JST: Engineering and Technology for Sustainable Development
Volume 32, Issue 2, April 2022, 065-073

Control of Semi-Active Suspension System Using Kalman Observer

Nguyen Trung Kien[1,2*], Dam Hoang Phuc[1], Lai Nang Vu[3], Vu Hai Thuong[2]
[1]Hanoi University of Science and Technology, Hanoi, Vietnam
[2]Nam Dinh University of Technology Education, Nam Dinh, Vietnam
[3]Technical Department, Department of Defense, Hanoi, Vietnam
[*]Email: kien.nguyentrungspkt@gmail.com

Abstract

Optimal control methods are increasingly used in automatic control systems, especially in automotive suspension system. However, the optimal control algorithm only achieves the highest efficiency in suspension control system when the required number of sensors is sufficient, corresponding to the number of states in the system. The arrangement of sufficient number of sensors depends on the capacity, economic conditions and responsiveness of the sensor. The Kalman observer is designed to reliably estimate the required parameters in the control where the number of sensors is limited. The article focuses on analyzing the theory of building a quarter-car model, developing and determining the optimal control matrix, the Kalman observer design method. The findings of the article reveal the effectiveness of automotive body vibration suppression and the required force for control corresponding to LQG control and LQR control, under the influence of square pulse road surface, when using two similar sensors are installed on the sprung and unsprung, thereby providing a choice of sensor type and the location on the semi-active ¼ suspension.

Keywords: LQR, LQG, observer, Kalman, a quarter-car model

1. Introduction

In automotive engineering system, the suspension system plays an important role in stability and comfort of a vehicle as well as passengers since it is responsible for the vehicle's body vibration. Such system can be classified into three group, including passive, semi-active and active suspensions. Among them, the semi-active configuration is preferred due to its cost effectiveness and controllability. Different from the passive suspension, the semi-active one, which includes an actively variable damping coefficient, has better vibration isolation, meanwhile, it requires less energy than the active configuration does. The semi-active system can change the viscosity of the dampers instead of increasing the stiffness of the elastomer. Research on semi-active suspension is continuously developed to create the highest efficiency, bridge the gap between semi-active and fully active suspension systems. The semi-active suspension system controls the damping force to improve the smoothness and safety of the automobile's movement. Damping force is changed through damping coefficient or flow through the orifice on the damper piston.

Currently, there are various research works relating to suspension control in literature. The study on the linear quadratic optimal control technique (LQR) [1] compared the vibrations between the passive suspension and the controlled suspension on different types of road surface. The findings evaluated the system efficiency between controlled and uncontrolled condition. In such system, the road surface is a state variable and the signal needed in the control included five parameters (body and wheel displacement, body and wheel oscillation speed, road surface profile), which required a large sensing system (five parameters were equivalent to five sensors). To reduce the number of sensors in the system, the research work in [2] used an algorithm that predicts the state of the suspension in response to road input with a Kalman filter and cruise control of the suspension system between the suspended and unsuspended masses. The Kalman filter was used as an observer that observes the states of the system and predicts the next states of the model. The study used the LQR [1] together with the Kalman filter, forming the linear quadratic gaussian (LQG), to control and observe the suspension space, which reduced the number of sensors (without body and wheel displacement sensors). The estimation and calculation of control parameters through available sensors were also mentioned by many studies. The estimation method could be done with a small number of sensors, but the amount of information was sufficient for control [3]. The study focused on estimating the vertical velocity of the chassis and relative velocity between the chassis and the wheel. The input to the estimator was a signal from the wheel displacement sensors and from the accelerometer sensors located in the chassis. In

ISSN 2734-9381
https://doi.org/10.51316/jst.157.etsd.2022.32.2.9
Received: February 15, 2022; accepted: February 27, 2022

