Determining the Hypothetical Hydrostatic Free Surface of the Closed Tank Filled Fully with Fluid Moving with an Acceleration to Calculate the Force Acting on Its Wall Applied in Fluid Transport

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Abstract

The hypothetical hydrostatic free surface is a new concept used to represent pressure distribution, calculate the pressure exerted on the wall of a moving closed tank with acceleration, translation or rotation around a fixed axis. The paper presents a common fluid mechanical problem, which has specific applications in liquid transport practice. The solution using the hypothetical hydrostatic free surface for a closed vessel is argued on the basis of mechanical theory. The article cited several new solutions and showed numerical examples to clearly see the results and assess more accurately the harmful effects of increasing pressure when the liquid transport vehicle suddenly accelerates, brakes especially in traffic crashes.

Keywords: Hypothetical hydrostatic free surface, closed tanks moving, pressure distribution.

1. Introduction

In fluid mechanics, the representation of pressure distribution and the force acting on a moving tank with acceleration, is extremely essential and practical. For an opening container with free surface, the free surface is the constant-pressure face, so the determination of pressure distribution and force on its walls is relatively easy. However, with a closed tank filled liquid, moving in acceleration, determination of its pressure distribution has a lot of debate. In this paper, author presents a concept of the “hypothetical hydrostatic free surface”, and how to define it to helps to represent the distribution of pressure and calculate the pressure on the tank wall more accurately. This is a new method that no author has mentioned so far.

In liquid transportation, the factor greatly affects the transport quality is acceleration, which changes the pressure in liquid. This is still a complicated issue that many scientific institutions, many scientists have mentioned but not enough. The method will evaluate the impact of increasing pressure when a vessel is in accelerated, braked, especially collided condition.

2. General problem

The tank is filled with homogeneous liquid (density ρ, kg/m^3), pressurized with pressure p_o *(N/m^2)*. The middle part of the tank is a circular cylinder of length L(m), two ends with flanges A and B are semi-spherical with radius *R(m)* as shown in Figure 1. The tank is placed on a steadily moving vehicle, then suddenly brake with acceleration is a (m/s ^{2}).

Determine the residual pressure changes in the vessel by plotting the pressure acting on the sides of the vessel on the vertical cross section of the vessel as shown.

Calculate the residual pressure components of the fluid acting on the bridge caps A and B of the tank when braking.

Example: Calculated for tank truck size L= 2m, R = 1m, ρ = 1000 kg/m^3, a = 7m /s^2 and a = 25m/s^2.

3. Solution

We use the new concept of "the hypothetical hydrostatic free surface". For an opening container, the free surface is constant - pressure surface, exposed to the air, and its residual pressure is zero. For a closed container filled fluid, it has not the free surface, but has the constant - pressure surface. We assume that there is a wider homogeneous liquid field, including the tank’s liquid and having the pressure distribution as in the closed tank; The surface, with zero residual pressure of this liquid field, is called the hypothetical hydrostatic free surface.

With the new concept mentioned above, and how to determine it as presented below, we can completely represent the pressure distribution in the

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assumed liquid field in general and the liquid portion in a tank in particular, and also from this, we accurately calculate the residual pressure due to the liquid acting on its walls at any position in this field.

To demonstrate the advantages of the hypothetical free surface in general, we investigate the specific cases. At first, the tank has no pressure, just full of water.

3.1. The tank is in stationary state or moving at a constant velocity in the gravitational field

In this case, the constant - pressure surfaces are the horizontal planes. The tank just filled with liquid without pressurizing, the hydrostatic hypothetical free surface is the horizontal plane going through the highest point C as shown in the Figure 2.

The pressure distribution in the tank increases with the depth of the water due to the gravitational acceleration, and is calculated as follows:

$$p = \gamma h = \rho g h = -\rho g z$$

where

h is the depth of the investigated point from the free surface

The highest point C has the minimum gauge pressure $p_C = 0$.

The lowest point D has the maximum gauge pressure $p_D = \rho g 2R$.

The pressure distribution diagram on its walls is shown in the Figure 2.

3.2. *The vehicle is brake at acceleration a <0:*

To understand the pressure change in the fluid, in the mechanical aspect, we can consider the inertial acceleration $\vec{a}$ like the gravitational acceleration g, and their directions are only different.

Assume that it is not affected by the gravitational acceleration, g = 0 (for example in the universe). When it is static, or moves with the constant velocity, its entire volume has no pressure.

