetilde{f}_{j,\theta}[n_1,n_2] = W\left(\widetilde{U}_{j,\theta}f\right)[n_1,n_2]$, Where the range for n_1 and n_2 is now $0 \le n_1 \prec L_{1,j}$ and $0 \le n_2 \prec L_{1,j}$ (for θ in the range $(-\pi/4, \pi/4)$;

4. Apply the inverse two dimensional fast Fourier transform (2D-IFFT) to each $\widetilde{f}_{j,\theta}$, hence collecting the discrete coefficients $c^D(j,\theta,k)$.

This algorithm has computational complexity of $O\left(n^2 \log n\right)$.

3. Proposed method

The objective of image thresholding is used to extract objects or regions of interest in an image from the background. Especially for discontinuos curve that we have to preserve. The thresholding is based on its gray level distribution. Image histograms are a useful tool to help discover some properties from images, and even directly obtain thresholds from them. We use the histogram approach. In this approach, the gray level distribution of pixels and the average gray level distribution of their neighborhood are used to select the optimal threshold vector. The algorithm for histogram segmentation is as follows:

Algorithm 2:

1. **I** = Noise input Image;

2. Calculate the histogram values h_i, bin width w_i, $i = 1..N$ of the image **I**, where N is number of bins of gray of the image;

3. Set the initial threshold value: $T_{init} = \frac{\sum_{i=1}^{N} h_i \mathrm{w}_i}{\sum_{i=1}^{N} h_i}$;

4. Segment the image using T_{init}. This will produce two groups of pixels $\mathbf{I}_1$ and $\mathbf{I}_2$;

5. Repeat step 2 and step 3 to obtain the new threshold values for each group: T_{group1} and T_{group2};

6. Compute the new threshold value: $T_{new} = \frac{(T_{group1}+T_{group2})}{2}$;

7. Repeat the steps from 2 to 6 until the difference in T_{init} for successive iterations is small enough;

8. Apply Curvelet shrinkage with threshold T_{new};

Schematic diagram of proposed method is shown on Fig. 3.

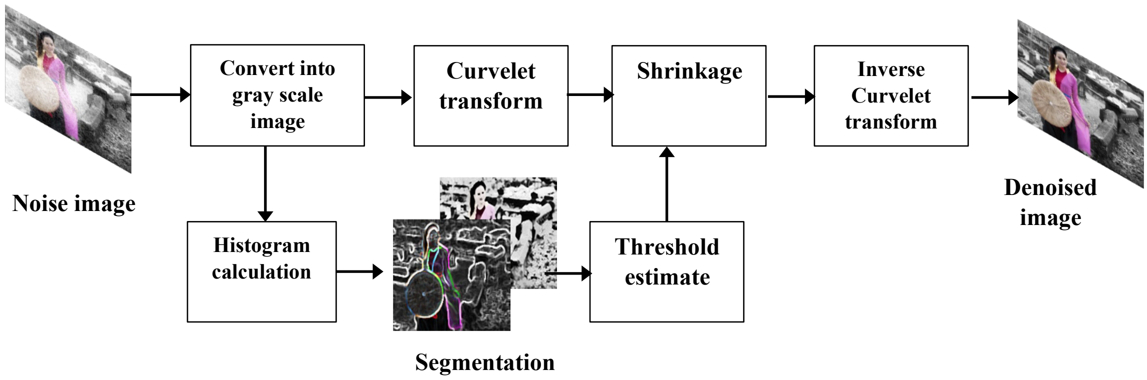


Fig. 3. Schematic diagram of proposed method for image denoising using Curvelet transform and histogram segmentation

4. Simulations and results

4.1. Evaluation Parameters

Image enhancement quality is difficult to assess. For the problem of estimating the distortion or the loss of information, we use PSNR parameter. PSNR is used for measuring the quality of the image and involve deviation of the enhanced image from the original image with respect to the peak value of the color level that affects the fidelity of its representation. It is an approximation to human sensitivity of reconstruction quality. A higher PSNR generally indicates that the reconstruction is of higher quality. PSNR is most easily defined via the mean squared error (MSE) as

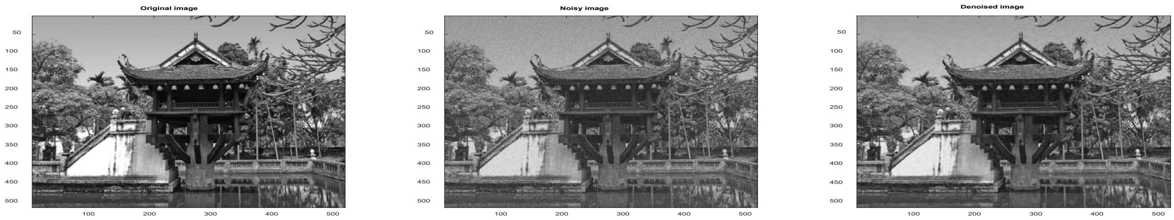
$$PSNR = 20Log_{10}\left[\frac{2^n - 1}{\sqrt{MSE}}\right] \text{ (dB)} \qquad (7)$$

where n is the number of bits/pixel used in representing the pixel of the image and the mean squared error is defined as

