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Identifying the Key Parameters in Mathematical Model to Simulate the Main Nutrients Transformation Process in Vietnamese Catfish *(Pangasianodon Hypophthalmus)* Ponds

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Abstract

Pangasius farming is a process of putting a large amount of material into the pond but then only harvesting a small amount of the farmed fish and discharging the rest as waste into the aquatic environment. This practice pollutes water sources and degrades the water environment in nearby rivers and canals. To address this issue, it is necessary to establish a mathematical model that can simulate the nutrient transformation processes in the pond. This study aims to set up a mathematical model to simulate the variation in the compositions of major nutrients in Pangasius ponds, determine the key parameters affecting the model, optimize the culture regime, enhance fish quality, and minimize environmental impact. The results have shown that the model's output data after calibration is relatively close to reality and the key parameters of the model were defined.

Keywords: Pangasius, modelling, nutrient, sensitive analysis, calibration.

1. Introduction

For many years, thanks to the favor of nature, pangasius and catfish *(Pangasius bocourti)* farming has been one of the aquaculture sectors with a high export value that is growing rapidly in the Mekong River Delta provinces. According to the Vietnam Association of Seafood Exporters and Producers (VASEP), in 2019, Vietnam exported pangasius to 131 markets, including eight main markets: China - Hong Kong, USA, EU, ASEAN, Mexico, Brazil, Colombia, and Japan, contributing to a total export revenue of 1.61 billion USD [1].

With a large amount of residues and high concentration of pollutants, the waste from Pangasius ponds has been negatively impacting the surrounding environment and the aquaculture sector, including fish health and yield [2]. When producing 1 ton of final Pangasius product, the amount of water discharged is 4,023 m^3. Out of this, 63% of the water is taken from the river, 19% from the main canal, and 11% from the field or garden. To produce 300 tons of final fish product, farmers need to use an average of 450 - 480 tons of feed. However, only about 75% of this feed is actually consumed by the fish. The remaining feed becomes leftover and rotting food and is then deposited on the bottom of the pond (pond farming) or rivers [3].

According to [4], a pond with a yield of 300 tons/ha, each farming season will release to the environment about 2,677 tons of wet sludge (equivalent to 937 tons of dry sludge) and 77,930 m^3 of wastewater. This amount of waste is discharged directly into the environment, causing depression of water quality, environmental pollution, and disease outbreaks, thereby reducing the sustainability of Pangasius farming. When feeding fish with industrial feed, only 37.5% of the accumulated nutrients end up in the fish meat. Out of this, dry matter such as nitrogen and phosphorus account for 32.6% and 42.7%, respectively. The remaining 24.7% is made up of other nutrients accumulated in the fish. Approximately 67.4% of the dry matter that is discharged into the environment consists of 5.03% in water, 45.63% in bottom sludge, and 16.74% loss due to evaporation or osmosis). As a result, there is a high accumulation of nutrients in the sludge with organic content accounting for about 10.5 - 11.7% [4], total nitrogen (TN) accounting for about 0.5%, and total phosphorus (TP) accounting for about 0.22% [5].

Many models have been developed for the management of rivers, lakes, and ponds in the world. Janse (1990) [6] modelled phosphorus fluxes in the hypertrophic Loosdrecht lakes located in the Netherlands. A dynamic and deterministic model is presented to simulate the phosphorus cycle and

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plankton growth in shallow, hypertrophic waters both before and after restoration measures. Carbon and phosphorus were described independently so that the dynamics of the P/C ratios could be modelled. In 2004, Janse [7] also used Model PC Lake to study the eutrophication of shallow lakes and ditches. The model describes a completely mixed water body that comprises both the water column and the top sediment layer, with the most important biotic and abiotic components. The model relied on several inputs, including water inflow, infiltration or seepage rate (if applicable), nutrient loading (such as Nitrogen (N) and Phosphorus (P)), particulate loading, temperature, light, dimensions of the lake (depth and size), size of the marsh zone, sediment features, and loading history (initial conditions). As outputs, the biomass and concentrations of all state variables and some derived variables and fluxes are calculated. In the USA, George (2005) [8, 9] researched a eutrophication model for Lake Washington. This model was used to test alternative managerial schemes, including multiple elemental cycles (organic Carbon (org.C, Nitrogen (N), Phosphorus (P), Silica (Si) , Oxygen (O)), as well as various functional groups of phytoplankton (diatoms, green algae, and cyanobacteria) and zooplankton (copepods and cladocerans) groups. *Li et al.* [10] studied the effects of nutrients on the growth of tilapia in the Nile River. The researchers developed a model using STELLA II software based on a field experiment. The experiment aimed to determine the limiting nutritional factors for fish growth in fertilized ponds. Simulation results indicated that supplementary feeding compensates for the nutrient deficiencies found in natural food. Results also revealed that protein supplements were necessary for increasing fish yields in fertilized ponds. Up to now, there have been several studies on fish farming modelling worldwide [12-18], but there has been no study on establishing models to determine the nutrients content in catfish farming ponds in Vietnam.

Therefore, this study aims to set up a mathematical model that can simulate changes in the composition of primary nutrients in Pangasius ponds. On that basis, the key parameters that affect the model and optimize the culture regime was determined, helping to improve fish quality and minimize environmental impact.

