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Theoretical Approach to the Performance Analysis of a Low-Specific Speed Pump as Turbine Based on Hydraulic Losses

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Abstract

This paper focuses on building a theoretical method, based on the calculation of hydraulic losses to predict the energy performance of a Low-specific speed Pump as Turbine (PaT) quickly and accurately that supported for the PaT's impeller design. The Euler equation is built with the analysis of hydraulic loss calculation and flow phenomena passing on the machine. The trust of this approach is validated by comparing with the available experimental data. The results show that the theoretical energy curves of the PaT are in a good agreement with the tendency of the experimental results in both pump and turbine modes in vicinity of design point. Thereby, we estimated the head loss distributions in the flow system of PaT, including: the total of the head loss, the impeller loss, the disk friction and spiral casing losses. From these results, to improve and harmonize the efficiency of reversible impeller in both pump and turbine modes, the designer is recommended to decrease the diameter D_2 and increase the impeller widths b_1, b_2 for improvement.

Keywords: Pump as turbine, reversible impeller, hydraulic losses, turbomachine, storage hydropower

1. Introduction

A centrifugal pump has been used as turbine (pump as turbine – PaT) application in pumped storage hydropower plants since 1950s [1]. The prediction of the hydraulic characteristics of PaT is still very difficult problem. Several research works have been suggested to predict the turbine efficiency based on the data of the pump efficiency at the best efficiency point (BEP) [2,3] or pump geometric parameters [4-6]. However, it is very complex and difficult to find a general relation that can cover behaviors of all pumps in a reverse mode. Gülich [6] and Chapallaz [7] reported that the relationship between the pump and turbine efficiency is not the same for all types and sizes of pump, but it depends on the flow pattern through the machine, expressed by the specific speed and losses. In order to predict the performance of PaT with a low-specific speed ($n_s \le 150$), we have to evaluate the loss distributions in the machine.

The main purpose of this work is to build a theoretical method basing on the calculation of hydraulic losses, then apply for predicting the energy characteristic curves of the PaT with a low-specific speed ($n_s \le 150$) and discussing for the developing in design. The calculation of the head loss components including the major hydraulic losses in the volute casing (h_{cas}), the impeller (h_{im}) and the draft tube (h_{dr}), the disk friction losses (h_{disk}) and the volumetric loss (Q_{leg}) must be carried out. Finally, the net head and the overall efficiency equations are set up. For validation, we make a comparison with the available experimental data for estimating the precision [8]. By the result, the loss distributions in different zones and the geometrical relationship of the impeller will be also discussed for improving efficiency in design of PaT.

2. Hydraulic losses in comprehension

The theoretical head of the impeller is used in this present study is based on basic Euler equation (1), [6]

$$gH_{theo} = u_2 c_{u2} - u_1 c_{u1} \quad (1)$$

Here, H_{theo} is theoretical head [m]; c is absolute velocity [m/s]; u is circumferential velocity [m/s]; *1* is marked for location at the leading edge of the blade; *2* is marked for location at the trailing edge of the blade.

2.1. The flow phenomenon

The flow through the impeller channel with the limited blade number and thickness will be slipped and blocked.

2.1.1. The flow phenomenon in the pump mode

The slip phenomenon

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Due to difference between the flow and blade angles, the flow at the inlet section of the impeller is affected by the slip factor. Gülich [6] and Shi et al. [9] gave the formulas to calculate the slip factor γ. However, it is so difficult to apply because of many unknown parameters. An alternative method for calculating this factor is proposed by Gülich [6] for radial pump as equation (2):

$$\gamma = f_1\left(1-\frac{\sqrt{\sin\beta_B}}{Z^{0.7}}\right)k_w \tag{2}$$

$$k_w = 1-\left(\frac{d_m^* - \varepsilon_{Lim}}{1-\varepsilon_{Lim}}\right)^3 \tag{3}$$

$$\varepsilon_{Lim} = \exp\left\{-\frac{8.16\sin\beta_B}{Z}\right\} \tag{4}$$

Here, for for radial pump, f_l=0.98, d^* =D_1/D_2; Z is the number of the impeller's blade and β_B is an angle between the relative velocity *w* and the circumferential velocity *u*.

