JST: Smart Systems and Devices
Volume 33, Issue 1, January 2023, 064-071

Influence of Shape Factor of a Beam Cross-Section on a Heat Transfer Process of the Electrothermal V-Shaped Micro Actuators

Kien Trung Hoang[1], Phuc Hong Pham[2*]
[1]Le Quy Don University of Technology, Ha Noi, Vietnam
[2]Hanoi University of Science and Technology, Ha Noi, Vietnam
[*]Corresponding author email: phuc.phamhong@hust.edu.vn

Abstract

This paper proposes a method to determine the shape factor in the heat transfer model of silicon electrothermal V-shaped micro actuators by using ANSYS WORKBENCH. The rectangular cross-section model (w-width and h-thickness) is built to investigate the heat transfer from the beam to the substrate through a thin layer of air. Moreover, this model can be used to determine the influence of beam width and beam thickness or height on its shape factor. Through simulation, the lowest heat emission generated on the beam when the height-to-width ratio (h/w) is about from 3 to 4. The shape factor is proportional to the ratio of beam height to beam width as well as the air gap between the V-beam and the substrate. The objective of the paper is to provide an effective method to determine the shape factor in the theoretical heat transfer model of the electrothermal V-shaped micro actuators and can be also applied to similar heat transfer models of other actuators.

Keywords: Shape factor, heat transfer model, electrothermal V-shaped micro actuator.

1. Introduction

Electrothermal V-shaped micro actuator is the element that generates motion and force to drive MEMS devices or micro systems. Due to some outstanding advantages such as a large driving force, low driving voltage and stable working, the V-shaped micro actuator has been used in micro devices and systems such as micro gripper [1], nano material testing system [2], thermal safety device [3], linear micro motor [4], etc. This actuator works on the principle of thermal expansion of the thin V-beam system when applying a voltage. Thus, a mathematical model is needed to describe the thermal-electrical energy conversion processes. This process is often complicated due to the influence of many nonlinear factors such as the thermal conductivity of the material, environment, working space, beam dimension. It is also a goal that is interesting in research, as well as the topic of many publications recently. Mathematical methods frequently used to describe heat transfer models can be listed as direct analysis [5, 6], finite difference [7, 8], nodal point [9], equivalent circuit [10], etc.

Heat transfer model using direct analysis method is applied to determine the temperature distribution on thin beams. The steady-state heat transfer model in the form of second-order differential equations has been elaborated in [11]. In this work, the author mentioned the effect of heat transfer through the air gap between the device layer and the substrate. The heat transfer model has been applied in some later publications such as [12, 13]. In [5], the authors improved the method by adding heat loss through the shuttle to substrate. To describe the phenomenon of heat transfer through the air gap between the device layer and the substrate, the above publications have used the shape factor of the beam [5, 11-13]. The shape factor is dimensionless quantity representing the ability of transfer heat from the beam to the substrate through the air gap. Thus, the shape factor is an important parameter of the mathematical models describing the heat transfer process of the micro-actuator. In the microsystems with feature size of micrometers, determining this coefficient experimentally is quite difficult.

This work focuses on establishing a formula to calculate the shape factor of the V-shaped micro actuator structure with an air gap changing from 1 to 4 µm, device layer thickness varying from 6 to 30 µm, and the beam width ranging from 3 to 10 µm. Furthermore, the influence of beam width w and beam thickness h on its shape factor is also presented.

2. Heat Transfer Model of Electrothermal V-Shaped Actuator

The configuration of the V-shaped actuator is shown in Fig. 1. The structure of the micro actuator consists of three main parts: a shuttle 1, inclined beam system 2, and fixed electrodes 3. The shuttle is placed at the center of the actuator and suspended by a symmetrical V-beam system. The beams have an incline angle θ with the X direction, the other end of the beam engages with two electrodes.

ISSN: 2734-9373
https://doi.org/10.51316/jst.163.ssad.2023.33.1.9
Received: July 13, 2022; accepted: November 29, 2022