addition, the control signal was used as the input to the estimator. The Kalman filter was analyzed in the frequency domain and compared with a conventional filter solution that includes both displacement and acceleration signals inference. The study showed the accuracy and reliability of the estimation compared with the experiment. In the above studies, the road surface was used as a state variable, which required a sensor to determine the road surface profile. To replace this, the road surface condition estimation method [4] controlling the suspension using MR dampers was introduced. In study [5], a real-time open loop estimate of the disturbance displacement input to the tire and an external disturbance force. This estimate is achieved with two acceleration measurements as inputs to the estimator; one each on the sprung and unsprung masses. Each vehicle can effectively estimate the road profile based on its own state trajectory [6]. By comparing its own road estimate with the preview information, preview errors can be detected and suspension control quickly switched from preview to conventional active control to preserve performance improvements compared to passive suspensions. The study indicated the desired road surface to increase the comfort to users is MR damping. The research outcome was the road surface satisfying the comfort of the automobile body. The findings of the mentioned studies clearly showed the good controllability of the LQR and LQG algorithms in the efficiency of vibration suppression. The design of an observer using Kalman tool is necessary in control to reduce the number of sensors in the system. There have not been many studies evaluating the control efficiency of the LQG algorithm on the semi-active suspension system based on the type and number of sensors. Therefore, in this study, we use the LQR method [1] applied on the ¼ suspension model, but consider the road surface as a noise signal, not a state variable [4]. We select simulation, evaluate the effect of vibration suppression and desired control force with the case of using two sensors in control compared to the case of four sensors and passive suspension system using Kalman filter to design state estimators [2],[3],[7], and estimate four states of the suspension system ¼ from two states (equivalent to two sensors). Therefore, the system model will become simpler and straightforward, therefore reduce the amount of information to be measured and improve the level of calculation. We focus on the simulation and evaluation of the effectiveness of body vibration suppression on the quarter car model when using the LQG algorithm and two input sensors (the displacement or oscillating velocity sensors installed on body and wheels). The research results, through evaluating the efficiency of vibration suppression and the energy used in the control, evaluate the influence of the control algorithm corresponding to the type and number of sensors used in the system, thereby proposing sensor type and location in semi-active suspension.

2. System Design

2.1. System Model

A quarter-car model only considers the vertical displacement of the suspended and unsuspended parts, regardless of movements in other directions such as the lateral and longitudinal roll of the automobile. The quarter-car model using semi-active damping is shown in Fig. 1.

The relationship between the state variables of the model and the physical variables of the suspension is shown as follows:

$$x = \begin{vmatrix} z_b & \dot{z}_b & z_{\mathrm{w}} & \dot{z}_{\mathrm{w}} \end{vmatrix}^T \tag{1}$$

where $z_b; \dot{z}_b; z_w; \dot{z}_w$ are displacement and velocity of the body and wheel, respectively.

In this study, we consider road surface as the disturbance of the system, so the state equation is written as follows:

$$\begin{cases} \dot{x} = Ax + Bf + Gz_{\mathrm{r}} \\ y = Cx \end{cases} \tag{2}$$

where *y*: output signal; *A*: physical matrix of the system; *B:* control matrix; *C*: output signal matrix; *G*: input disturbance matrix; z_r: road profile; *f*: control force; *x*: state variables

Specifically:

$$A = \begin{vmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k}{m_b} & -\dfrac{c}{m_b} & \dfrac{k}{m_b} & \dfrac{c}{m_b} \\ 0 & 0 & 0 & 1 \\ \dfrac{k}{m_{\mathrm{w}}} & \dfrac{c}{m_{\mathrm{w}}} & -\dfrac{k+k_t}{m_{\mathrm{w}}} & -\dfrac{c}{m_{\mathrm{w}}} \end{vmatrix}; \; B = \begin{vmatrix} 0 \\ \dfrac{1}{m_b} \\ 0 \\ -\dfrac{1}{m_{\mathrm{w}}} \end{vmatrix}$$

$$C = \begin{vmatrix} c_1 & c_2 & c_3 & c_4 \end{vmatrix}^T; \; G = \begin{vmatrix} 0 & 0 & 0 & k_t/m_{\mathrm{w}} \end{vmatrix}^T$$

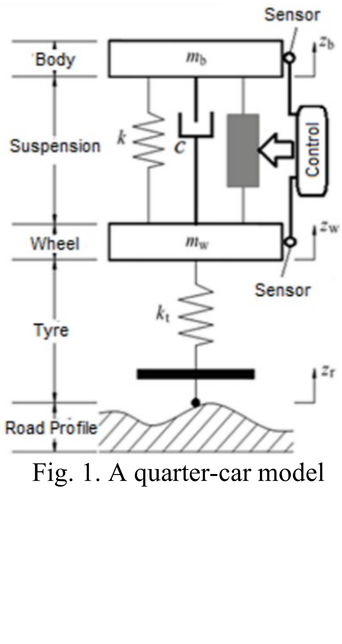

Fig. 1. A quarter-car model

Table 1. Parameters of the quarter-car

Parameter	Notation	Measure	Value
Body mass	m_b	*kg*	342
Wheel mass	m_w	*kg*	38
Spring stiffness	c	*N/m*	12500
Tire stiffness	k_t	*N/m*	76000
Damping coefficient	c	*Ns/m*	1500

Physical parameters of the suspension system model are shown in Table 1

2.2. Controller Design

2.2.1. LQG controller

The optimal controller LQG is a combination of the optimal state feedback controller LQR and the Kalman observer. In the LQG controller, the influence of the noises z_r, f will be monitored (or filtered) by the Kalman observer and gives the best state signal $x \approx z$. The state signal from the observer will be fed to the optimal state feedback controller LQR to generate the most optimal control signal $f(t)$. The selection of values in the observer (Kalman algorithm) depends on the type of sensor used in the model. The block diagram of semi-active suspension control system according to the LQG algorithm is shown in Fig. 2.