The effect of the blade blockage

Due to the thickness *e* and the finite blade number *Z*, the blockage phenomenon will appear at the inlet and the outlet sections and restrict the flow through channel. As a result, the flow velocity increases and has effects on the distribution of velocity in cross section [6]. The blade blockage factor is defined as equation (5) following:

$$\tau = \left\{1-\frac{Ze}{\pi D\sin\beta_B\sin\lambda}\right\}^{-1} \tag{5}$$

with *D* is impeller's diameter and λ is an angle between the blade and side disk.

2.1.2. The flow phenomenon in the turbine mode

Gülich [6] indicated that the effects of the blade blockage in the pump and turbine modes are similar according to equation (5), while the effects of the slip factor in the turbine mode can be ignored. This is because the flow approach angle in the turbine mode depends mainly on the flow and the cross-sections of the guide vanes.

2.2. Theoretical head

2.2.1. Theoretical head of the pump's impeller

In order to determine the theoretical head according to equation (1), the velocity components at the inlet and outlet sections must be identified. In theory, based on the velocity triangles (Fig.1a), we have:

$$c_{1u} = \frac{c_{1m}}{\tan\alpha_1} = \frac{Q_p}{A_{1m}\tan\alpha_1} \tag{6}$$

$$c_{2u} = u_2 - w_{2u} = u_2 - c_{2m}\cot\beta_2 \tag{7}$$

When considering the influences of the slip factor and the blade blockage, equation (7) can be derived:

$$c_{2u} = \gamma_2 u_2 - \frac{Q_P\tau_2\cot\beta_{2B}}{A_{3m}} \tag{8}$$

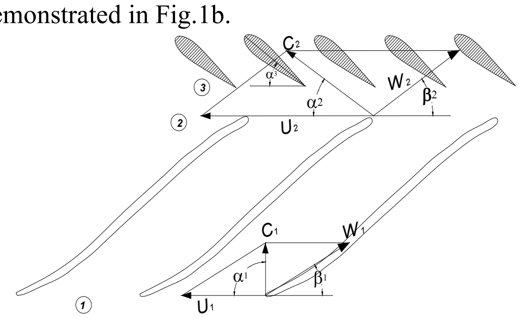
From the equations (6) and (8), equation (1) for the pump mode can be derived:

$$H_{im}^{theo,P} = \frac{\gamma_2 u_2^2}{g} - \frac{u_2 Q_p\tau_2}{gA_{3m}\tan\beta_{2B}} - \frac{u_1}{g}\frac{Q_p}{A_{0m}\tan\alpha_1} \tag{9}$$

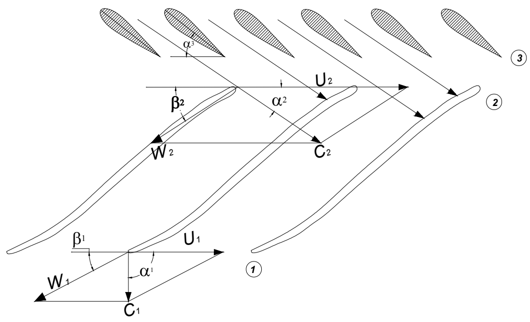
A_{im} is area of the local cross section at radius R_1, R_2 and R_3 correspondingly positioning of impeller leading edge, trailing edge and vane's inlet.