2. Material and Method

2.1. Data Collection

Data were collected and analyzed in four striped catfish ponds with an average area of 200 m^2 each and a depth of 2.5 m in Cai Rang District, Can Tho city. Water was exchanged daily about 30% each time by pumping from the sedimentation pond (3,000 m^2).

The experiment had two treatments with two replications of each. The control treatment (CT) involved not using phytase in feed, while the experimental treatment (ET) involved using phytase 0.01% in feed.

Samples were collected at the same point under the fish-feeding bridge at all times. Dissolved oxygen (DO) concentration was directly measured in the ponds every day (at 8 am and 4 pm) by the HANNA analyzer. Samples of other factors such as TAN (Total Ammonium Nitrogen), P-PO_4^{3-}, TKN (Total Kjeldahl Nitrogen), TP, N-NO_2^- and N-NO_3^- were collected before raising and every ten days in the morning time (a total of 13 times) from May to November 2016. Water was analyzed at the Water Quality Laboratory - College of Aquaculture and Fisheries, Can Tho University. The sample reserve and analysis method followed the instructions and standard [11].

2.2. Data Processing Method

Data were collected from Pangasius ponds in the field and in previous studies. The mathematical model was used to simulate the processes occurring in Pangasius ponds. The equations were solved numerically and then coded by Matlab 2018. The final step was to use datasets and results from experiments or previous studies to validate the model.

2.3. Model Assumption

The object of the study is a closed pond for intensive catfish farming in the Mekong Delta of Vietnam, which has a simple physical structure that only represents the epilimnion pond. Nutritional resources for Pangasius fish originated from autotrophic and heterotrophic food sources in ponds. The protein concentration of phytoplankton was assumed to be constant. Water was exchanged 30% daily from the sedimentation pond assuming there is no water loss due to leakage or evaporation. There was no exchange of heat and light at the water surface in the pond. The survival rate of fish remained constant, and catfish ponds were homogeneous blocks.

2.4. Relation Matrix

Integrated data on water environmental parameters in catfish ponds in the Mekong Delta, Vietnam was used to build a relation matrix of mathematical equations. The data was inherited from models of eutrophication [8] and semi-intensive fish ponds. The matrix shows the correlation of the equations and variables, which is presented in the Table 1.

Table 1. The relation matrix of the mathematical equations

Functions	Variables													
	N	***P***	***d***	***T***	***gr***	***PH***	***ZO***	***Gra***	***Vs***	***li***	***pre***	***Neg***	***Peg***	***Ce***
PHYT	1	1	0	1	1	1	1	1	1	1	0	0	0	0
ZOOP	0	0	0	1	0	0	1	1	0	0	1	0	0	0
Ammonium	1	0	0	0	0	1	0	0	0	0	0	1	0	0
Nitrate	1	0	0	0	0	1	0	0	0	0	0	0	0	0
DON	1	0	0	0	0	1	1	0	0	0	0	1	0	0
PON	1	0	1	0	0	1	1	0	0	0	0	1	0	0
Phosphorous	0	1	0	0	0	1	1	0	0	0	0	0	1	0
DOP	0	1	0	0	0	1	1	0	0	0	0	0	0	0
POP	0	1	0	0	0	0	1	0	0	0	0	0	0	0
DOC	0	1	0	1	0	1	1	0	0	0	0	0	0	0
POC	0	0	1	1	0	1	1	0	0	0	0	0	0	1
DO	0	0	0	0	1	1	1	0	0	0	0	0	0	0

Note: 1: Relation 0: Non-relation

Explanation: *PHYT (function)*: Phytoplankton; *ZOOP (function)*: Zooplankton; *DON*: Dissolved organic nitrogen; *PON*: Particulate organic nitrogen; *DOP*: Dissolved organic phosphorus; *POP*: Particulate organic phosphorus; *DOC*: Dissolved organic carbon; *POC*: Particulate organic carbon; *DO*: Dissolved oxygen

N: Nitrogen; *P*: Phosphorous; *d*: depth; *T*: Temperature; *gr*: growth; *li*: light; *Gra*: Grazing rate of zooplankton; *Vs*: Settling velocity; *pre*: zooplankton predation; *Neg*: $N_{egestion}$ - Nitrogen excreted by zooplankton; *Peg*: $P_{egestion}$ - Phosphorus excreted by zooplankton; *Ce*: $C_{egestion}$ - Carbon excreted by zooplankton; *PH (variable)*: Phytoplankton; *ZO (variable)*: Zooplankton.

2.5. Equations

Based on the relation matrix (Table 1), the correlation between the components in the pond was defined according to functions and variables. The equation describing biological processes in Pangasius ponds was referenced from [8] and set up as follows.

a) Phytoplankton:

The equation that describes the changes in phytoplankton in fish ponds is:

$$\frac{\partial PHYT_{(i,x)}}{\partial t} = growth_{max(i)} \times f_{nutrient(i,x)} \times f_{light(i,x)} \times f_{temperature(i,x)} \times PHYT_{(i,x)} - bm_{ref(i)} \times e^{ktbm(i)(T(x)-Tref(i))} \times PHYT_{(i,x)} - V_{settling(i)} \times f_{temperature(x)} \times PHYT_{(i,x)} \times f_{depth(x)} - \sum_{j=cop,clad} Grazing_{(i,j,x)} \times ftemperature(j,x) \times ZOOP_{(j,x)} - outflows \times PHYT_{(i,EPI)} \quad (1)$$