When the vehicle is decelerated with a negative aceleration, due to the inertial force, the liquid is pushed forward in the x direction, and the pressure at the point A is zero, $p_A = 0$. The constant-pressure surface is the vertical face perpendicular to the x axis, and the hypothetical free surface passes through point A.

The change of the pressure distribution in the tank on the x direction can be found as follows:

$$p = p_0 - \rho a x \quad (1)$$

where p_0 is the pressure value of the original O.

Set the coordinate origin to point A, we have:

$$p_0 = p_A = 0 \quad (2)$$

The maximum pressure on its wall at the point B is calculated as follows:

$$p_B = -\rho a.\overline{AB} = -\rho a(L + 2\mathrm{R}) \quad (3)$$

In general, when the tank suffers both the gravitational acceleration and inertial acceleration. The pressure distribution in the tank on both directions can be determined by the following formula.

$$p = p_0 - \rho g z - \rho a x \quad (4)$$

The constant-pressure surface including the free surface inclines an angle from the horizontal plane [1,4, 5,7,8].

$$tg\alpha = -a / g \quad (5)$$

As defined above, the hypothetical hydrostatic free surface is the surface in the hypothetical liquid field with the zero pressure, so we will find a hypothetical point, with zero pressure, (possibly outside the tank) in the assumed liquid field when accelerated with a < 0 and g > 0.

Combining the effect of the gravitational acceleration g (Figure 2) and the inertial acceleration a (Figure 3) on the fluid field we see that the point O is the intersection of two hypothetical hydrostatic free surface (perpendicular lines through A and horizontal lines via C). (Figure 4).

The pressure augment at any direction in the assumed fluid field can be obtained.

On horizontal direction, we have $p = -\rho a x$:

On vertical direction, we also have $p = \rho g h$:

On direction perpendicular with the hypothetical free surface, we have $p = \rho q n$:

where $\vec{q}$ is the synthetic acceleration and can be determined from the formula.

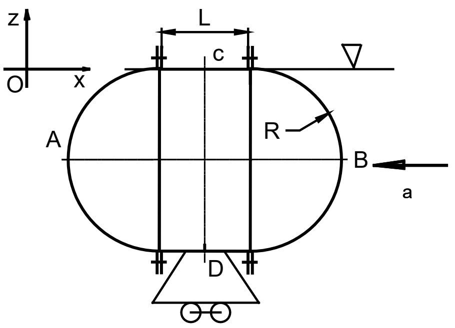

Fig. 1. Model of a tank on a moving vessel

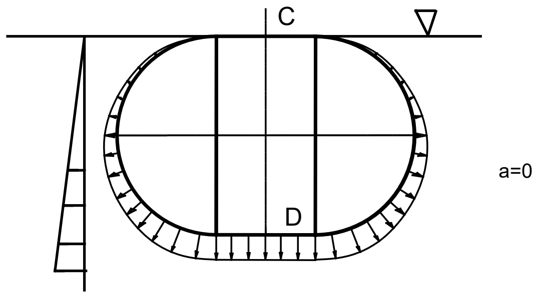

Fig. 2. The pressure distribution on tank's wall when it is in stationary state

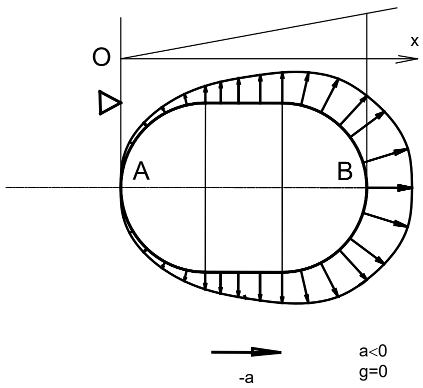

Fig. 3. Pressure distribution on tank's wall with zero weight and in moving acceleration

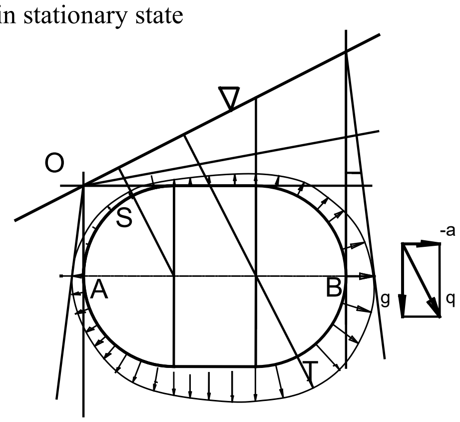

Fig. 4. Pressure distribution on the tank's wall with gravitational and inertial acceleration (g>0, a<0).