2.2.2 Theoretical head of the turbine impeller

In the turbine mode, the absolute flow angle α_2 is important and affects greatly the velocity triangle at the impeller inlet. Gülich [6] and Chapallaz [7] showed that this angle can be determined from the guide vanes or spiral casing geometry. An approximation of the inflow angle α_2 can be calculated from the cross section of the vane throat as demonstrated in Fig.1b.



a) Pump mode



b) Turbine mode

Fig. 1. Determination of the outflow angle from the throat area, applicable to a guide vane

From the inlet velocity triangle of the impeller in the turbine mode as Fig.1b, we have:

$$c_{1u} = u_1 - c_{1m}\cot\beta_1 \tag{10}$$

When considering the influence of the finite number of blades, we have:

$$c_{1u} = u_1\left(1+\frac{\pi\sin\beta_{1B}}{Z}\right) - c_{0m}\tau_1\cot\beta_{1B} \tag{11}$$

$$c_{2u} = \frac{R_3}{R_2}c_{3u} = \frac{R_3}{R_2}\cot\alpha_{3B}c_{3m} \tag{12}$$

$$\Rightarrow c_{2u} = \frac{R_3}{R_2}\cot\alpha_{3B}\frac{Q_T}{A_{3m}} \tag{13}$$

From the equations (11) and (13), equation (1) for the turbine mode can be derived as following (14):

$$H_{im}^{theo,T} = \frac{R_3\cot\alpha_{3B}Q_T}{R_2 g}\frac{u_2}{A_{3m}} - \frac{\pi\sin\beta_{1B}u_1^2}{gZ} + \frac{c_{0m}\cot\beta_{1B}u_1\tau_1}{g} - \frac{u_1^2}{g} \tag{14}$$

2.3. Determination of the hydraulic losses in the impeller

2.3.1. Hydraulic losses in the impeller of the pump mode

Friction losses:

The friction loss is defined as the linear loss caused at the wall boundary layer of the blade, the impeller chamber and so on. Under the effects of fluid viscosity, the friction loss is defined as equation (15), [6]:

$$h_{fr} = \frac{\xi_{fr}u_2^2}{2g} \tag{15}$$

$$\xi_{fr} = 4c_d\frac{L_{av}}{D_h}\left(\frac{w_{av}}{u_2}\right)^2 \tag{16}$$

where C_d is the dissipation coefficient:

$$c_d = \left(c_f + 0.0015\right)\left(1.1+\frac{4b_2}{D_2}\right) \tag{17}$$

Friction coefficient c_f and Reynolds number R_e are given as:

$$c_f = \frac{0.136}{\left(-\log\left(0.2\dfrac{\varepsilon}{L_{av}+\dfrac{12.5}{\mathrm{Re}}}\right)\right)^{2.15}} \tag{18}$$

$$\mathrm{Re} = \frac{w_{av}L_{av}}{\nu} \tag{19}$$

L_{av} is the average length of space between the blades; D_h is the equivalent hydraulic diameter of the impeller as equation (20) and w_{av} is the average relative velocities as equation (21):

$$D_h = \frac{2\left(a_2b_2 + a_1b_1\right)}{a_2 + b_2 + a_1 + b_1} \tag{20}$$

$$w_{av} = \frac{2Q}{Z\left(a_1b_1 + a_2b_2\right)} \tag{21}$$

Incidence loss at the impeller inlet

When the flow rate is not equal to the designed flow rate, incidence at the inlet can lead to flow separation on the blade surface, which will then cause incidence loss. The incidence loss of blade inlet is defined by Bing et al. [5] as equation (22):

$$h_{in} = \frac{f_{inc}w_{1u}^2}{2g} \tag{22}$$

where f_{inc} is an incidence loss coefficient and value vary in the range of 0.5 to 0.7.

Inlet recirculation loss

The appearance of the inlet recirculation usually encounters in the pump modes, while this loss is ignored in the turbine mode due to effects of the guide vanes at the inlet section. The head loss due to recirculation is given by Djebedjian [10], then:

$$h_{rec} = 0.005\frac{\omega^3 D_1^2}{\gamma Q}\left(1-\frac{Q}{Q_{BEP}}\right)^{2.5} \tag{23}$$

The recirculation loss depends on the inlet geometry of the impeller and the flow rate. A default value of 0.005 for the loss coefficient is taken.