The growth-limiting functions of phytoplankton is:

$$growth_{(i)} = growth_{max\,(i)} \times f_{nutrient(i)} \times f_{flight(i)} \times f_{temperature(i)}$$

b) Zooplankton

The equation that describes the changes of Zooplankton in fish ponds is:

$$\frac{\partial ZOOP_{(j,x)}}{\partial t} = gref_{(j,x)} \times f_{temperature(j,x)} \times \left(\sum_{i=diat,green,cyan} Grazing_{(i,j,x)} + Grazing_{detritus(j,x)}\right) \times ZOOP_{(j,i)} - bm_{ref} e^{ktbm(j)(T(x)-Tref(j))} \times ZOOP_{(j,x)} - predation_{(j,x)} - outflows \times ZOOP_{(j,EPI)} \quad (2)$$

c) Ammonium:

The concentration of NH_4^+ in the water layer was influenced by various processes, including uptake by phytoplankton for photosynthesis and biomass synthesis, the excretion of phytoplankton and fish excretion, and the nitrification reaction. The equation below describes the change of ammonium in fish ponds:

$$\frac{\partial NH_{4(x)}}{\partial t} = -\sum_{i=diat,green,cyan} prefNH_{4(i,x)} \times N_{up(i,x)} \times N_{fb(i,x)} \times PHYT_{(i,x)} - nitrification_{(x)} + \sum_{i=diat,green,cyan} FBM_{NH_{4(i)}} \times N_{(i,x)} \times bm_{ref(i)} e^{ktbm(i)(T(x)-Tref(i))} \times PHYT_{(i,x)} + \sum_{j=cop.clad} FBM_{NH_{4(j)}} \times \frac{N}{C_{(j)}} \times bm_{ref(j)} e^{ktbm(j)(T(x)-Tref(j))} \times ZOOP_{(j,x)} + KN_{mineral(x)} \times DON_{(x)} + \sum_{j=cop,clad} FE_{NH_4(j)} \times Negestion_{(j,x)} - outflows \times NH_{4(EPI)} \quad (3)$$

d) Nitrate:

The concentration of NO_3^- in the water layer changed due to NO_3^- absorption by phytoplankton for photosynthesis and biomass synthesis, nitrification reaction, and denitrification reaction. The equation below describes the change of Nitrate in fish ponds:

$$\frac{\partial NO_{3(x)}}{\partial t} = -\sum_{i=diat,green,cyan}\big(1 - prefNH_{4(i,x)} \times N_{up(i,x)} \times N_{fb(i,x)} \times PHYT_{(i,x)} + nitrification_{(x)} - denitrification_{(x)}\big) - outflows \times NO_{3(EPI)} \quad (4)$$

$prefNH_4 = 1 - exp(-\Psi_i * NH_4)$, describe the algae's preferential absorption of ammonium over nitrate (i);

e) Dissolved organic nitrogen (DON)

The concentration of DON in the water layer was influenced by several processes, including phytoplankton excretion and metabolism, oxidation (mineralization) of organic nitrogen in water, solubilization (decomposition) of organic nitrogen debris in the water layer into DON. The following equation describes the change of DON in fish ponds:

$$\frac{\partial DON_{(x)}}{\partial t} = \sum_{i=diat,green,cyan} FBM_{DON(i)} \times N_{(i,x)} \times bm_{ref(i)} e^{ktbm(i)(T(x)-Tref(i))} \times PHYT_{(i,x)} + \sum_{j=cop,clad} FBM_{DON(j)} \times \frac{N}{C_{(j)}} \times bm_{ref(j)} e^{ktbm(j)(T(x)-Tref(j))} \times ZOOP_{(j,x)} + KN_{dissolution(x)} \times PON_{(x)} - KN_{mineral(x)} \times DON_{(x)} + \sum_{j=cop,clad} FE_{DON(j)} \times Negestion_{(j,x)} - outflows \times DON_{(EPI)} \quad (5)$$

f) Particulate organic nitrogen (PON):

There were different processes that lead to an increase in PON in fish ponds. These included the death and excretion of phytoplankton, decay and excretion of fish, and residual feed. On the other hand, decomposing organic matter dissolves POC into DOC, fish feeding, and sedimentation reduce PON in the water layer. The equation below describes the variation of PON in fish ponds:

$$\frac{\partial PON_{(x)}}{\partial t} = \sum_{i=diat,green,cyan} FBM_{PON(i)} \times N_{(i,x)} \times bm_{ref(i)} e^{ktbm(i)(T(x)-Tref(i))} \times PHYT_{(i,x)} + \sum_{j=cop,clad} FBM_{PON(j)} \times N / C_{(j)} \times bm_{ref(j)} e^{ktbm(j)(T(x)-Tref(j))} \times ZOOP_{(j,x)} - KN_{dissolution(x)} \times PON_{(x)} - \sum_{j=cop,clad} Grazing_{detritus(j,x)} \times PON_{(x)} / POC_{(x)}(N / C_{(cop)}) \times f_{temperature(j,x)} \times ZOOP_{(j,x)} - VP_{seltling} \times f_{temperature(x)} \times PON_{(x)} \times f_{depth(x)} + \sum_{j=cop,clad} FE_{PON(j)} \times Negestion_{(j,x)} - outflows \times PON_{(EPI)} \quad (6)$$

g) Phosphorous:

The equation describes the change in phosphorus in fish ponds:

$$\frac{\partial PO_{4(x)}}{\partial t} = -\sum_{i=diat,green,cyan} P_{up(i,x)} \times P_{fb(i,x)} \times PHYT_{(i,x)} + \sum_{i=diat,green,cyan} FBM_{PO_{4(i)}} \times P_{(i,x)} \times bm_{ref(i)} e^{ktbm(i)(T(x)-Tref(i))} \times PHYT_{(i,x)} + \sum_{j=cop,clad} FBM_{PO_4(j)} \times P/C_{(j)} \times bm_{ref(j)} e^{ktbm(j)(T(x)-Tref(j)} \times ZOOP_{(j,x)} + KP_{mineral(x)} \times DOP_{(x)} + \sum_{j=cop,clad} EF_{PO_4(j)} \times Pegestion_{(j,x)} - outflows \times PO_{4(EPI)} \quad (7)$$

h) Dissolved organic phosphorus (DOP):

The following equation describes the transformation of DOP in fish ponds:

$$\frac{\partial DOP_{(x)}}{\partial t} = \sum_{i=diat,green,cyan} FBM_{DOP_{(i)}} \times P_{(i,x)} \times bm_{ref(i)} e^{ktbm(i)(T(x)-Tref(i))} \times PHYT_{(i,x)} + \sum_{j=cop,clad} FBM_{DOP(j)} \times \frac{P}{C_{(j)}} \times bm_{ref(j)} e^{ktbm(i)(T(x)-Tref(i)} \times ZOOP_{(j,x)} + KP_{dissolution(x)} \times DOP_{(x)} - KP_{mineral(x)} \times DOP_{(x)} + \sum_{j=cop,clad} EF_{DOP(j)} \times Pegestion_{(j,x)} - outflows \times DOP_{(EPI)} \quad (8)$$

i) Particulate organic phosphorus (POP):

The equation below describes the transformation of POP in fish ponds:

$$\frac{\partial POP_{(x)}}{\partial t} = -\sum_{i=diat,green,cyan} FBM_{POP(i)} \times P_{(i,x)} \times bm_{ref(i)} e^{ktbm(i)(T(x)-Tref(i))} \times PHYT_{(i,x)} + \sum_{j=cop,clad} FBM_{POP_{(j)}} \times P / C_{(j)} \times bm_{ref(j)} e^{ktbm(j)(T(x)-Tref(j))} \times ZOOP_{(j,x)} - KP_{dissolution(x)} \times POP_{(x)} - \sum_{j=cop,clad} Grazing_{detritus(j,x)} \times POP_{(x)} / POC_{(x)} *\times f_{temperature(j,x)} \times ZOOP_{(j,x)} - VP_{seltling} \times f_{temperature(x)} \times POP_{(x)} \times f_{depth(x)} + \sum_{j=cop,clad} FE_{POP(j)} \times Pegestion_{(j,x)} - outflows \times POP_{(EPI)} \quad (9)$$

j) Dissolved organic carbon (DOC):

The equation below describes the change of DOC in the fish pond:

$$\frac{\partial DOC_{(x)}}{\partial t} = \sum_i \left[FBM_{DOC(i)} + \big(1 - FBM_{OC(j)}\big) \times \frac{KH_{EXUD(i)}}{KH_{EXUD(i)} + DO_{(x)}}\right] \times bm_{ref(i)} e^{ktbm(i)(T(x)-Tref(i))} \times PHYT_{(i,x)} + \sum_j \left[FBM_{DOC(i)} + \big(1 - FBM_{OC(j)}\big) \times \frac{KH_{EXUD(j)}}{KH_{EXUD(j)} + DO_{(x)}}\right] \times bm_{ref(j)} e^{ktbm(j)(T(x)-Tref(j)} \times ZOOP_{(j,x)} + \sum_{j=cop,lad} FE_{DOC} \times Cegestion_{(j,x)} - \frac{DO_{(x)}}{DO_{(x)} + KH_{OOXRESP}} \times K_{respdoc(x)} \times DOC_x + KC_{dissolution(x)} \times POC_{(x)} - denitrification_{(x)} / DENIT_{\frac{NO_3}{DOC}} \quad (10)$$

k) Particulate organic carbon (POC):

The equation that describes the change of POC in fish ponds is as follows:

$$\frac{\partial POC_{(x)}}{\partial t} = -\sum_{i=diat,green,cyan} FBM_{POC(i)} \times bm_{ref(i)} e^{ktbm(i)(T(x)-Tref(i))} \times PHYT_{(i,x)} + \sum_{j=cop,clad} FBM_{POC(j)} \times bm_{ref(j)} e^{ktbm(j)(T(x)-Tref(j))} \times ZOOP_{(j,x)} - KC_{dissolution(x)} \times POC_{(x)} - \sum_{j=cop,clad} Grazing_{detritus(j,x)} \times f_{temperature(j,x)} \times ZOOP_{(j,x)} - VP_{seltling} \times f_{temperature(x)} \times POC_{(x)} \times f_{depth(x)} + \sum_{j=cop,clad} FE_{POC(j)} \times Cegestion_{(j,x)} \quad (11)$$

l) Dissolved oxygen (DO):