$$\vec{q} = \vec{g} + \overrightarrow{-a}$$
$$q = \frac{g}{\cos\alpha} = \frac{a}{\sin\alpha} \qquad (6)$$

The pressure of the point A is due to the gravitational acceleration g.

$$p_A = p_O + g.\overline{OA} = \rho g R \qquad (7)$$

The total pressure of the point B is due to the gravitational and inertial acceleration.

$$p_B = p_A - \rho a.\overline{AB}$$
$$= \rho g R - \rho a.\left(L + 2\text{R}\right) \qquad (8)$$

The point, having the maximum gauge pressure, is the point C due to the negative inertial acceleration

$$p_C = p_0 - \rho a.\overline{OC}$$
$$= -a\left(R + \frac{L}{2}\right) \qquad (9)$$

The lowest point, having the maximum gauge pressure, is the point D due to the gravitational and inertial acceleration.

$$p_D = p_C + \rho g.\overline{CD}$$
$$= -a\left(R + \frac{L}{2}\right) + \rho \text{g}2\text{R} \qquad (10)$$

The point S, the nearest hypothetical free surface, on the tank's wall has the minimum pressure

The point T, the farthest hypothetical free surface, on the tank's wall has the maximum pressure

When the hydrostatic hypothetical free surface is obtained, it is easy to determine the force acting on the tank's sides.

3.3. The tank is pressurized with $p_0 \neq 0$

Compared with a vessel without initial pressure, in the initial pressure vessel the pressure at each point either increases by the same amount p_0>0 or decreases by the same amount p_0 <0.

In this case, the hypothetical hydrostatic free surface (∇') parallels to the surface with $p_0 = 0$ and translates in the acceleration inertial direction one distance $x_{\_} = p_o / \rho.a$, or in the acceleration gravitational direction amount $h_0 = p_0/\rho g$, or in the q direction a mount $\mathrm{n_o} = \mathrm{p_o}/(\rho.\mathrm{q})$ in which the q is calculated from the Formula 7.

Thus, we have taken the relative hydrostatic problem (the tank moving with the acceleration) to the absolute hydrostatic problem with the acceleration $\vec{q}$ and the hydrostatic hypothetical free surface (∇'). The $\vec{q}$ is calculated from the Formula 6 and 7.

3.4. Using the hypothetical hydrostatic free surface to calculate the gauge pressure of the liquid acting on the A and B flange.

When there is a hypothetical hydrostatic free surface, we consider the given problem into a closed tank submerged in the liquid, and can be calculated as any curved surface with a note that the fluid pressure is always directed at surface of the tank.

The force acting on the curved surface at any direction can be calculated as follows [1]:

$$P_s = \gamma_s . V_s = \rho . q_s . V_s \qquad (11)$$

Where, q_s is the component of an acceleration on the s direction, and Vs is the body creates pressure in the direction s of the surface (the volume of fluid that exerts pressure on the surface in the direction s).

We can now calculate the gauge pressure of the liquid on the A and B flange without pressurizing when the vehicle is braked with the positive gravitational acceleration and negative inertial acceleration.

The force on the vertical direction can be define.

$$P_{zA} = P_{zB} = P_z = \rho g V \qquad (12)$$

where, V is the volume of the half sphere.

The force on the horizontal, with the direction perpendicular with the drawn plane, is zero.

$$P_y = P_{yA} = P_{yB} = 0 \qquad (13)$$

The force on the x direction is the same direction of acceleration a, so we have [1]

With the flange A, the force is directed in the left

$$P_{xA} = -\rho . a . V_A \qquad (14)$$

where, V_A is the pressure object of sphere A in the direction of acceleration a, and S_x is the area of the cylinder bottom shown in the Figure 6.

$$V_A = S_x \cdot \left(R + \frac{R}{tg\alpha} \right) - V \qquad (15)$$

With the flange B, the force is directed to the right.

$$P_{xB} = P_{xA} - \rho . V_b . a \qquad (16)$$

where, V_b is the tank's volume

$$V_b = 2V + S_x . L \qquad (17)$$

Example: Calculating for the truck's tank filled water, having two meters of the length and one meter of the radius shown in the Figure 6, without pressurizing, is to illustrate the effect of the acceleration on the pressure distribution for three cases.