As shown in Fig. 2, the input to the LQG controller is the output signal of the suspension. This output signal depends on the matrix *C* (the output signal matrix). The selection of the values of the matrix *C* corresponds to the number and type of sensors used in the system. The output of the LQG controller is the value of the desired force applied to the suspension. This desired force can be from a semi-active damping, or a controllable elastomer.

The input signal to the LQG controller is the input signal to the Kalman observer. The Kalman observer estimates or filters these signals (depending on the number of sensors selected; when choosing enough sensors in the system, the Kalman observer functions as a filter). The output signal from the LQG controller is the output from the force controller according to the LQR algorithm. The relationship between the LQR controller and the Kalman observer is the estimated signal and the desired force *(f*)*. That is, the output from the Kalman observer will be the input to the LQR controller, and vice versa. This is a closed loop, ensuring the principles in the automatic control system. The calculation and construction of the LQR controller and the Kalman observer are completely independent.

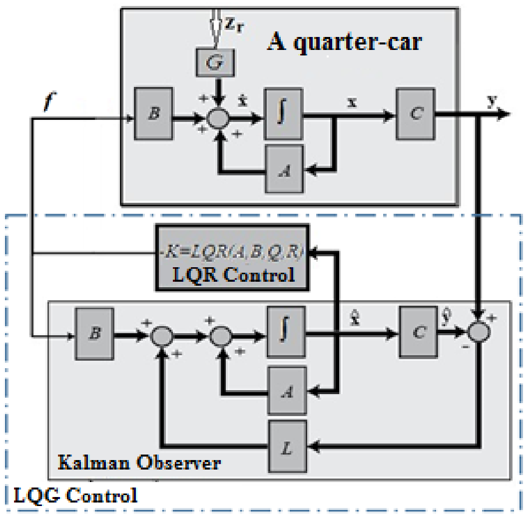

Fig. 2. Diagram of suspension system control according to the LQG algorithm

Fig. 3 is the layout of the LQG control for the semi-active suspension.

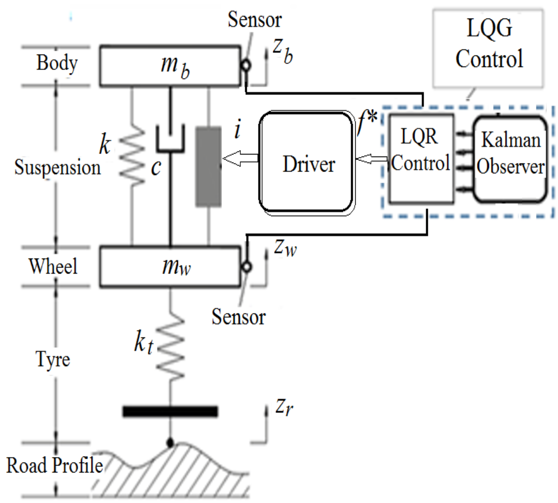

Fig. 3. Layout of the semi-active suspension system using the LQG controller

2.2.2. LQR controller

The goal of the control problem here is to determine the component f so that with an unknown external influence on (z_r) the state variable vector z needs to be quickly returned to the origin. In other words, it is necessary to quickly suppress the oscillations in the system caused by external forces over time. The diagram of suspension system with optimal control LQR is shown in Fig. 4.

The LQR algorithm determines the control signal f so that the objective function has the following quadratic form:

$$f = \int_0^{\infty} (x^T Q\, x + f^T R\, f) dt \quad (3)$$

where Q and R are weight matrices based on the time balance to make the system stable in quality and the control energy dissipation .