Diffusion loss

Due to the thickness of the blade tail, the fluid experiences a process of sudden expansion, which leads to the Jet-Wake structure in the channel blades. If the ratio of the relative velocity at the inlet w_1 and the outlet w_2 exceeds a value of 1.4, the separation may appear in the impeller at any point. This loss is also identified by Bing et al. [5] as equation (24):

$$h_{dif} = \left(1-\frac{B}{1-\varepsilon}\right)^2\frac{c_{2m}^2}{2g} \tag{24}$$

where B is the ratio of diffuser vane inlet width to impeller outlet width, ε is the wake factor defined as equation (25):

$$\varepsilon = 1 - \frac{w_2}{w_0}\left(\frac{w_0}{w_2}\right)_{crit} \quad (25)$$

Here, $(w_0/w_2)_{crit}$ is the critical velocity ratio when fluid has flow separation to lead to Jet-Wake structure, the value generally is selected of 1.4.

Circulation loss

When the impeller rotates, the relative velocity (W) at the suction surfaces of the blades increases and W at the pressure surfaces of the blades decreases. As a result, at the closed impeller channel, the circulatory flow will happen. This loss head is given by Djebedjian [10] as equation (26):

$$h_{cir} = \frac{u_2^2\left(1-\gamma_2\right)+u_1^2\left(1-\gamma_1\right)}{g} \quad (26)$$

2.3.2. Hydraulic losses in the impeller of the turbine mode

Hydraulic loss has direct relationship with the geometrical shape of flow channel. If no test data are available, the turbine characteristics are often estimated the statistical correlations from a particular centrifugal pump [6]. In the turbine mode, noted that the energy performance is mostly determined by the inlet triangle with the governing element of the volute casing (angle α_3). So, the circulation loss now occurs at the inner periphery of the impeller.

2.3.3. The losses in the spiral casing, vane, draft zones and other losses [6, 11]:

The loss in the spiral casing

The losses from the spiral casing and vane are given as equation (27)

$$h_{cas} = \xi_{vol}\frac{u_2^2}{2g} \quad (27)$$

$$\xi_{cas} = \frac{1}{Qu_2^2}\sum\left(c_f + 0.0015\right)c^3\Delta A \quad (28)$$

where ΔA is the wetted surface

Loss in the vane diffuser

+ Friction in the vaneless diffuser with constant width

$$\xi_{fv} = \frac{2c_f R_2}{b_3\sin\alpha_3\cos^2\alpha_3}\left(\frac{c_{2u}}{u_2}\right)^2\left(1-\frac{R_2}{R_4}\right) \quad (29)$$

+ Shock losses

$$\xi_{sv} = \phi_2^2\left(\tau_2 - \frac{b_2}{b_3}\right)^2 \quad (30)$$

$$\phi_2^2 = \frac{c_{2m}}{u_2} \quad (31)$$

$$h_{van} = \left(\xi_{fv} + \xi_{sv}\right)\frac{u_2^2}{2g} \quad (32)$$

Loss in the space zone

In structure of PaT, there are spaces between the blades, the casing and vane zones. Under the pressure difference between the two surfaces of the blades (pressure and suction sides), there are two stages of flow process, which are sudden compression and sudden expansion which cause clearance loss and has calculated by [11]:

$$h_{spa} = 0.6\frac{a_{sp}}{b_2}\frac{c_{u2}}{g}\sqrt{\frac{2\pi}{Zb_2}\left(\frac{R_{1t}^2 - R_{1h}^2}{R_{2t} - R_{1t}}\right)c_{2u}c_{m1}} \quad (33)$$

Where *spa* means space between zones.