The equation below describes the change in DO in fish ponds:

$$\frac{\partial DO_{(x)}}{\partial t} = \sum_{i=diat,green,cyan} (1.3 - 0.3 \times prefNH_4) \times Growth_{(i,x)} \times RESP_{DO/C} \times PHYT_{(i,x)} - \sum_i \frac{DO_{(x)}}{KH_{EXUD(i)} + DO_{(i)}} \times bm_{ref(i)} e^{ktbm(i)(T(x)-Tref(i))} \times RESP_{DO/C} \times PHYT_{(i,x)} - \sum_j \frac{DO_{(x)}}{KH_{EXUD(j)} + DO_{(x)}} \times bm_{ref(j)} e^{ktbm(j)(T(x)-Tref(j))} \times RESP_{\frac{DO}{C}} \times ZOOP_{(j,x)} - \frac{DO_{(x)}}{DO_{(x)} + KH_{OOXRESP}} \times K_{respdoc(x)} \times RESP_{\frac{DO}{C}} \times DOC_{(x)} - nitrification_{(x)} \times NITRIF_{\frac{O}{NH_4}} + \frac{K_{reaeration} \times Surface\ area}{Epilimnion\ Volume} \times \left(DO_S - DO_{(epi)}\right) \quad (12)$$

2.6. Model Solving Method

Differential equations cannot be solved by conventional analytical methods but must be approximated by numerical methods as Picard approximation, Taylor series, power series, Euler, Runge - Kutta method. Among these methods, the Runge - Kutta method is the most effective: it is both highly accurate, the algorithm is not too complicated, and it is applied extensively to solve differential equations. The time step is *h*/2.

The equations after solving were coded and simulated by the Matlab software.

3. Results and Discussion

3.1. Model Calibration

Initial data were taken at the beginning of fish stocking. The environmental parameters were sampled and analyzed once every 10 days. Table 2 shows the initial data used to simulate and solve the mathematic equations.

Table 2. Initial data of water environment at time *t* = 0

No	Parameter	Unit	Sample	
			CT1	**CT2**
1	COD	mg/L	22.1	25.0
2	TSS	mg/L	24.0	29.0
3	TON	mg/L	0.455	0.449
4	TOP	mg/L	0.055	0.067
5	NH_4^+	mg/L	0.146	0.146
6	NO_3^-	mg/L	0.018	0.021
7	PO_4^{-3}	mg/L	0.043	0.047
8	DO	mg/L	6.5	6.5

Note: CT1, CT2: control treatment pond 1,2

The model used a total of 94 parameters, whose values had been referred from previous studies [8]. These values will be used for sensitivity analysis and model calibration. Table 3 presents the values of some parameters, while the full values of all parameters can be given in [8].

The sensitivity analysis method involves changing a parameter *X* by ±10%, while keeping the initial values of all other parameters constant. The next step is to calculate the relative error *(RE)* of the state variables that are affected by changing values of *X*. Each parameter *X*, when changed, results in a different error value (RE_i) for each state variable at a particular time (*i*). The sensitivity of a parameter is evaluated based on the maximum, minimum, and average *RE* values associated with that parameter. In which:

$$RE_{X,i} = \frac{|Y_{bđ}(i) - Y_X(t)|}{Y_{bđ}} * 100\%;$$

where, $Y_{bd(i)}$ is the state variable value of the the model output with the initial parameter value, and $Y_{x(t)}$ is the value of the state variable of the model output with the parameter *X* value changed within ± 10% at time *t*.

According to the sensitivity analysis results, none of the parameters have the most sensitive to two and/or more state variables simultaneously. This indicates that the mathematical equation system used in the catfish pond model is built on a conceptual model that has been inherited and developed from existing ecological models to ensure linear independence. The linear independent system gives the most compact results, hence it is more reasonable and convenient for model calibration.

Table 3. Model parameters used in the model

Parameter	Unit	Value	Description
$growthmax_{(diat)}$	day^{-1}	2.2	Maximum growth for diatoms
$growthmax_{(green)}$	day^{-1}	1.8	Maximum growth for greens
$growthmax_{(cyan)}$	day^{-1}	1.2	Maximum growth for Cyanobacteria
$Nupmax_{(i)}$	mgN/mgC.day	0.16	Maximum nitrogen uptake rate
$Nmax_{(i)}$	mgN/mgC	0.18	Maximum phytoplankton internal N
$Nmin_{(i)}$	mgN/mgC	0.08	Minimum phytoplankton internal N
$KN_{(diat)}$	mgN/m^3	65	Half saturation constant for nitrogen uptake by diatoms
$KN_{(green)}$	mgN/m^3	45	Half saturation constant for nitrogen uptake by greens

Calibration method: Parameter *X* has the greatest sensitivity to the state variable *Y*, changing its value within a specified range will calculate the error compared to actual samples and the model's correlation coefficient. The values for higher correlation coefficients and smaller errors are further adjusted using the algorithm available in RStudio. Model testing and calibration are carried out using foreign data sets and measurement data sets in Pangasius ponds in Vietnam to adjust parameters to suit the climate conditions in Vietnam.