First, the truck is in stationary or moving state with the constant velocity a = 0

Second, the truck is is suddenly braked safely. Because the adhesive coefficient of the truck's tires is calculated as $\varepsilon = a / g$, and taken in the range of 0.7 - 0.8, the acceleration in the second case is of 7 m/s^2 .

The last, the truck is suddenly collided with the acceleration a of 25 m/s^2.

The calculation results are shown in the table 1.

Table 1. The change of pressure and force of the tank's flange

No	Parameter	Symbol	Unit	Acceleration m/s^2		
				0	7	25
1	At the point A	p_A	kN/m^2	9,81	9,81	9,81
2	At the point B	p_B	kN/m^2	9,81	37,81	109,81
3	At the flange A	P_{XA}	kN	30,80	38,13	56,97
4	At the flange B	P_{XB}	kN	30,80	111,39	318,63

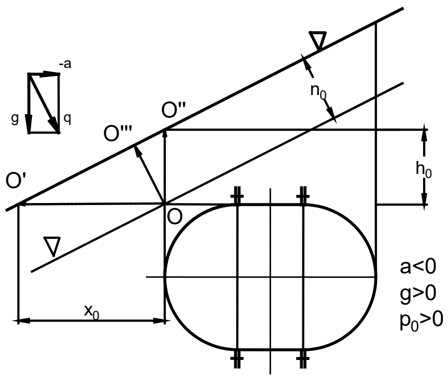

Fig. 5. The change of the hypothetical hydrostatic free surface when the tank is pressurized g > 0; a < 0, $p_o > 0$.

From the above example, we can see that the pressure and force of the closed tank filled fluid, go up rapidly when the acceleration increases. Especially for vehicles carrying water, gasoline tanks braked suddenly, or collided unexpectedly, the pressure in the tank can increase many times. This can easily cause explosion or damage the container structure.

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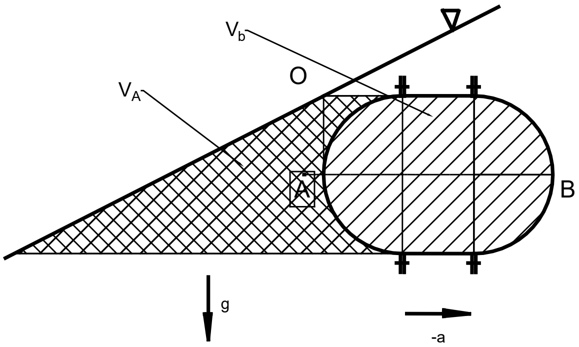

Fig. 6. Method to calculate the force acting on the bottle flanges

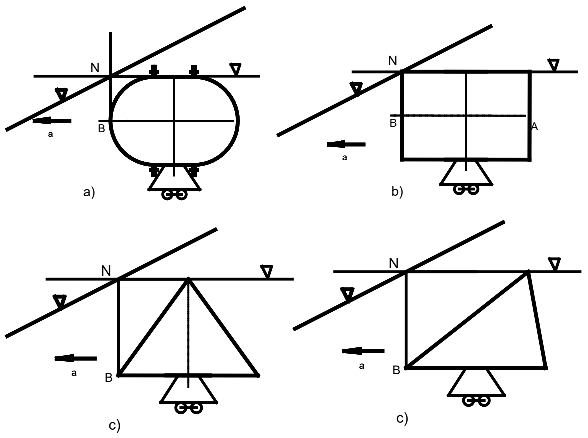

Fig. 7. Some examples determine the hypothetical hydrostatic free surface of the closed tank with different jar structures.

3.5. Some examples determine the hypothetical hydrostatic free surface of the closed tank with different jar structures

In order to clearly see the huge difference in pressure and force values of the sealed vessel with different structures when calculated according to the new method, we can name some common structural cases in practice as shown in Figure 7.

4. Conclusion

The paper has presented the concept of the hypothetical hydrostatic free surface, and the method of determining it with closed containers containing fully liquid and moving with translational acceleration. With this concept, we can easily calculate the distribution pressure and the force acting on any surface of a liquid reservoir.

The concept of "the hypothetical hydrostatic free surface" is also applied to closed containers filled fully fluid when rotating around the vertical axis with an acceleration.

The results of this paper are also applied to calculate pressure in ship propellers and thrust propeller.

Acknowledgments

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