The loss in the draft tube

In the draft tube, the total losses are made of friction (h_{fr}), diffusing (h_{pd}), and the kinetic losses (h_{pc}) as equation (34) [11]. In this case of pump mode, the flow is gradual contraction loss, while it is gradual expansion loss in turbine mode.

$$h_{dr} = h_{fr} + h_{pd} + h_{pc} \quad (34)$$

$$h_{fr}^{dr} = \frac{\lambda L}{8tg\frac{\theta}{2}D_1}\frac{c_3^2 - c_5^2}{2g} \quad (35)$$

$$h_{pd}^{P} = 0.8\left(\sin\frac{\theta}{2}\right)\frac{\left(c_3 - c_5\right)}{2g} \quad (36)$$

$$h_{pd}^{T} = 3.2\left(tg\frac{\theta}{2}\right)^{1.25}\frac{\left(c_3 - c_5\right)^2}{2g} \quad (37)$$

$$h_{pc} = \frac{c_5^2}{2g} \quad (38)$$

Disk friction loss

The head loss due to the disk friction is calculated from Djebedjian [10]:

$$h_{disk} = 0.5C_M\frac{\rho\omega^3\left(0.5D_2\right)^2}{\gamma Q} \quad (39)$$

The disk friction coefficient is calculated from:

$$C_M = \left(\frac{k_s}{0.5D_2}\right)^{0.25}\left(\frac{s}{0.5}\right)\mathrm{Re}^{-0.2} \quad (40)$$

where: k_S - the disk surface roughness and s - the axial gap.

Leakage loss

The leakage flow is calculated as equations (41) and (42) by Gulich [6]:

- Leakage loss at impeller

$$\frac{Q_{\mathrm{Im}}^{leg}}{Q_{BEP}} = \frac{aZ_H}{n_s^m} \tag{41}$$

- Leakage loss at seal

$$\frac{Q_{se}^{leg}}{Q_{BEP}} = \frac{5,5}{n_s^{1.8}} \tag{42}$$

2.4. The overall efficiency in energy equation

2.4.1. The overall efficiency of the pump mode

The overall efficiency of pump is computed by equation (43):

$$\eta_P = \frac{H_{im}^P}{H_{im}^{theo,P} + h_{cas}^P + h_{spa}^P + h_{van}^P + h_{dr}^P + h_{disk}^P} \frac{Q^P}{Q^P + Q_{leg}^P} \tag{43}$$

in which, H_{im}^P is the actual head developed by the pump at any discharge rate:

$$H_{im}^P = H_{im}^{theo,P} - h_{cir}^P - h_{in}^P - h_{fr}^P - h_{rec}^P - h_{dif}^P \tag{44}$$

2.4.2 The overall efficiency of the turbine mode

The overall efficiency of turbines is computed by equation (45)

$$\eta_T = \frac{H_{im}^T}{H_{im}^{theo,T} + h_{cas}^T + h_{spa}^T + h_{van}^T + h_{disk}^T} \frac{Q^T - Q_{leg}^T}{Q^T} \tag{45}$$

in which, H_{im}^T is the actual head developed by the turbine at any discharge rate:

$$H_{im}^T = H_{im}^{theo,T} - h_{in}^T - h_{fr}^T - h_{rec}^T \tag{46}$$

2.5. Applied model in analysis

In this paper, we refer to the available experimental data from a PaT model in the research of Yang et al. [8] for validation of our theoretical approach. The same mode and parameters are used in this study are shown in Table 1. This is a single stage centrifugal PaT with rated speed of 150 rpm in both turbine and pump modes.