According to the diagram, the LQR controller is replaced by K matrix. The control law has the following form:

$$f = -Kx \quad (4)$$

where the state feedback matrix is determined from the following Ricatti equation:

$$-KA - A^T K - Q + KBR^{-1}B^T K = 0 \quad (5)$$

According to the diagram in Fig. 2 and the method of setting state variables, the matrix K is a 4×4 matrix with the input of four state variables. The matrix K has a variable value, when the weight matrices take into account the control efficiency and the level of energy dissipation in the control change. Thus, for each fixed system, this K value does not change during the control process. With the physical values in Table 1 and the selection of the weight matrix Q and R, the matrix K with the following values is determined:

$$K = \left| 21191 \quad 2593 \quad -21638 \quad -234 \right|^T$$

According to the state variable setting method, the LQR controller needs four state parameters of the system: displacement of the body and wheel; displacement velocity of body and wheel. Therefore, to apply LQR to control the suspension system, it is necessary to equip four sensors corresponding to four state parameters. In this study, to match the assembly ability and economic conditions, we selected 2/4 sensors. Because the number of selected sensors is less than required by the LQR algorithm, it is necessary to design a state estimator (observer) so that the information from two sensors can be converted into the information of four sensors according to the optimal LQR controller requirements. This is calculated and built through the Kalman observer.

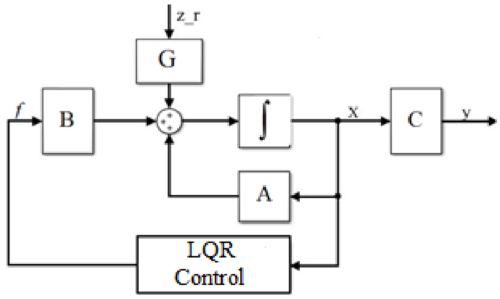

Fig. 4. Diagram of suspension system control according to LQR

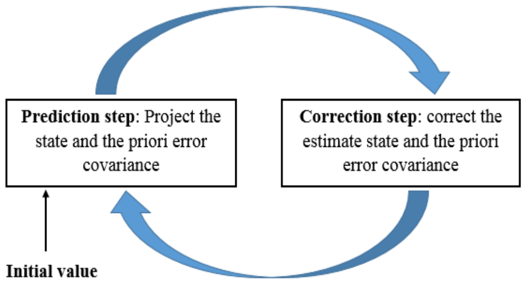

Fig. 5. The prediction and correction steps of Kalman filter

2.2.3. Kalman observer

The Kalman filter is a remarkable method to predict and estimate the state of a stationary process by minimizing the mean square error.

The results of Kalman filter have very small error. The Kalman filter has applications in spacecraft orbit determination, estimation and prediction of target trajectories, simultaneous localization and mapping,

The discrete Kalman filter cycle is shown in Fig. 5. It consists of two steps:

• Prediction step. In prediction step, the goal is to obtain the predicted state for next time step by forward projection of the current state and error covariance estimates.

• Correction step. In correction step, the aim is to correct the estimate state and error covariance.

The purpose of the estimator is to estimate the working states of the suspension system based on the model state variable setting method. The LQR controller estimates the value of the state vector x in the system. From the estimated x, the damping resistance f will be calculated through the control matrix.

The estimated values are based on the output signal from the actuator in the semi-active suspension (semi-active damping), and the sensor signal from the sensors located on the wheels and the body of the vehicle. The diagram of Kalman observer connection in semi-active suspension control is shown in Fig 6.

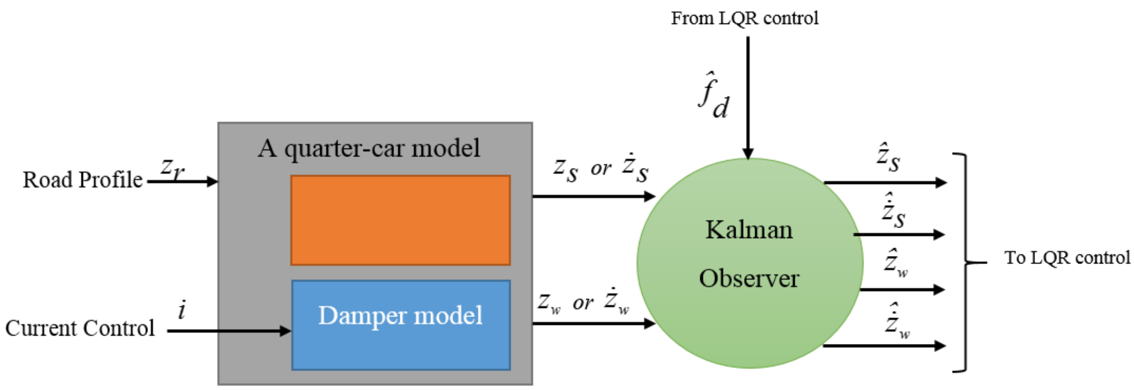

Fig. 6. Diagram of Kalman observer connection in semi-active suspension control

Kalman observer is responsible for estimating and calculating the output signal for LQR controller. LQR controller needs four state variable parameters.

