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Simplified Trajectory-Tracking Method for an Under-Actuated USV

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Abstract

Traditional control methods designed for the trajectory tracking problem of under-actuated vehicles often aim to directly stabilize the tracking-error system of differential equations to the origin. This often results in complex control algorithms. This paper introduces a simplified control method for the trajectory tracking problem of an under-actuated Unmanned Surface Vehicle (USV). The proposed method consists of a guidance law for intermediate variables, which are the pseudo-yaw angle and the pseudo-surge velocity, and a Proportional-Integral (PI) controller. The control term for surge velocity is designed to be bounded, which reduces undesired instability. A model is built based on linear dynamics equations and parameters of an USV. The proposed tracking method is applied to the model to track different trajectories in simulations. The simulation results show that the proposed method effectively tracks different trajectories under various initial conditions. The control term for surge velocity remains stable despite the relatively large initial tracking-error condition.

Keywords: Trajectory tracking, unmanned surface vehicle, USV.

1. Introduction

In the last two decades, there have been many researches on developing autonomous Unmanned Surface Vehicles (USVs) [1], [2]. While USVs' configurations may vary, guidance, navigation and control systems remain to be key components to an autonomous USV [3]. Most USVs' control systems are designed to satisfy one of four types of control objectives: set-point regulation, trajectory tracking, path following, and path maneuvering [3].

A trajectory tracking problem can be defined as forcing the USV's state to track a time-varying desired state. In a comprehensive review of USV development [2], Z. Liu *et al.* remarked that the trajectory problem for fully-actuated USVs has been reasonably understood [4]. Trajectory tracking problem for under-actuated USV, however, are still an active research topic, due to challenges in the nonholonomic constraints [5].

Many researches on the trajectory tracking problem for under-actuated USVs derive the tracking-error equations from the USV's dynamic and kinematic equations, and design a controller that globally stabilizes the tracking-error at the origin. [6], [7], [8].

In [9], H. Huang *et al.* introduced a novel method with reduced complexity that is not based on the tracking-error equations. Instead, intermediate variables, called pseudo-variables (pseudo-yaw angle and pseudo-surge velocity), are designed to be direct functions of the real state, desired state, and tracking-error. The controller's objective is to converge the USV's yaw angle and surge velocity to the pseudo-variables.

In [9], the pseudo-surge velocity term, however, is not bounded, which might cause undesired saturation of the actuators. Motivated by H. Huang *et al.*'s work, in this paper, we take a similar approach and propose a bounded pseudo-surge velocity term, which can be implemented with a simple Proportional-Integral (PI) controller.

This paper is organized as follows. Section 2 specifies the considered USV's configuration and the governing dynamic and kinematics equations. In Section 3, the guidance law for yaw angle and surge velocity is designed. Section 4 specifies the controller. In Section 5, the simulation results illustrating the proposed controller's effectiveness are provided. The conclusions are drawn in Section 6.

2. Kinematic and Dynamic Models

The USV configuration considered in this paper is a twin-hull vessel with two independent motors or thruster attached to each hull (shown in Fig.1).

The USV's dynamic model used in this paper is described by the following linear maneuvering equations [3]:

$$(\boldsymbol{M_{RB}} + \boldsymbol{M_A})\dot{\boldsymbol{\nu}} + (\boldsymbol{C_{RB}} + \boldsymbol{C_A} + \boldsymbol{D})\boldsymbol{\nu} = \boldsymbol{\tau} \quad (1)$$

where $\boldsymbol{\nu} = [u \quad v \quad r]^T$, u, v, r denote the surge velocity, sway velocity, and yaw velocity, respectively, $\boldsymbol{\tau} = [F_x \quad F_y \quad N]$ denotes the forces and moments applied on the USV, $\boldsymbol{M_{RB}}$ is the rigid-

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body mass matrix, $\boldsymbol{M_A}$ is the added mass matrix. $\boldsymbol{C_{RB}}$ is the rigid-body Coriolis and centripetal matrix, $\boldsymbol{C_A}$ is the linear hydrodynamic Coriolis and centripetal matrix. $\boldsymbol{D}$ is the linear damping matrix.

$$\boldsymbol{M_{RB}} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & mx_G \\ 0 & mx_G & I_z \end{bmatrix}$$

$$\boldsymbol{M_A} = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & -Y_{\dot{r}} \\ 0 & -N_{\dot{v}} & -N_{\dot{r}} \end{bmatrix}$$

$$\boldsymbol{C_{RB}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & mU \\ 0 & 0 & mx_G U \end{bmatrix}$$

$$\boldsymbol{C_A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -Y_{\dot{v}}U \\ 0 & 0 & -Y_{\dot{r}}U \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & -Y_r \\ 0 & -N_v & -N_r \end{bmatrix}$$

where m is the USV's mass, x_G is the x_b - axis coordinate of the USV's center of gravity (CG), I_z is the moment of inertia about the z_b axis. $X_{\dot{u}}$, $Y_{\dot{v}}$, $Y_{\dot{r}}$, $N_{\dot{v}}$, $N_{\dot{r}}$, X_u , Y_v , Y_r , N_v , N_r are the hydrodynamic derivatives. U is the cruise speed, about which $\boldsymbol{C_A}$ and $\boldsymbol{D}$ matrices are linearized.