Out of 94 parameters used in the model to analyze the sensitivity, 15 parameters with high sensitivity (>5%) will be selected for calibration (Fig. 1). The remaining 15 parameters with low sensitivity (<0.001%) and 64 parameters with medium sensitivity (0.001-5%) will not be selected for calibration.

Table 4 presents the results of 15 model parameters with the highest sensitivity to the state variables of nutrient concentrations in ponds. The listed parameters have the greatest sensitivity to one of the monitored state variables, these parameters will be adjusted to match the real model of catfish ponds.

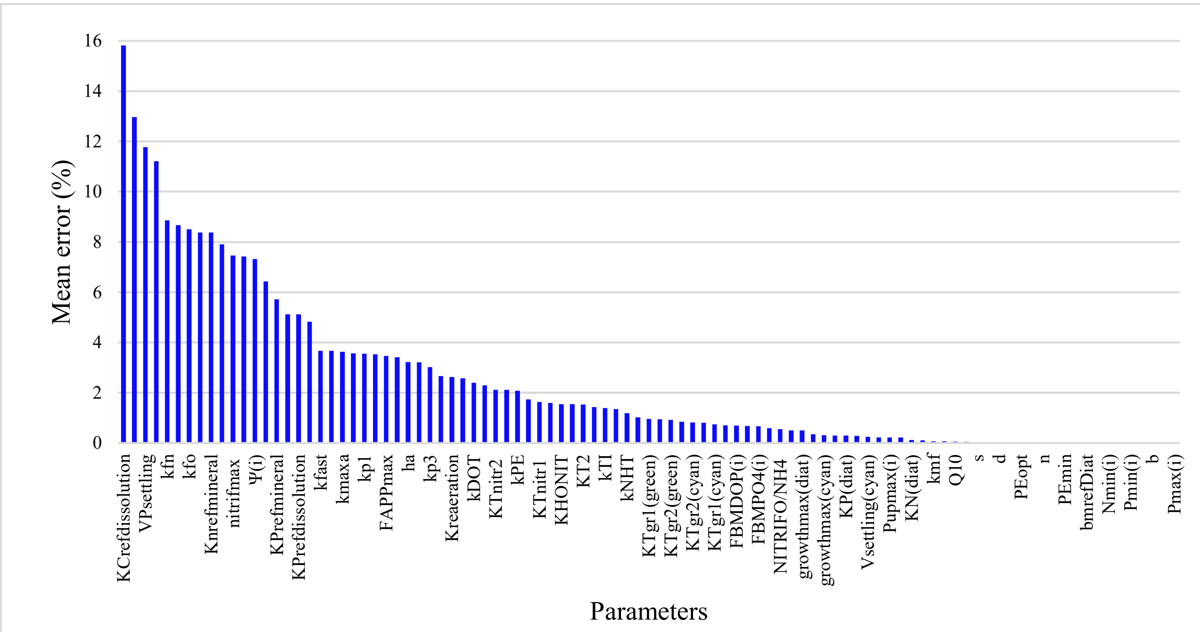

Fig. 1. The error of the parameters for the nutrient variables

Table 4. The results of the model parameters with the highest sensitivity

No	Parameter	Value range (%)	The mean error of state variable (%)							
			COD	TSS	TN	TP	NH_4^+	NO_3^-	PO_4^{3-}	DO
1	h_h	10	4.075	21.635	17.430	1.670	0.842	6.479	0.126	8.644
		-10	3.776	36.502	12.293	2.567	0.734	4.075	0.207	5.330
2	$VP_{settling}$	10	8.644	20.961	15.866	19.114	6.832	8.644	1.406	2.407
		-10	10.677	34.002	19.981	23.771	5.330	5.154	2.004	3.530
3	$KC_{refdissolution}$	10	67.998	34.742	6.821	1.917	2.004	3.339	0.749	15.866
		-10	43.797	42.717	7.742	2.520	1.390	3.506	0.898	17.101
4	$FBM_{DOC(i)}$	10	26.755	19.114	5.154	2.735	1.663	7.630	0.912	5.447
		-10	23.206	20.148	4.833	2.655	1.605	7.130	0.855	4.274
5	$FBM_{POC(i)}$	10	35.168	26.028	9.282	2.450	1.657	4.384	0.833	9.722
		-10	28.797	31.405	8.794	3.900	0.950	4.528	0.767	10.826
6	$K_{refrespDOC}$	10	36.515	32.192	5.535	1.043	3.248	12.956	0.540	8.064
		-10	38.165	29.727	6.479	1.298	3.979	16.873	0.224	10.675
7	k_{fn}	10	2.833	3.027	13.877	0.767	20.356	21.155	0.066	2.853
		-10	4.106	3.133	15.310	0.680	22.008	25.304	0.093	3.057
8	$KN_{refdissolution}$	10	3.805	2.651	15.609	0.341	9.128	6.822	0.056	1.367
		-10	6.482	2.710	18.167	0.583	8.392	4.075	0.078	1.674
9	$KN_{refmineral}$	10	4.590	2.734	13.098	0.100	34.123	13.337	0.092	1.398
		-10	5.219	2.830	15.046	0.059	28.037	12.008	0.077	1.213
10	$\Psi_{(i)}$	10	8.916	5.075	0.072	0.155	19.002	20.379	0.056	8. 939
		-10	7.598	4.468	0.446	0.201	19.047	20.984	0.045	10.770
11	$nitrif_{max}$	10	5.680	6.061	9.202	0.398	17.430	15.7658	0.046	4.622
		-10	4.810	9.507	6.515	0.010	19.916	13.752	0.166	5.523
12	k_{fp}	10	0.998	0.166	1.200	25.040	1.012	0.262	19.700	2.316
		-10	1.000	1.164	0.084	26.077	1.566	0.450	20.222	1.557
13	$KP_{refmineral}$	10	2.498	5.681	0.110	18.484	0.081	0.041	15.021	2.643
		-10	2.180	5.200	0.383	20.833	0.047	0.050	17.154	1.021
14	k_{fo}	10	14.253	14.590	6.933	1.309	2.651	5.989	1.106	21.185
		-10	18.950	12.197	7.138	1.454	2.735	6.305	1.068	18.171
15	$K_{reaeration}$	10	1.264	1.006	0.226	0.576	3.147	1.957	0.842	14.807
		-10	1.602	1.952	0.392	0.045	1.786	2.449	0.734	9.305