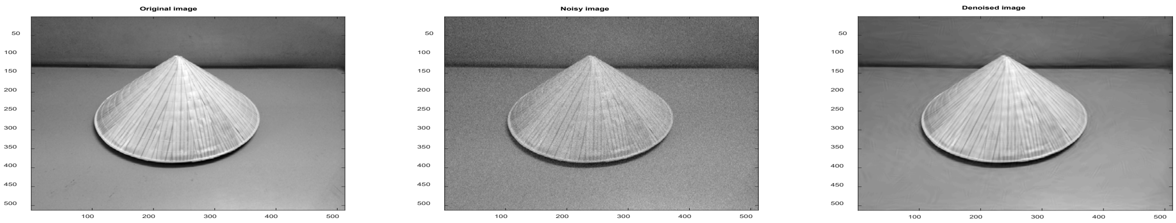
$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\left[I(i,j) - K(i,j)\right]^2 \qquad (8)$$

where m, n represent number of rows and columns in the input image. $I(i,j)$ and $K(i,j)$ denotes the noise free and noisy pixel image respectively.

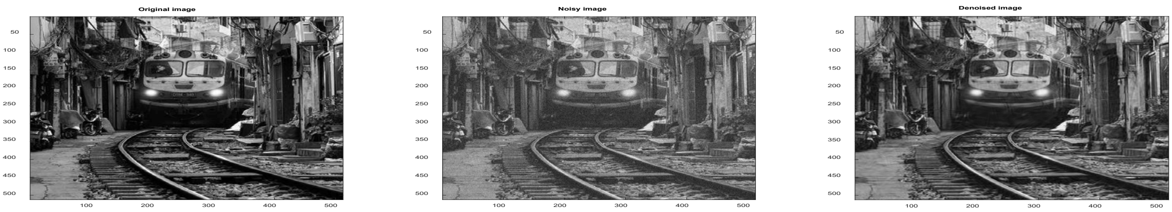
PSNR = 29.26



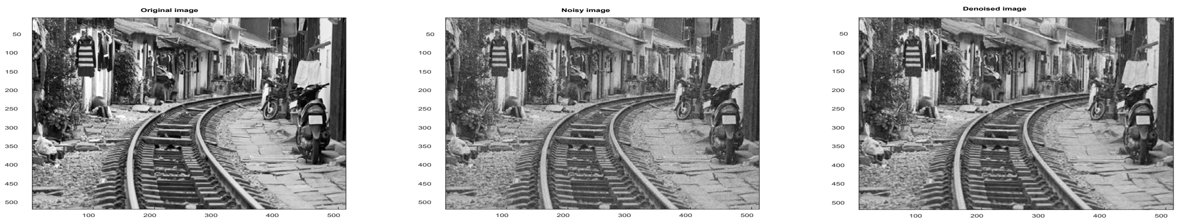
PSNR = 27.32



PSNR = 35.8965



PSNR = 28.53



PSNR = 27.76

Fig. 4. Test images with Gaussian noise (σ = 15) and corresponding PSNR; (left) original input image i.e. without noise, (center) image contaminated with white Gaussian noise, (right) denoised image of the proposed method.

4.2. Experiment results

We test our algorithm with images that contain circles and curves, especially the forth image with train rail. The tested images shown on Fig. 4. are corrupted with Gaussian white noise with noise standard deviation (σ) varies. In the pictures with curves, we see that the curve is preserved clearly.

We proceeded to eliminate noise of 8-bit gray level images by proposed method, with each being a different noise variance (sigma). The results are obtained in the following table

Table 1. Denoising results expressed by PSNR parameter

Sigma	Lady with circle hat	One Pilar Pagoda	Tranditional Vietnamese Hat	Train	Train Track
15	29.38	27.46	35.97	28.62	27.89
25	26.72	24.69	33.75	25.80	24.93
35	25.10	23.17	32.04	24.18	23.18
45	24.06	22.18	30.71	23.06	22.04
55	23.16	21.36	29.90	22.20	21.14
65	22.55	20.75	28.87	21.55	20.43
75	21.97	20.33	28.13	20.91	19.83

In order to prove the effectiveness of the proposed image denoising method, we made a comparison with some other denoise methods such as median filter, Wiener filter, Hard thresholding, Soft thresholding. And this is the result of implementing the above methods:

Table 2. Comparision of different denoising methods

Noise image	Wiener filter	Median	Hard thresholding	Soft threshoding	Proposed method
28.23	33.28	32.12	32.67	32.90	34.20

5. Conclusion

In this paper, we presented an innovative image denoising method using Curvelet transform and histogram segmentation. Image denoising is one of the important fields in the restoration area because the degradation of images will affect the processes of feature recognition, segmentation, edge detection, etc. The comparison of the denoised image from the proposed method was made. Based on the experimental results, Curvelet transforms give far better performance than the wavelet transforms. Although Curvelet transforms is promising and efficient for noise removal, still onedrawback arises must be regarded in future. The drawback is the quality of the reconstructed plat areas in the images. Scraches apprears in reconstructed plat areas.

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