The kinematics are defined as follows

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} cos(\psi) & -sin(\psi) & 0 \\ sin(\psi) & cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} u \\ v \\ r \end{bmatrix} \quad (2)$$

where x, y are the USV's inertial coordinates. ψ is the yaw angle.

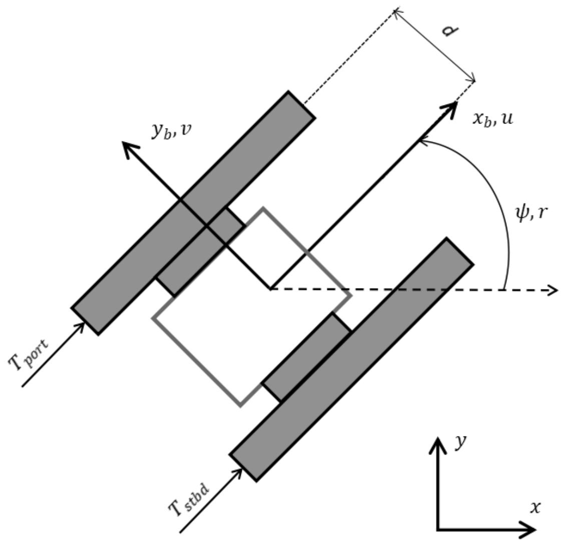

Fig. 1. The USV's configuration and dynamics

3. Guidance Law

Based on the desire trajectory $x_d(t), y_d(t)$ the look-ahead coordinates are defined as follows

$$\begin{cases} x_{target} = x_d(t) + \Delta . cos(\psi_d) \\ y_{target} = y_d(t) + \Delta . sin(\psi_d) \end{cases} \quad (3)$$

where $\psi_d = atan2(\dot{y_d}(t), \dot{x_d}(t)) \in [-\pi, \pi]$, $\Delta > 0$ is the look-ahead distance.

The along-track error x_e and cross-track error y_e are defined as follows

$$\begin{bmatrix} x_e \\ y_e \end{bmatrix} = \begin{bmatrix} cos(\psi_d) & sin(\psi_d) \\ -sin(\psi_d) & cos(\psi_d) \end{bmatrix} . \begin{bmatrix} x_d(t) - x \\ y_d(t) - y \end{bmatrix} \quad (4)$$

The proposed pseudo-yaw angle and pseudo-surge velocity terms are as follows

$$\begin{cases} \psi_{pseudo} = atan2(y_{target} - y, x_{target} - x) \\ u_{pseudo} = (u_d + \Delta u . k_{x_e}) . k_{y_e} . k_{e_\psi} \end{cases} \quad (5)$$

where $u_d = \sqrt{\dot{x_d}(t)^2 + \dot{y_d}(t)^2}$, $\Delta u > 0$

$$\begin{cases} k_{e_\psi} = \dfrac{a}{e_\psi{}^2 + a} , k_{e_\psi} \in (0,1], \\ k_{x_e} = \dfrac{x_e |x_e|}{x_e{}^2 + b} , k_{x_e} \in (-1,1), \\ k_{y_e} = 1 + (k_{y_e} max - 1) \dfrac{y_e{}^2}{y_e{}^2 + c} , k_{y_e} \in [1,2). \end{cases}$$

where $e_\psi = \psi_{pseudo} - \psi$ denotes the heading error.

e_ψ and $e_u = u_{pseudo} - u$, which denotes the surge velocity error, are then used to drive the heading and surge velocity controller.

Note that in the event of abnormally large value of $|x_e|$ and $|y_e|$, $u_{pseudo} \in [0, (u_d + \Delta u) k_{y_e} max]$ remains bounded, which reduces instabilities.

4. Controller

A Proportional-Integral (PI) Controller is implemented, the control thrust $T_{control}$ and control yaw moment $N_{control}$ are as follows

$$\begin{cases} T_{control} = K_{p_u} . e_u + K_{i_u} . \int e_u \, dt \\ N_{control} = K_{p_\psi} . e_\psi + K_{i_\psi} . \int e_\psi \, dt \end{cases} \quad (6)$$

Thrusts on the starboard-side motor and the port-side motor are allocated as follows

$$\begin{cases} T_{stbd} = T_{control}/2 + N_{control}/2d \\ T_{port} = T_{control}/2 - N_{control}/2d \end{cases} \quad (7)$$

where d is the distance from each motor to x_b- axis.

The thrust on each motor is saturated by the motors' maximum thrust T_{max} .

The resultant force and moment are

$$\begin{cases} T = T_{stbd} + T_{port} \\ N = d.(T_{stbd} - T_{port}) \end{cases} \tag{8}$$

In this paper, only the motors' thrusts are taken into account. Environmental forces are neglected.

$$\boldsymbol{\tau} = [T \quad 0 \quad N]^T \tag{9}$$

5. Simulations

The proposed tracking method is validated with simulation results. The USV model is constructed with the parameters specified in [10] by Klinger *et al.* The USV's dynamics, as discussed in Section 1, is linearized about the cruise speed U. Therefore, some parameters used in simulations are approximated from the original parameters.