The Kalman observer is used to estimate the working states of the suspension system based on the estimation algorithm according to the available sensors. The observer determines the process of changing state from the time *(k-1)* to the time *(k)* according to the formula:

- Time update:

$$\begin{cases} \hat{x}_k^- = A\hat{x}_{k-1} + Bf_k \\ P_k^- = AP_{k-1}A^T + Q_e \end{cases} \tag{6}$$

- Measured value update:

$$\begin{cases} K_k = P_k H^T (HP_k H^T + R_e)^{-1} \\ \hat{x}_k = \hat{x}_k^- + K_k(\hat{x}_k - H\hat{x}_k^-) \\ P_k = (I - K_k H)P_k^- \end{cases} \tag{7}$$

where A is a time-variant matrix relating the state at the previous time step *(k-1)* to the state at the current step *(k)* and H is a time-variant matrix relating the state to the measurement (they are assumed to be constant; $\hat{x}_k^-$ is the predicted state vector containing the state variables of interest, $\hat{x}_{k-1}$ is the previous state vector, f_k is the input vector; P_k^- is the priori error covariance. It is used to calculate the Kalman gain in the correction step. The correction update steps by equation (7); Q_e is the noise covariance matrix; R_e is the noise measurement covariance matrix; K_k is the Kalman gain that minimizes the posteriori error covariance; P_k is posteriori error covariance and I is unit matrix.

When selecting two displacement sensors, choose: Q_e =30, R_e =1. And two speed sensors, choose: Q_e =1, R_e =1.09

3. Simulation and Survey

3.1. Simulation Scenario

In this study, we selected two sensors. The sensor type is displacement sensor or oscillating velocity sensor. Simulation results compare the efficiency of vibration suppression and desired control force in control options when using two sensors (LQG algorithm) and four sensors (LQR algorithm) compared to passive ones.

The simulation plan is presented in Table 2.

Table 2. Simulation scenario

Input / Option	Road surface	Sensor				Evaluation criteria	Note
		z_b	z_w	$\dot{z}_b$	$\dot{z}_w$		
1	Square bump			X	X	-Efficiency of vibration suppression - Desired control force	LQG1
2		X	X				LQG2
3		X	X	X	X		LQR
4							Passive

3.2. Simulation Results

Simulation results evaluate the effectiveness of automobile body vibration suppression under different control options. Fig. 7 shows the vibration of the automobile body. According to the graph, when using the LQG controller (LQR + Kalman observer), the oscillation suppression efficiency of the LQG1 controller is highest. The oscillation suppression efficiency is expressed through the maximum amplitude of vibration and the time to suppress the oscillation. Fig. 8 shows the maximum vibration amplitude of the automobile body corresponding to the control options.

When using the LQG control option with two velocity sensors, the maximum amplitude reduction effect is highest, while when using two displacement sensors, the efficiency is slightly lower.

To evaluate the reduction in amplitude of body oscillation of each option, we develop a formula to determine the percentage of reduction in amplitude of the control options compared to the passive suspension system. The formula is as follows:

$$\delta = \frac{x_{p_{max}} - x_{C_{imax}}}{x_{p_{max}}} \times 100(\%) \qquad (8)$$

where δ: percentage of reduction in amplitude; x_{pmax}: maximum amplitude of body in the passive state; x_{Cmax}: maximum amplitude of body in the i^{th} option