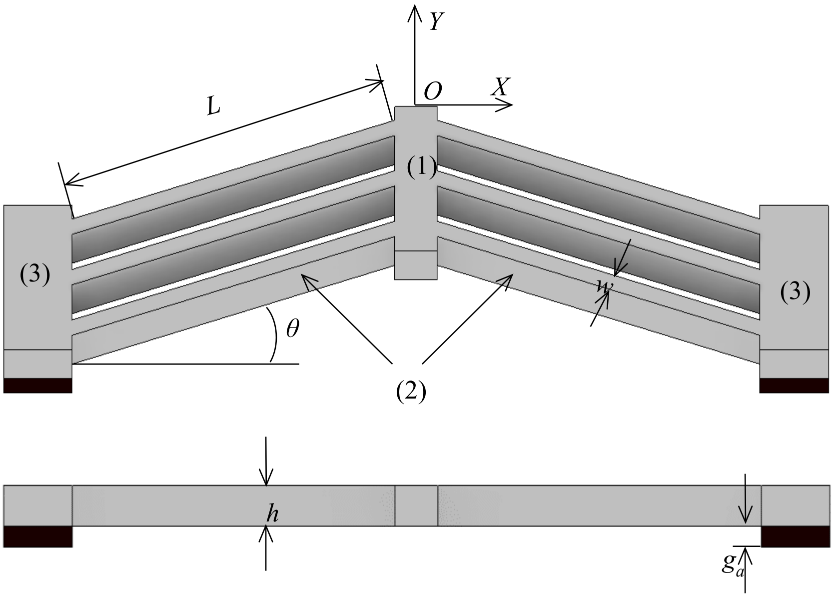

Fig. 1. The configuration of the electrothermal V-shaped actuator.

In Fig. 1, L, h and w are the length, the height (or the thickness of device layer), and the width of a single beam, respectively; g_a is the air gap between a device layer and substrate.

When voltage is applied to the two electrodes 3, the current will be conducted through the thin beams and generate heat. The beams will heat up and expand in the axial direction, creating a force that pushes the shuttle to move in the Y direction. When the voltage drops to zero, the temperature on the beams gradually decreases to ambient temperature causing the beams to shrink and pull the shuttle back to the original position.

To simplify the mathematical model, the assumptions for the analytical heat transfer model are: material properties to be constant during temperature changes; neglecting the heat transfer from the shuttle 1 to the substrate through the air gap; assuming the temperature on the electrodes 3 and the substrate always equal to the ambient temperature; neglecting heat transfer due to convection and radiation in the air; heat transfer is being considered one-way and along the length of the V-beams (regardless of the influence of beam angle).

The equivalent beam model of the pair of V-beam is shown in Fig. 2.

The energy balancing equation in x-direction of a beam element at x is as follows:

$$dq_{st} = dq_{cd} + dq_{ls} + dq_e \tag{1}$$

where, q_{st}, q_{cd}, q_{ls} and q_e are the heat stored, heat conduction, heat loss through the air gap and heat generated by applying voltage in a beam element, respectively. They are expressed as:

$$dq_{st} = C_p D_s \frac{dT}{dt} A dx \tag{2}$$

$$dq_{cd} = k_s A \left.\frac{dT}{dx}\right|_{x+dx} - k_s A \left.\frac{dT}{dx}\right|_{x} \tag{3}$$

$$dq_{ls} = -\frac{k_a S}{g_a}(T - T_0) w dx \tag{4}$$

$$dq_e = \frac{U^2}{4\rho_0 L^2} A dx \tag{5}$$

where, C_p, D_s and k are the specific heat, density and thermal conductivity of silicon, respectively; k_a is the thermal conductivity of air; S is the shape factor of beam; U is the driving voltage; ρ_0 is the resistivity of silicon at room temperature; T and T_0 are the temperature of beam element and ambient. A is the section area of the beam.

The equation describing the heat transfer process in the equivalent heat state of V-beams (Fig. 2) has the following form [6]:

$$C_p D_s \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} - \frac{k_a S}{h g_a}\left(T - T_0\right) + \frac{U^2}{4\rho_0 L^2} \tag{6}$$

From (6), the heat density loss through the air gap on a beam cross-section is expressed as:

$$q_{ls} = \frac{k_a S}{h g_a}(T - T_0) \tag{7}$$

From (7), the shape factor S can be inferred as follows:

$$S = q_{ls} \frac{h g_a}{k_a (T - T_0)} \tag{8}$$

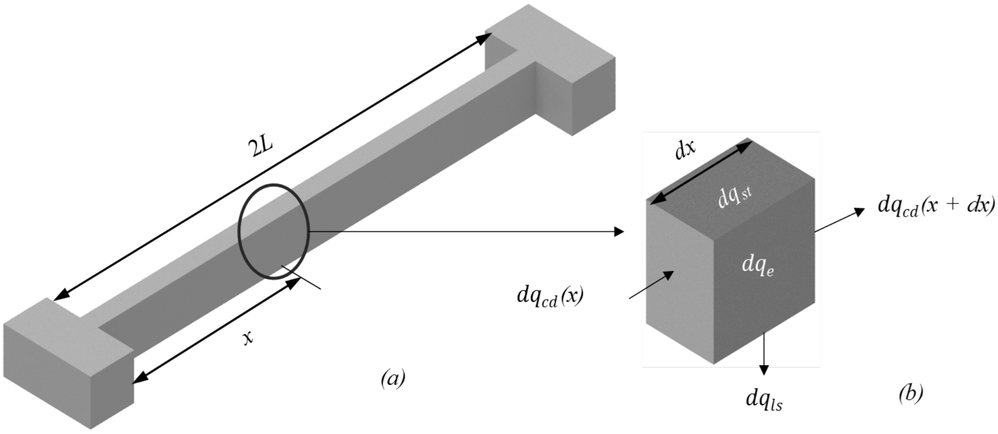

Fig. 2. The equivalent beam model (*a*) and one beam element (*b*)

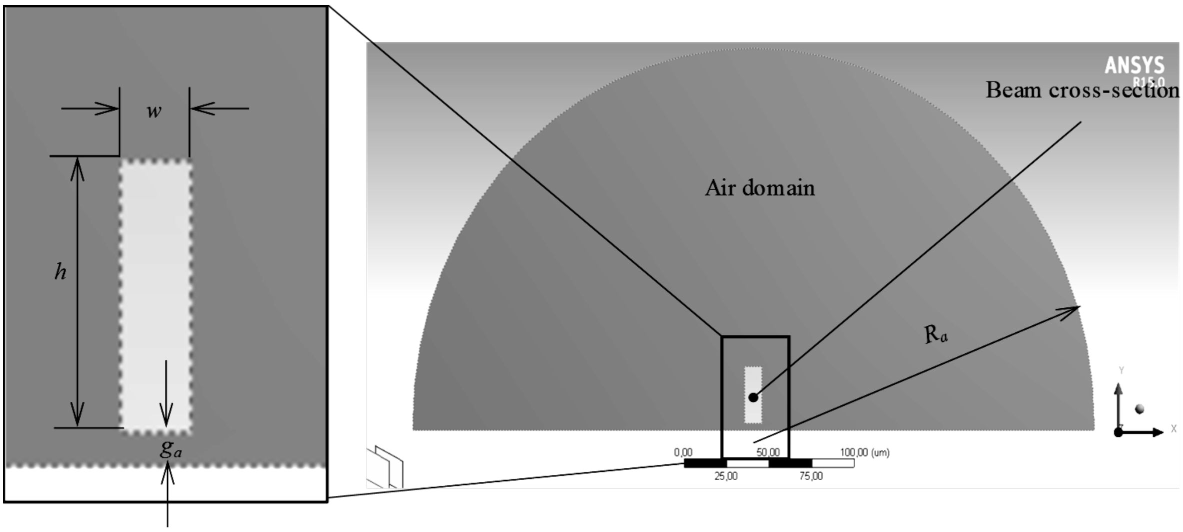

Fig. 3. The 2D model of a beam section in an air domain

According to (8), when the parameters such as g_a, h, k_a, the temperature T and heat density q_{ls} are clear, the shape factor S will be completely determined.

To determine the value of the shape factor, a FEM simulation method has been proposed in [11], in which the shape factor is examined and established as a formula for the range of air gap from 0.5 to 3 µm. Various formulas of shape factors have also been determined by FEM simulation and used in the heat transfer model of the others [6, 7, 13]. Although these formulas are suitable for the mathematical, structural and material models of the above publications, they have different calculation methods and tend to have different values of the shape factor. Therefore, it is difficult to choose one present formula of the shape factor for the electrothermal micro actuator fabricated by SOI-MEMS technology.

3. The Model of Beam Cross-Section in ANSYS WORKBENCH

The 2D geometric model of a rectangular beam cross-section placed in an air area with radius R_a is designed in ANSYS WORKBENCH as Fig. 3.

To determine the shape factor S, the *Steady-State Thermal* model in ANSYS (steady thermal state simulation) will be used. The properties of the beam material and air are given in Table 1.

In practice, the air domain has no size limitation, but computational or simulation modeling is finite. Therefore, if the radius of the air domain R_a is too small, it does not match the requirements of the problem (only considering heat transfer through the air gap). This means that the value of radius R_a needs to be investigated and selected to suit the requirements of each heat transfer problem.