The proposed tracking method is tested with variations in type of trajectory, trajectory's scale, speed, and initial condition.

The control parameters are as follows:

$$\begin{cases} \Delta = 5 \\ \Delta u = 2 \\ k_{y_e}max = 1.2 \\ [a \quad b \quad c] = [2 \quad 60 \quad 50] \\ [K_{p_u} \quad K_{i_u}] = [70 \quad 10] \\ [K_{p_\psi} \quad K_{i_\psi}]_{t=0} = [30 \quad 0.04] \end{cases}$$

5.1. Straight Trajectories

The desired trajectories and initial conditions are as follows:

Simulation Run #1

$$\begin{cases} x_d(t) = 5 + 0.8t \\ y_d(t) = 10 + 0.6t \\ [u \quad v \quad r]_{t=0} = [0 \quad 0 \quad 0] \\ [x \quad y \quad \psi]_{t=0} = [0 \quad 0 \quad \pi/4] \end{cases}$$

Simulation Run #2

$$\begin{cases} x_d(t) = 50 + 0.5t \\ y_d(t) = -30 + 0.5t \\ [u \quad v \quad r]_{t=0} = [0 \quad 0 \quad 0] \\ [x \quad y \quad \psi]_{t=0} = [0 \quad 0 \quad 3\pi/4] \end{cases}$$

Simulation Run #3

$$\begin{cases} x_d(t) = 50 + 0.5t \\ y_d(t) = 30 + 0.5t \\ [u \quad v \quad r]_{t=0} = [0 \quad 0 \quad 0] \\ [x \quad y \quad \psi]_{t=0} = [0 \quad 0 \quad 3\pi/4] \end{cases}$$

Simulation results of Run #1, Run #2, Run #3 are shown in Fig. 2, Fig. 3, Fig. 4, respectively.

The USV's trajectory, desired trajectory, along-track error, cross-track error, heading error, surge velocity, sway velocity and yaw rate of each simulation run are shown in their respective figure.

Fig. 2 shows the tracking performance of a straight trajectory with relatively small initial along-track and heading error condition. The USV model tracks a smooth trajectory. The cross-track error quickly converges to the origin, and the along-track error asymptotically decrease to a steady state error of 3 m. The surge velocity and the yaw angle smoothly stabilize about the desired surge velocity and the desired yaw angle.