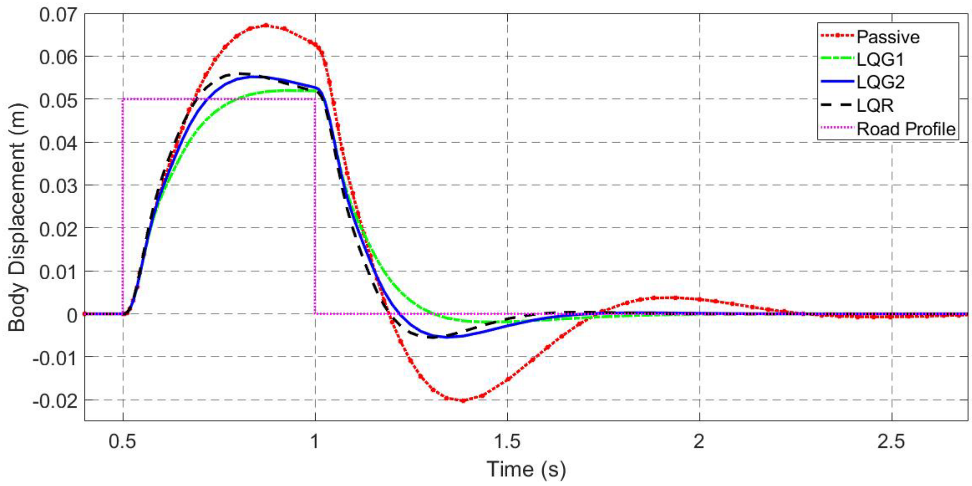

Fig. 7. Body displacement with different control options

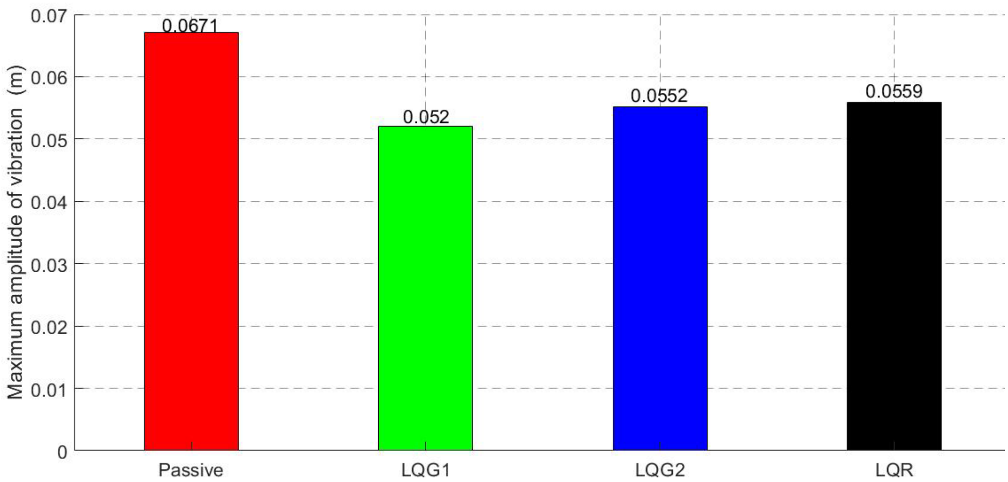

Fig. 8. Maximum amplitude of vibration with different control options

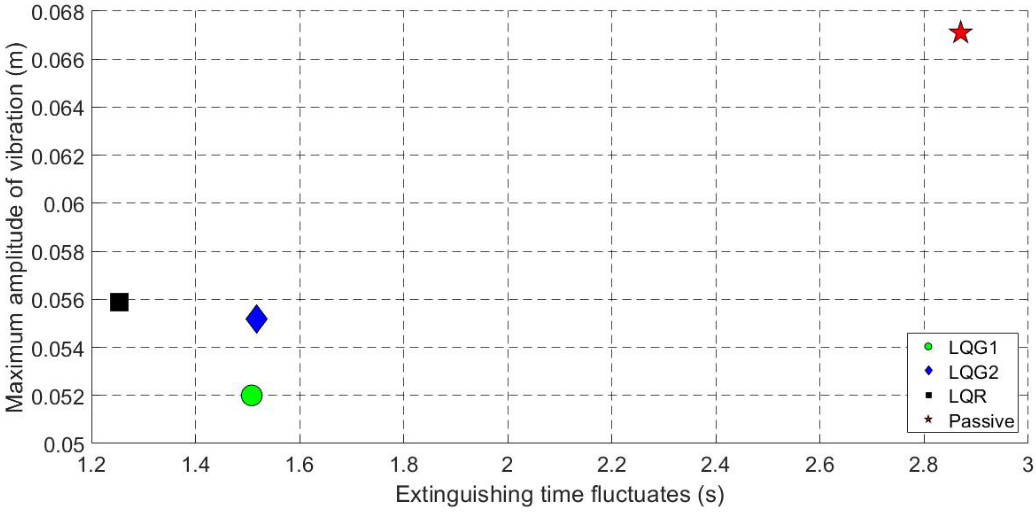

Fig. 9. The comparison of vibration suppression efficiency with different control options