High sensitivity (>5%): 15 parameters

Medium sensitivity (0,001-5%): 64 parameters

Small sensitivity (<0,001%): 15 parameters

Through sensitivity analysis for the model's parameters, among 15 parameters with high sensitivity (Table 4), it was found that $VP_{settling}$, $KC_{refdissolution}$, $KN_{refdissolution}$, $KN_{refmineral}$, and $KP_{refmineral}$ had the greatest influence on the modelled environmental parameters in the pond. Specifically, the parameters with the most significant influence on COD, TSS are $VP_{settling}$, $KC_{refdissolution}$, $FBM_{DOC(i)}$, $FBM_{POC(i)}$, $K_{refrespDOC}$, and k_{fo}. Meanwhile, the parameters that have the greatest influence on TN are $VP_{settling}$, k_{fn}, $KN_{refdissolution}$, and $KN_{refmineral}$. For TP, the parameters that have the greatest influence are $VP_{settling}$ and $KP_{refmineral}$. Similarly, k_{fn}, $KN_{refmineral}$, $\Psi_{(i)}$, and $nitrif_{max}$ have the significant impact on NO_3^- and NH_4^+, while k_{fp} and $KP_{refmineral}$ have the strongest impact on PO_4^{3}. Lastly, $KC_{refdissolution}$, k_{fo}, and $K_{reaeration}$ are the parameters that have the most significant impact on DO.

in these:

$VP_{settling}$ - Settling velocity of particles at reference temperature;

$KC_{refdissolution}$ - Particulate carbon dissolution/ hydrolysis rate at reference temperature;

$KN_{refdissolution}$ - Particulate nitrogen dissolution/ hydrolysis rate at reference temperature;

$KN_{refmineral}$ - Nitrogen mineralization rate at reference temperature;

$K_{refrespDOC}$ - Respiration rate of dissolved organic carbon at reference temperature;

$KP_{refmineral}$ - Phosphorus mineralization rate at reference temperature;

$FBM_{DOC(i)}$ - Fraction of basal metabolism excreted as *DOC*;

$FBM_{POC(i)}$ - Fraction of basal metabolism excreted as *POC*;

k_{fo} - DO coefficient for fish catabolism;

$K_{reaeration}$ - Re-aeration coefficient.

The values of the parameters after calibration are shown in Table 5.

Table 5. The value parameters with the highest sensitivity after calibration

No	Parameters	Unit	Value before calibration	Value after calibration
1	h_h	day^{-1}	0.05	0.095
2	$VP_{settling}$	m/day	0.9	0.5
3	$KC_{refdissolution}$	day^{-1}	0.008	0.02
4	$FBM_{DOC(i)}$	-	0.2	0.4
5	$FBM_{POC(i)}$	-	0.5	0.7
6	$K_{refrespDOC}$	day^{-1}	0.0024	0.01
7	k_{fn}	gN/kcal	0.017	0.025
8	$KN_{refdissolution}$	day^{-1}	0.0005	0.01
9	$KN_{refmineral}$	day^{-1}	0.0045	0.01
10	$\Psi_{(i)}$	(mgN/m^3)$^{-1}$	0.3	0.65
11	$nitrif_{max}$	mgN/m^3.day	0.15	0.85
12	k_{fp}	gP/kcal	0.002	0.001277
13	$KP_{refmineral}$	day^{-1}	0.04	0.07
14	k_{fo}	gDO/kcal	0.28	0.493
15	$K_{reaeration}$	m/day	2.0	2.4

Explanation:

- The difference in the values of $KN_{refdissolution}$ and $KN_{refmineral}$ parameters after calibration due to there is a natural amount of nitrogen (N) present in Lake Washington, while in Vietnamese pangasius ponds, N is formed both naturally and supplied through outside food.

- The values of $KN_{refmineral}$ and $KP_{refmineral}$ are also lower before calibration because the low water temperature in Lake Washington (< 20^{o}C), whereas the average temperature in Vietnamese pangasius ponds is always higher (ranging from 25^o - 30^o C). This difference in temperature leads to higher efficiency in the mineralization of N and P.

- $K_{reaeration}$ after calibration is also higher due to aeration of the pangasius pond; in addition, water was changed occasionally.

- $K_{refrespDOC}$ is different after calibration due to the characteristics of tropical ponds because pangasius has a large density and big size.