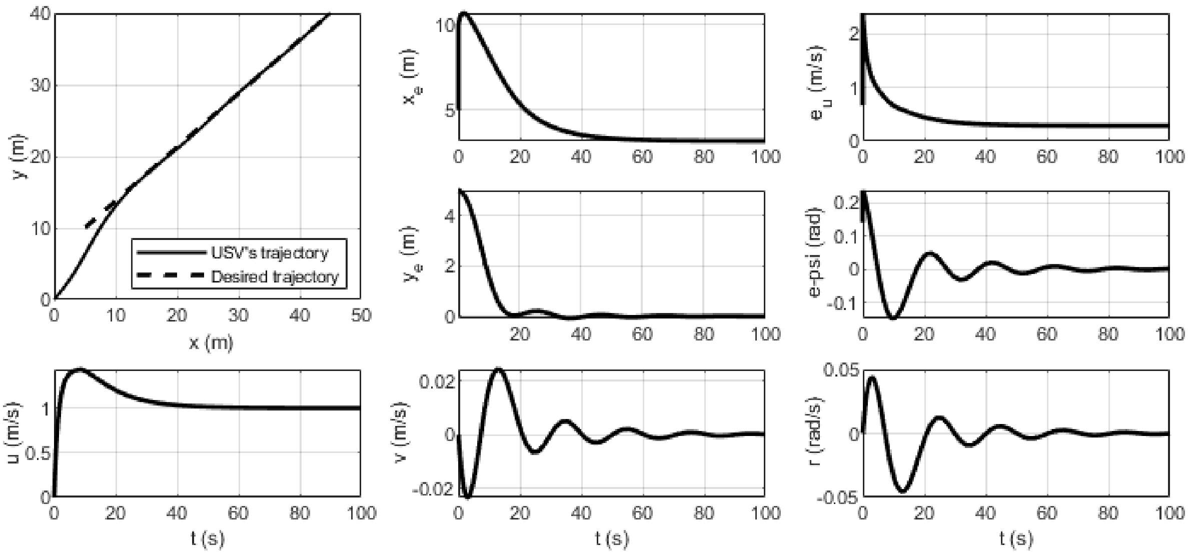

Fig. 2. Trajectory tracking performance - Simulation Run #1

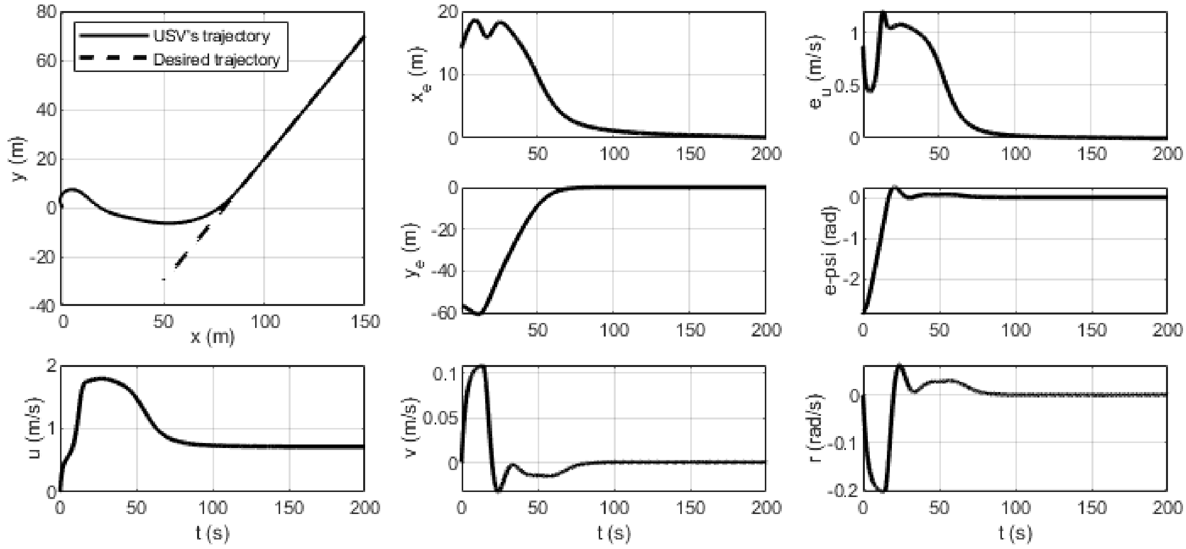

Fig .3. Trajectory tracking performance - Simulation Run #2

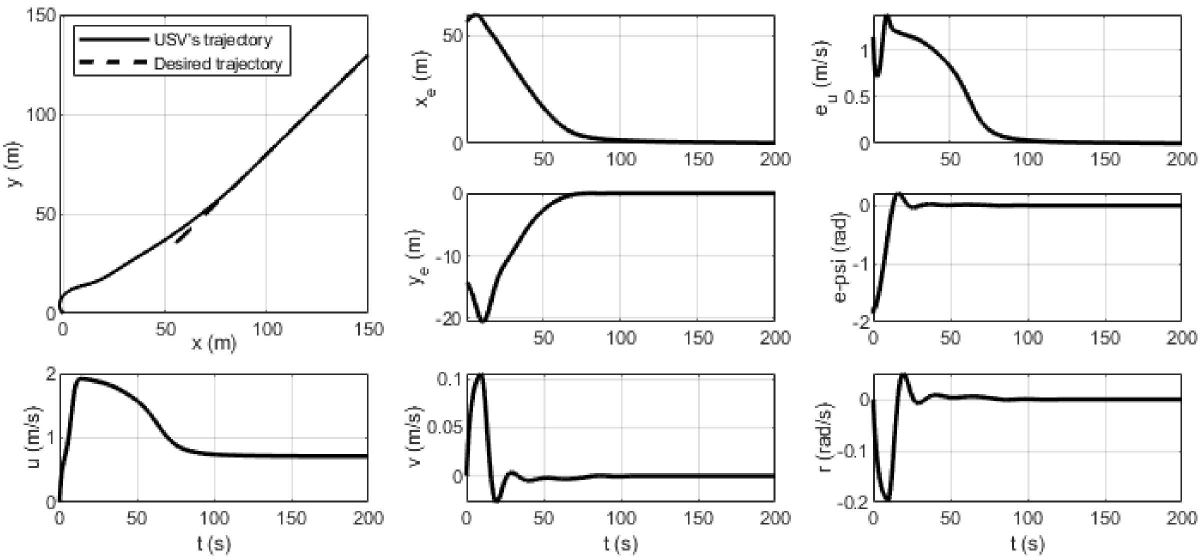

Fig. 4. Trajectory tracking performance - Simulation Run #3

Fig. 3 and Fig. 4 show the tracking performances of straight trajectories with relatively large initial along-track, cross-track and heading errors condition. The USV model, in both Fig. 3 and Fig. 4, tracks smooth trajectories. The cross-track error and the along-track error quickly converges to the origin. It can be noted that the k_{e_ψ} factor constrains u_{pseudo} when $|e_\psi|$ is large. The USV rapidly turns to the desired yaw angle at low surge velocity before pursuing at high surge velocity. Despite large initial along-track, cross-track and heading errors condition, the surge velocity is bounded. The sway velocity remains bounded within the magnitude of 0.12 m/s.

Overall, the proposed trajectory-tracking method gives good tracking performance of straight trajectories.