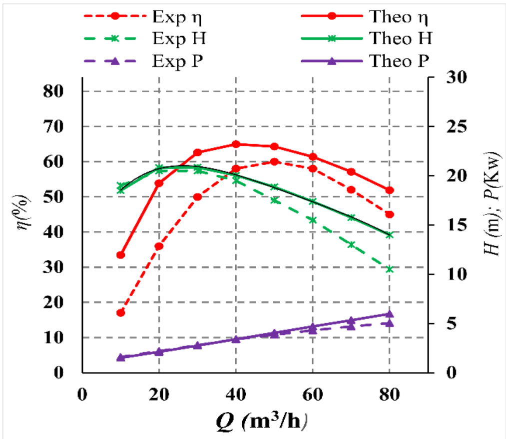
Table 1 Major geometric parameters of the PaT [8]

D_1	Z	e	β_{1B}	L	D_2	θ	β_{2B}	b_2
mm		mm	^{0}	mm	mm	^{0}	^{0}	mm
102	6	4	39	15	235	88.06	28.22	15.30

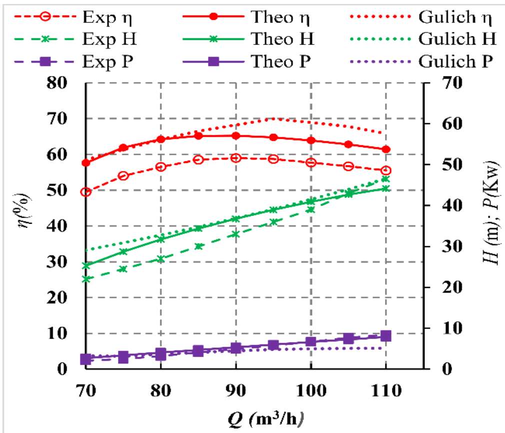
3. Results and discussion

3.1 Validation of the theoretical method

To validate the accuracy of the theoretical method, the experimental and theoretical energy curves of the impeller with diameter of 235 mm are presented and compared in Fig.2. The figure shows that the theoretical performance curves are in a good agreement in the tendency of the experimental results in both modes. But some gap between the efficiency curves should be considered. There are some loss components which could not be calculated by the theoretical method, such as the turbulent losses in the space between the impeller hub, shroud and casing or the sealing gap, the flow regime around the particular blade, mechanic transmission. In addition, in the lower and higher flow regions, the swirling and circulation regions appear that cause the significant loss in both modes. These losses are calculated difficultly by theoretical methods and should be considered more carefully in the future works.



a) Pump mode



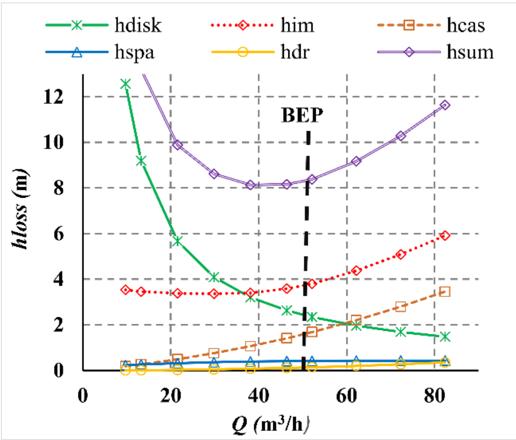
b) Turbine mode

Fig.2. Comparison of experimental and theoretical calculation curves (*Theo* – present approach, *Exp* - experimental data [8], Gulich calculation - [6])

Additionally, in the turbine mode, Gülich [6] introduced the steps to predict the turbine characteristics from the statistical correlations of a particular centrifugal pump. In which, the turbine characteristic curves $H_T = f(Q_T)$ and $\eta_T = f(Q_T)$ show relations to the BEP and runaway point. In this paper, those results of Gülich are also used to compare with present theoretical results as shown in Fig.2b. Accordingly, the present theoretical method shows more suitable with the experimental data than the calculation by Gülich [6] in order to predict the PaT's turbine mode performance.