Thus, the typical and suitable parameters for catfish ponds at the conditions of the Mekong Delta region are $KN_{refdissolution}$, $KN_{refmineral}$, $KP_{refmineral}$, $K_{reaeration}$, and $K_{refrespDOC}$. This is due to the high average temperature of the mineralization process, which speeds up the digestion and decomposition process. Moreover, frequent aeration, water exchange, food supply, and rapid growth of fish cause animal respiration faster.

3.2. Model Results

The results of the model are shown in Table 6.

The corrected graphs for PO_4^{3-} and TP show an average error rate of 30.33% and 30.83% (≤40%). The Mean Squared Error (MSE) and Root Mean Squared Error (RMSE) of PO_4^{3-} in actual pond are 0.159 and 0.399 respectively, while in model, they are 0.000 and 0.016. similarly, the MSE and RMSE of TP in the actual pond are 0.138 and 0.371, while in the model, they are 0.001 and 0.0107. The difference in the simulation model can be attributed to external factors such as changes in weather conditions, suboptimal temperature, or small number of fish deaths (0-6%) during the growth and development of fish. Other factors affecting the management of production include the addition and change of water.

During the first 70 days of the investigation, the average error of NO_3^- and NH_4^+ were 21.9% and 21.4%, respectively. After 30 days, the model's found value continued to increase gradually while the measured value from the actual decreased rapidly. The error in the last 30 days of NO_3^- correction showed a significant difference (> 40%). This difference could be partly due to the model's inability to simulate the impact of external factors in real conditions. The average error of TN was 14.6%, and the variation trend was similar between the reality and simulated data from the model. The MSE and RMSE of NO_3^- were 0.043 and 0.206 in reality, 0.045 and 0.212 in the model;. The MSE and RMSE of NH_4^+ in reality were 1.343 and 1.159, respectively, while in the model they were 0.67 and 0.82. The MSE and RMSE of TN in reality were 0.195 and 0.441, while in the model they were 0.001 and 0.031.

Table 6. Values of environmental parameters in model and real ponds

Day	0	10	20	30	40	50	60	70	80	90	100
DO reality (mg/L)	6.5	4.4	2.7	4.2	6.8	4.0	2.5	2.1	1.9	2.5	2.4
DO model (mg/L)	6.5	4.8	2.3	4.0	9.3	3.2	2.3	1.8	1.6	2.8	2.2
PO_4^{3-} reality (mg/L)	0.043	0.174	0.273	0.498	0.404	0.641	0.582	2.01	1.82	1.79	3.07
PO_4^{3-} model (mg/L)	0.043	0.209	0.391	0.574	0.621	0.842	0.953	1.55	1.552	1.754	1.956
TP reality (mg/L)	0.098	0.272	0.521	1.06	0.544	0.853	1.80	2.135	2.377	2.764	3.924
TP model (mg/L)	0.098	0.350	0.614	0.86	1.120	1.380	1.63	1.889	2.148	2.403	2.658
NO_3^- reality (mg/L)	0.018	0.033	0.160	0.236	0.333	0.524	0.80	0.588	0.259	0.230	0.232
NO_3^- model (mg/L)	0.018	0.099	0.181	0.262	0.343	0.485	0.721	0.588	0.452	0.359	0.334
NH_4^+ reality (mg/L)	0.146	0.2	0.81	0.92	1.77	2.18	1.88	2.83	2.58	2.27	1.16
NH_4^+ model (mg/L)	0.146	0.406	0.814	1.131	1.523	1.877	2.342	2.625	2.764	2.841	2.722
TN reality (mg/L)	0.442	0.66	0.91	2.2	2.77	3.85	3.59	4.71	5.55	5.43	5.22
TN model (mg/L)	0.442	0.89	1.56	2.07	2.57	3.16	3.71	4.22	4.79	5.33	5.85

4. Conclusion

The corrected graphs for PO_4^{3-} and TP have an average error of 30.33% and 30.83%, respectively, which is within the acceptable range of lower than 40%. The mean error for NO_3^- and NH_4^+ is 21.9% and 21.4%, respectively, while TN has an error of 14.6%. Although the model results after calibration are relatively close to reality, some errors may have occurred during the actual experiment due to factors such as weather, input water and so on that affect environmental conditions. However, the reality and model nutritional data are still within the range of fish viability and consistent with results from other researchers.

The model has been calibrated and tested using a dataset of real pangasius ponds located in Can Tho city. The key parameters of the model were defined as: $VP_{settling}$ (m/day); $KC_{refdissolution}$ (day^{-1}); $KN_{refdissolution}$ (day^{-1}); $KN_{refmineral}$ (day^{-1}), $KP_{refmineral}$ (day^{-1}), $FBM_{DOC(i)}$, $FBM_{POC(i)}$, $K_{refrespDOC}$ (day^{-1}), k_{fo} (gDO/Kcal), k_{fn} (gN/kcal), $\Psi_{(i)}$ (mgN/m^3)$^{-1}$, $nitrif_{max}$ (mgN/m^3.day), k_{fp} (gP/kcal), and $K_{reaeration}$ (m/day).

To control the water quality of the pond, a numerical model can be used to calculate the content of environmental parameters at different times during the farming process. This assists in forecasting the right time to improve the quality of the pond environment.

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