5.2. Circular Trajectories

The desired trajectories and initial conditions are as follows:

Simulation Run #4

$$\begin{cases} x_d(t) = 30sin(0.018t) \\ y_d(t) = -10 + 30cos(0.018t) \\ [u \quad v \quad r]_{t=0} = [0 \quad 0 \quad 0] \\ [x \quad y \quad \psi]_{t=0} = [0 \quad 0 \quad \pi/4] \end{cases}$$

Simulation Run #5

$$\begin{cases} x_d(t) = 20 + 30sin(0.03t) \\ y_d(t) = -40 + 30cos(0.03t) \\ [u \quad v \quad r]_{t=0} = [0 \quad 0 \quad 0] \\ [x \quad y \quad \psi]_{t=0} = [0 \quad 0 \quad \pi/4] \end{cases}$$

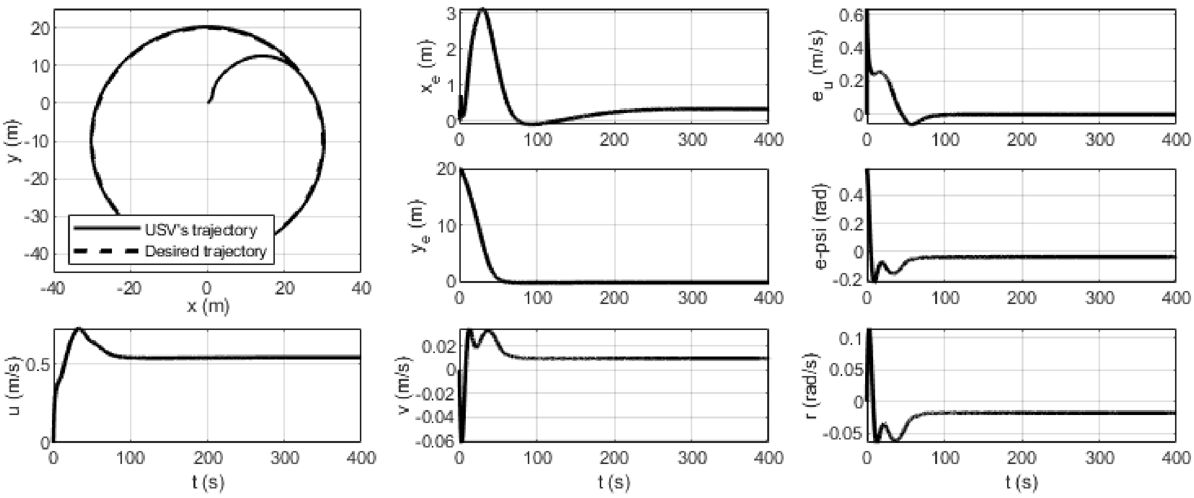

Fig.5. Trajectory tracking performance - Simulation Run #4