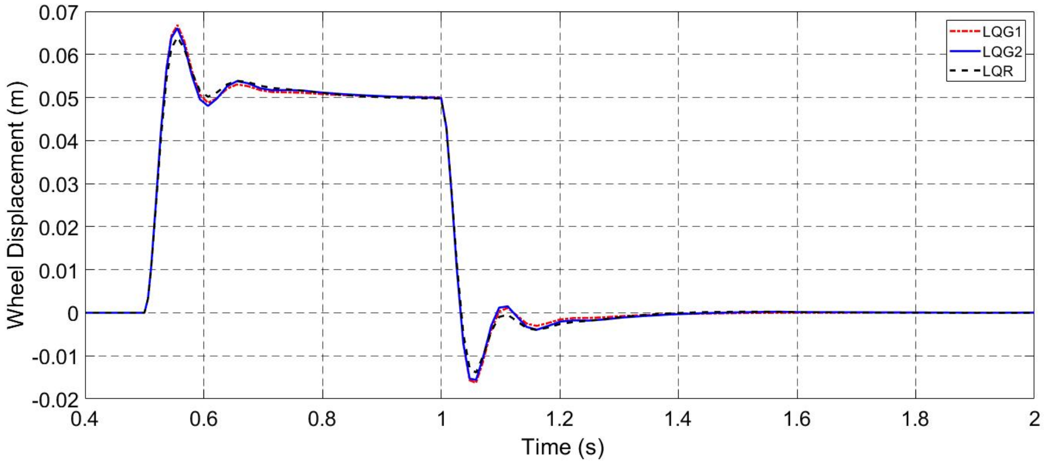

Fig. 10. The comparison of wheel displacement with different control options

The comparison of the vibration suppression efficiency (the reduction of maximum amplitude and the time of vibration suppression) among simulation options is shown in Fig. 9.

From the graph, we can see that the distance of the equilibrium positions among the three control options LQG1 (circle), LQG2 (rhombus) and LQR (square) are similar. In terms of oscillation time suppression efficiency, LQR controller is the best. Regarding amplitude reduction, LQG1 controller achieves the highest efficiency (22.5% reduction), LQG2 controller decreases by 17.73%, and LQR controller reduced the lowest amplitude (10.73% reduction). This shows that the control efficiency is different between the two controllers LQG and LQR. The LQG controller has better effect on attenuating the oscillation amplitute, whereas the LQR counterpart performs more effectively in reducing the fluctuation time. As for the LQG controller, the oscillation suppression time of the two options LQG1 and LQG2 is quite similar. This shows that, in terms of oscillation suppression efficiency, the LQG1 option (using two velocity sensors) is the best. In addition, the comparison of positions on the graph shows a clear effect between controlled and passive suspension (star shape).

The wheel oscillations corresponding to three simulation options LQG1, LQG2 and LQR are shown in the figure. According to the graph, the wheel oscillations of the two options LQG1 and LQG2 are quite similar in both amplitude and frequency. The maximum wheel amplitude according to the LQG1 option is 0.06683(m), LQG2 is 0.06609(m) and LQR is 0.06401(m). The difference in the maximum wheel oscillation amplitude between the LQR control algorithm and the two LQG1 and LQG2 algorithms is 2.82 mm and 2.08 mm, respectively.

Thus, in terms of wheel oscillation amplitude, the control efficiency according to the control law LQR is the best, and the LQG1 is the worst. This shows the rationality while controlling the suspension system, that is: to achieve the smoothness effect of the body, the wheels will vibrate more and will be more likely to separate from the road surface.

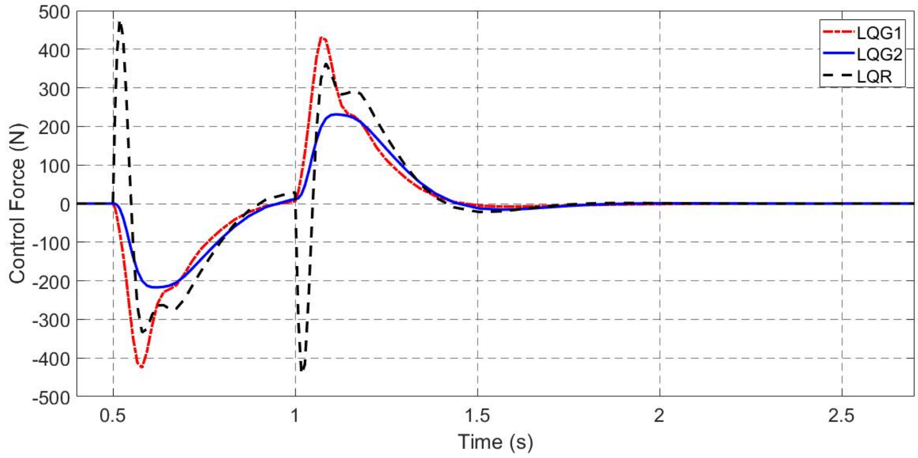

Fig. 11. Control force characteristics of suspension system ¼

In the control design, it is necessary to consider the quality of control because each different control algorithm will give different results. If the reduction of amplitude is satisfied, the oscillation suppression time will increase and vice versa. The choice of control quality depends on the control capabilities and sensors used in the system. The control quality depends on the responsiveness of the control energy or control force.