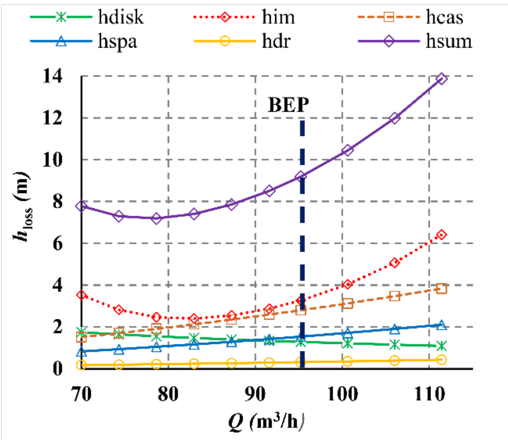
Table 2. Comparison of the experimental and theoretical results at BEPs

Results	Pump mode			Turbine mode		
	η	H	P	η	H	P
	%	m	kW	%	m	kW
Experiment	59.66	17.16	3.93	58.68	36.07	5.82
Theory	63.83	18.61	4.16	64.75	39.01	6.03
Error (%)	4.17	7.87	5.63	6.08	7.54	3.47



a)

Pump mode



b) Turbine mode

Fig.3. The loss distribution in different zones of the PaT system by present approach

The Table 2 lists the errors at BEP in two modes. As illustrated, the errors of the efficiency, head and shaft power are 4.17%, 7.87% and 5.63% respectively at the BEP in the pump mode, while those in the turbine mode are 6.08%, 7.54% and 3.47%.

3.2. Analysis the hydraulic loss distribution

The Figure 3 shows a comparison of the loss distribution in the different zones of two modes of pump and turbine by the theoretical method. The five loss components including the loss in the spiral casing (*hcas*), space (*hspa*), impeller (*him*), the draft tube (*hdr*), the disk friction (*hdisk*) and the sum of loss components (*hsum*) are presented. The results illustrate a significantly difference of these components in two modes. The pump impeller loss has the largest proportion with about 45.29%, followed by the disk friction loss and spiral casing with 27.96% and 25.07%, respectively. However, in the turbine mode, the impeller loss (35.45%) is 11.72% smaller than the spiral casing zone loss (47.16%), while the disk friction loss is only 13.95%. The draft tube has the smallest loss ratio with these figures not exceeding 5% in both modes. These results are relatively suitable with published results of Shi [9], Rawal and Kshirsagar [12] and Singh [13].

4. Conclusion

In present paper, this theoretical prediction method is derived from the basic formulas comprehensively for reversible impeller with a low specific speed (n_s ≤150). By the results, the conclusion are as follows:

1) This model can be used to predict the tendency of the energy characteristic curves of head, discharge, and efficiency quickly and acceptably. The errors in calculation of the efficiency, head and shaft power are 4.17%, 7.87% and 5.63% respectively at the BEP in the pump mode, while those values in the turbine mode are 6.08%, 7.54% and 3.47% respectively.

2) In PaT, the main head losses caused by the impeller zone, the spiral casing zone and the disk friction that occupy the largest ratio. In the pump mode, the impeller loss occupies the largest ratio with nearly 45.29%, followed by the disk friction loss and spiral casing with 27.96% and 25.07%. In the turbine mode, the impeller loss is 11.72% smaller than that of the spiral casing zone and space (35.45% comparing to 47.16%), while the disk friction loss is only 13.95%. Therefore, the improvements in designing the blade profile, the diameter D_2 and the dimension of the spiral casing are very important. For the harmonize of the impeller's efficiency in the pump

and turbine modes, the designer should decrease the diameter D_2 and increase the impeller width b_1, b_2.

3) The head losses in the spiral casing and the space regions depend mainly on the flow and rise rapidly once the flow capacity increases, while the head loss in the draft tube is affected insignificantly.

Although the loss components caused by particular flow regime around the blade, turbulent or swirling and circulation phenomena in the spaces of the flow system, the present theoretical approach can help the designer make predicting the energy performance quickly for the case of a low specific speed PaT at the early design stage to save time and cost.

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