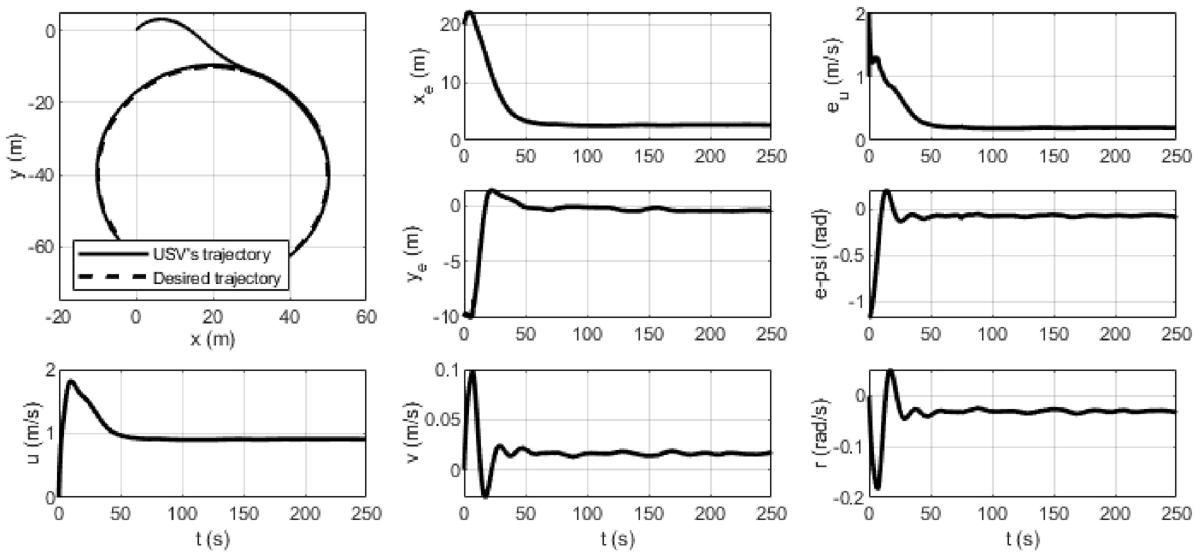

Fig.6. Trajectory tracking performance - Simulation Run #5

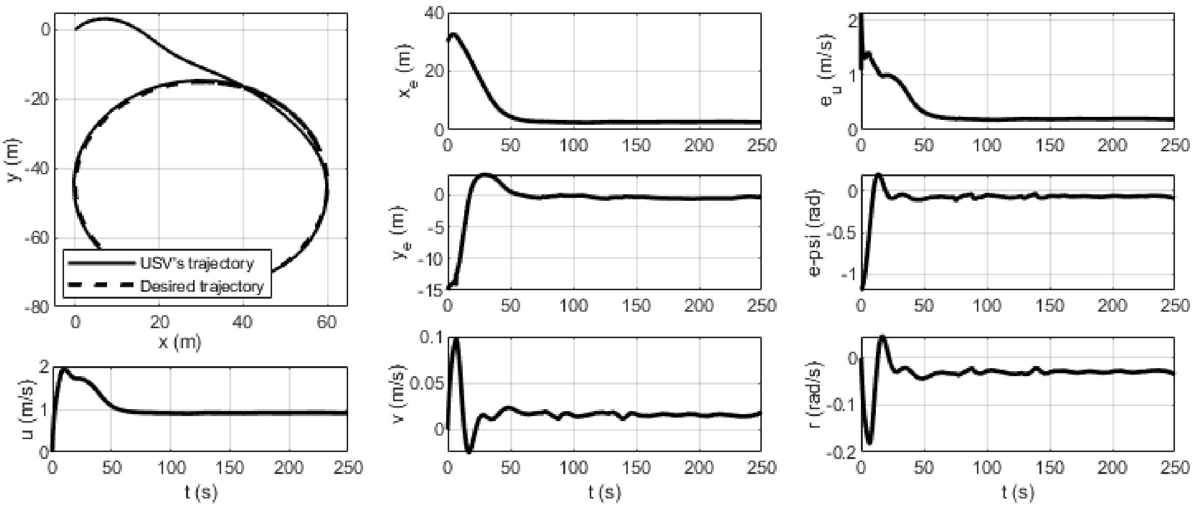

Fig.7. Trajectory tracking performance - Simulation Run #6

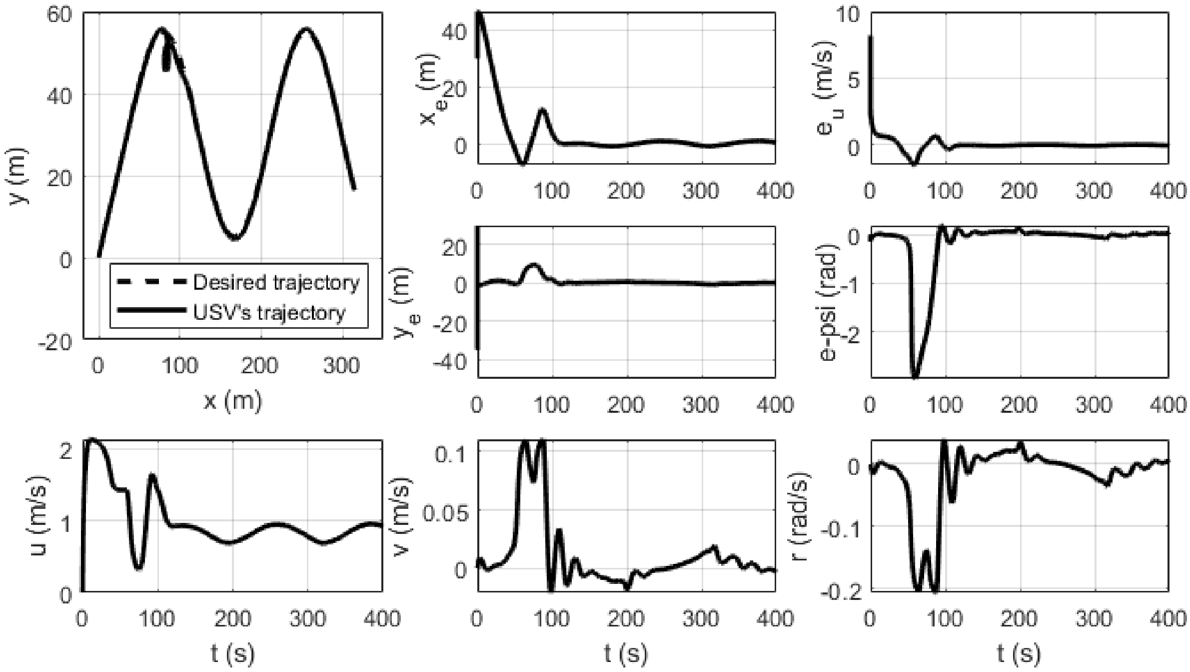

Fig.8. Trajectory tracking performance - Simulation Run #7

Simulation Run #6

$$\begin{cases} x_d(t) = 30 + 30sin(0.03t) \\ y_d(t) = -45 + 30cos(0.03t) \\ [u \quad v \quad r]_{t=0} = [0 \quad 0 \quad 0] \\ [x \quad y \quad \psi]_{t=0} = [0 \quad 0 \quad \pi/4] \end{cases}$$

Simulation results of Run #4, Run #5, Run #6 are shown in Fig. 5, Fig. 6, Fig. 7, respectively.