Fig. 11 shows the control force characteristics according to different control algorithms. According to Fig. 11, the maximum control force corresponds to the largest LQG1 controller (430N), the smallest LQG2 controller (231N). In Fig. 7, the amplitude reduction corresponding to the two cases of LQG1 and LQG2 controllers is about 5% different, but in Fig. 9, the maximum control force of the LQG1 controller is almost twice as large as that of the LQG2 controller. On the other hand, the characteristic curve of the LQG2 controller is much more linear than that of the LQG1 controller. This shows that the ability to generate and control the control force of LQG2 is easier than that of LQG1 controller.

Therefore, when designing the actuator to generate control force for the suspension system, the LQG2 controller is better and easier (small control force but good effect). As for the LQR algorithm, when all the four parameters from the sensor are sufficient, the control force characteristics are able to act faster and compatible with the actual impact of the road surface) and the control force in the compression stroke is larger than in the exhaust stroke. This clearly shows the advantage of LQR control in quick suppressing the oscillation time of the automobile body.

4. Conclusion

This paper focuses on theoretical analysis in building a semi-active suspension model using the LQG control algorithm. This algorithm is a combination of the LQR linear quadratic controller and the Kalman observer. The study has found the optimal set of control parameters (*K* matrix) as well as calculated and estimated input parameters through sensors used in the system. The article focusing on analyzing the control efficiency of the suspension system with two sensors in the system (compared with the optimal control requirement of four sensors) has compared the control quality through the oscillation suppression efficiency and desired control force according to each simulation option.

When controlling a semi-active suspension system under the condition that the driver has only two sensors, he should choose two sensors of the same type. This is consistent with reality, that means, choosing two displacement sensors or two velocity sensors. The sensor location is on both the body and the wheel. In this study, we find that the control method using the LQG2 controller (using the 02 displacement sensors) is the most effective, because the control force generated is small but still effective in suppressing the oscillation (the amplitude is 5% lower than that using the LQG1 controller (using the two velocity sensors), but the control force is nearly twice as small). The LQG2 controller is suitable for actuator design and development. Therefore, it is recommended choosing a displacement sensor for both the body and the wheel and use the LQG2 controller. This is completely consistent with the reality of using and developing sensor technology today, because displacement sensors are easier to manufacture and cheaper, especially when the output signal characteristics from the sensor are similar and linear. Thus, the algorithm to read data from the sensor is also simpler. The development of the Kalman algorithmic estimator needs to be studied to give the most accurate estimation parameters, which can be applied to the alternating arrangement between the types and the number of sensors (using oscillation sensors and velocity sensor alternately, or using one or three sensors...), thereby improving control efficiency as well as economic efficiency.

5. References

[1] Abdolvahab Agharkakl, Ghobad Shafiei Sabet, Armin Barouz, Simulation and analysis of passive and active suspension system using quarter car model for different road profile, International Journal of Engineering Trends and Technology, Vol.3 , no. 5., 2012

[2] Pawar, Shital M., A. A. Panchwadkar, Estimation of state variables of active suspension system using Kalman filter, International Journal of Current Engineering and Technology EISSN 2017: 2277-4106.

[3] Lindgärde, Olof. Kalman filtering in semi-active suspension control, IFAC Proceedings, Vol. 35.1 2002, pp. 439-444.

[4] Zuohai Yan Shuqi Zhao, Road condition predicting with kalman filter for magneto-rheological damper in suspension system, Blekinge Institute of Technology, 2012

[5] Jonathan Daniel Ziegenmeyer, Estimation of disturbance inputs to a tire coupled quarter-car suspension test rig, Master Thesis, Virginia Polytechnic Institute and State University, 2007.

[6] Rahman M, Rideout G, Using the lead vehicle as preview sensor in convoy vehicle active suspension control, Vehicle System Dynamics, Vol. 50, Issue 12, 2012, p. 1923-1948.

[7] Ying Fan, Hongbin Ren, Sizhong Chen, Yuzhuang Zhao, Observer design based on nonlinear suspension model with unscented Kalman filter, School of Mechanical Engineering, Beijing Institute of Technology, Beijing 100081, China, 2015
https://doi.org/10.1080/00423114.2012.707801