The USV's trajectory, desired trajectory, along-track error, cross-track error, heading error, surge velocity, sway velocity and yaw rate of each simulation run are shown in their respective figure.

Fig. 5. shows the tracking performance of circular trajectories with relatively small initial tracking-error condition. The USV model tracks a smooth trajectory. The along-track error stabilizes about steady state errors below 2.8 m. The cross-track error quickly converges to the origin. The sway velocity decreases asymptotically to a steady-state value of 0.01 m/s. The USV model experiences a mild overshooting during the pursuit phase.

Fig 6. and Fig. 7 shows the tracking performance of circular trajectories with relatively small initial tracking-error condition. The cross-track error quickly converges to the origin with mild perturbation. The sway velocity decreases to a steady-state value of 0.02 m/s with mild perturbation. The cross-track error converges to the origin with perturbation magnitude below 0.6 m.

Overall, the proposed trajectory-tracking method gives good tracking performance of circular trajectories.

5.3. Sinusoidal Trajectories

The desired trajectories and initial conditions are as follows

Simulation Run #7

$$\begin{cases} x_d(t) = 35 + 0.7t \\ y_d(t) = 30 + 25sin(0.025t) \\ [u \quad v \quad r]_{t=0} = [0 \quad 0 \quad 0] \\ [x \quad y \quad \psi]_{t=0} = [0 \quad 0 \quad \pi/4] \end{cases}$$

Simulation Run #8

$$\begin{cases} x_d(t) = 60 + 0.8t \\ y_d(t) = 60\sin(0.01t) \\ [u \quad v \quad r]_{t=0} = [0 \quad 0 \quad 0] \\ [x \quad y \quad \psi]_{t=0} = [0 \quad 0 \quad \pi/4] \end{cases}$$

Simulation Run #9

$$\begin{cases} x_d(t) = 0.7t \\ y_d(t) = 10 + 25\sin(0.025t) \\ [u \quad v \quad r]_{t=0} = [0 \quad 0 \quad 0] \\ [x \quad y \quad \psi]_{t=0} = [0 \quad 0 \quad \pi/4] \end{cases}$$

Simulation results of Run #7, Run #8, Run #9 are shown in Fig. 8, Fig. 9, Fig. 10, respectively.

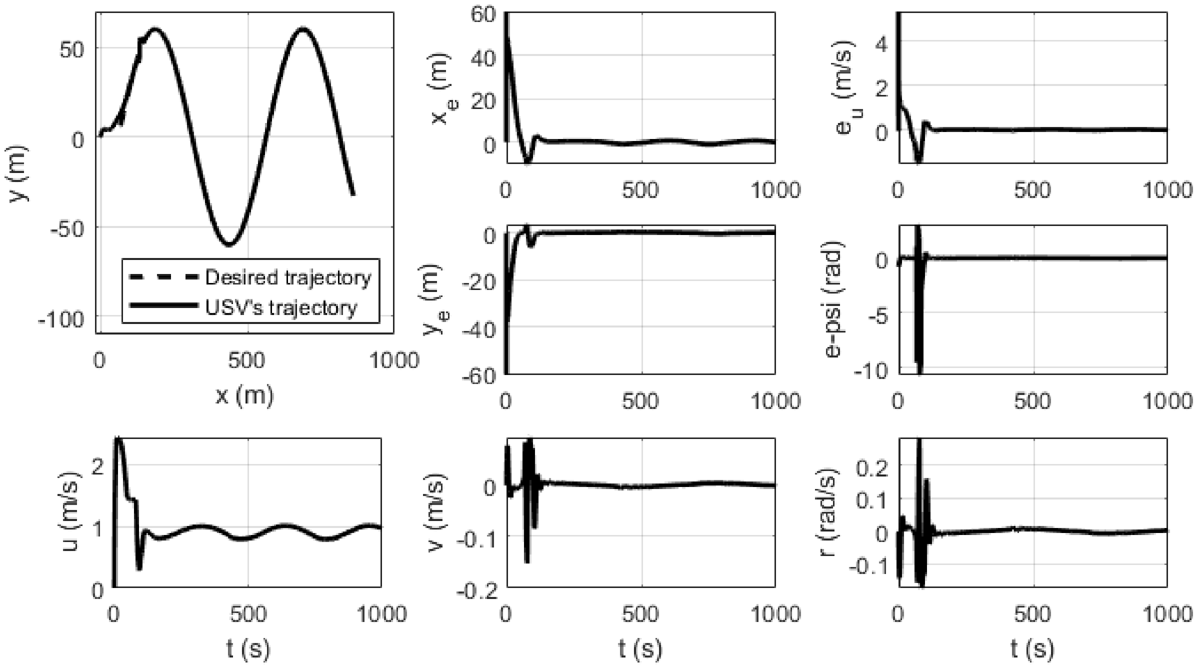

Fig. 9. Trajectory tracking performance - Simulation Run #8

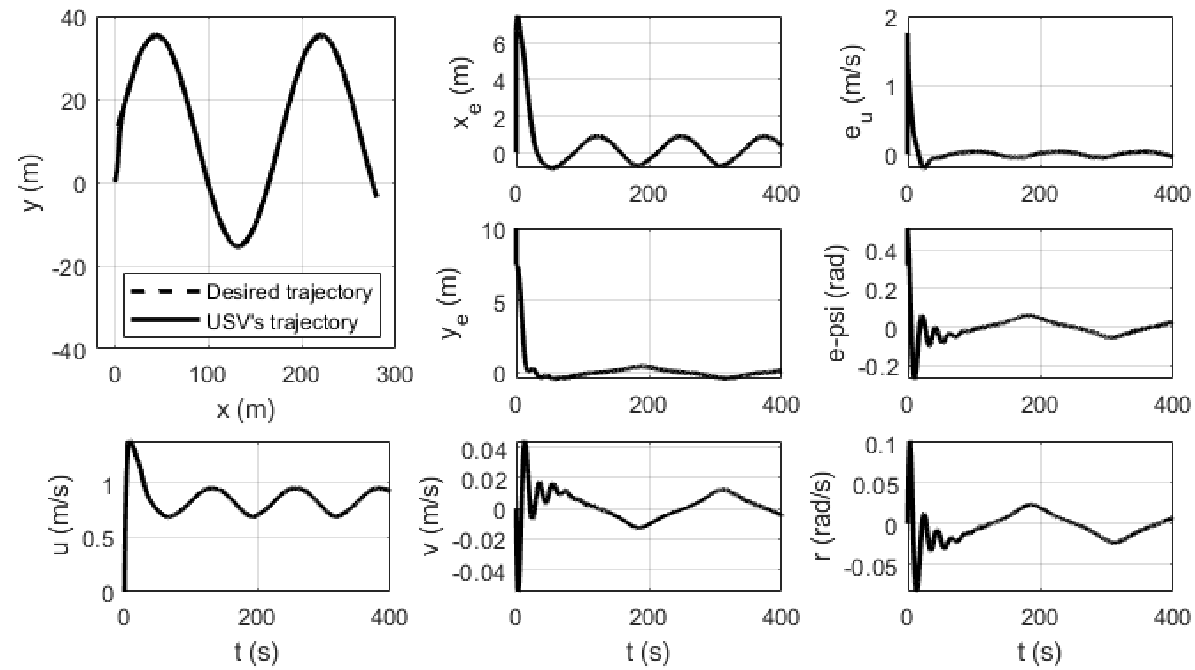

Fig. 10. Trajectory tracking performance - Simulation Run #9

The USV's trajectory, desired trajectory, along-track error, cross-track error, heading error, surge velocity, sway velocity and yaw rate of each simulation run are shown in their respective figure.

Fig. 8, Fig. 9, Fig. 10 show the tracking performance of a sinusoidal trajectories with relatively large initial along-track error. Cross-track errors in Simulation Run #7 and Run #8 converge to the origin. Cross-track error in Simulation Run #9 oscillates about the origin with magnitude under $0.5\,m$ and the sinusoidal trajectory's frequency.

In both Simulation Run #7 and Run #8, during initial pursuit phase, the along-track errors overshoot, which result in undesired instability. In Simulation Run #9, the along-track error oscillates about the origin with magnitude under $1\,m$.

Overall, the simulations show that u_{pseudo} remains bounded regardless of large tracking-error; k_{e_ψ} factor effectively constrains u_{pseudo} when $|e_\psi|$ is large; cross-track error effectively converges to the origin; along-track tracking performance suffers a certain degree of steady state error.

Instability caused by overshooting of the along-track error comes from the fact that each set of constant control parameters is suitable only for a specific range of desired surge velocity and yaw rate.

6. Conclusion

This paper proposes a trajectory tracking method for under-actuated USVs that can be implemented with a simple PI controller. A model is built based on linear dynamics equations and parameters of an USV. The proposed tracking method is applied to the model to track different trajectories in simulations. Simulation results show that the proposed trajectory tracking method can effectively track straight and circular trajectories.

The surge velocity control term is designed to overcome relatively large yaw error and large tracking-error conditions. Simulation results show good tracking performance in straight and circular trajectories despite relatively large initial tracking-error conditions. Under large tracking-error conditions, the surge velocity control term is bounded. Under large yaw error conditions, the surge velocity term is constrained so that the USV can rapidly turn to the desired yaw angle at low surge velocity before pursuing at high surge velocity.

Tracking performance in sinusoidal trajectories simulations show the limits of this control method. Sinusoidal trajectories simulations show overshooting during initial pursuit phase, which results in undesired instability. This overshooting is caused by the fact that the surge velocity control term is designed to operate only within a limited range of time-varying desired velocity. Further work needs to be done to calculate the maneuverability limit of the specified USV. In future work, we will conduct more rigorous analysis of the maneuverability limits of different sets of constant control parameters and design a more adaptive control method.

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