When the beam width w is fixed as 10 µm with the parameters in Table 1, the influence of the air radius R_a on the temperature of the beam cross-section is simulated and gives the results as shown in Fig. 4.

According to the graph in Fig. 4, the temperature of the beam cross section increases with the radius of the air layer and reaches saturation when the air radius is larger than 180 µm. Thus, the value of the air radius suitable for the simulation model is selected as R_a = 200 µm.

Based on the above geometry, material assignment, meshing and boundary condition setting are performed in the *Steady-State Thermal* model, respectively.

In this case, the boundary conditions proposed as the temperature on the boundary of the air layer assumed to be equal to the ambient temperature T_0; the heat density q_{ls} is assigned on the beam cross-section as shown in Fig. 5.

Table 1. The properties of beam material (silicon) and air

Name	Symbol	Value
The thermal conductivity of silicon	k_s	148×10^6 pW/µm
The thermal conductivity of air	k_a	0.0257×10^6 pW/µm
Ambient temperature	T_0	25°C
Heat density	q_{ls}	2.5×10^5 pW/µm^2

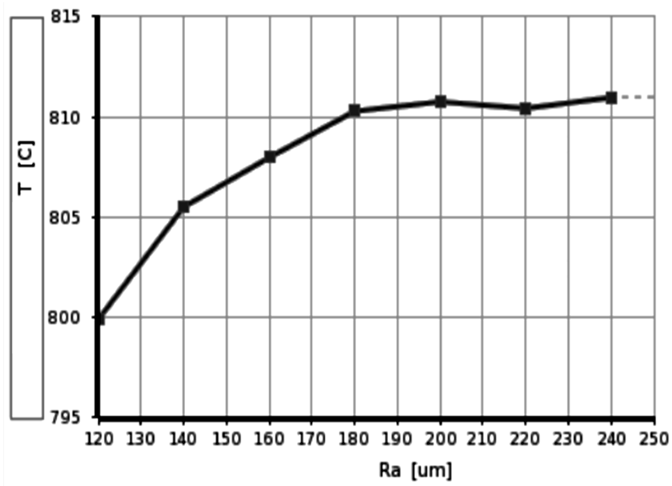

Fig. 4. The relation between R_a and the temperature of a beam element.

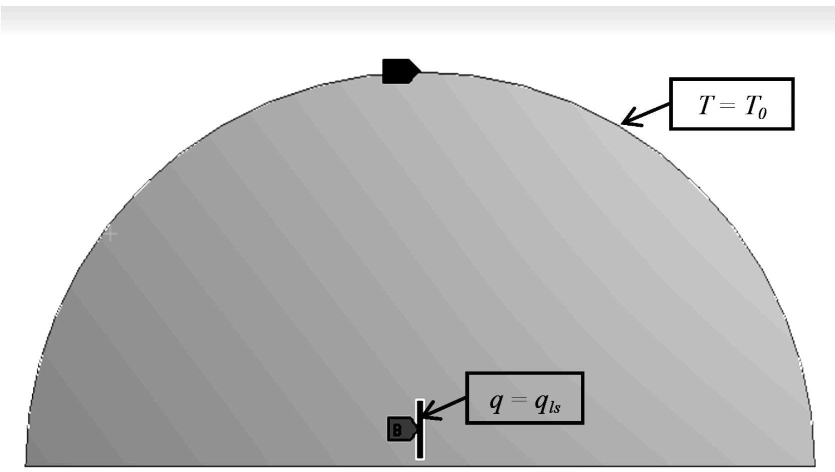

Fig. 5. Boundary conditions of the simulation model

4. Results and Discussion

To validate the exactness of the simulation model determining the shape factor in our work, it is compared with the shape factor of previous publication [11] as shown in Fig. 6 (at the same value g_a = 2 µm and h = 20 µm). We can see that the average deviation of the shape factor S between the two researches is not significant (lower than 3% as in Fig. 6b) when the ratio h/w changes from 2 to 10. Thus, the values of the shape factor determined by ANSYS model as mentioned above are acceptable and quite appropriate in the heat transfer problem for the V-shaped beam in the examined dimension range.

According to (8), with the given parameters in Table 1, the shape factor S will be completely found when the temperature of the beam cross-section is determined by simulation. The temperature results of the beam cross-section are determined by simulation when changing the design dimensions such as beam thickness h, beam width w and the air gap g_a as shown in Fig. 7 and Table 2. The relation between temperature and ratio h/w is illustrated in Fig. 8.

According to Fig. 8 and Table 2, the temperature of beam cross-section increases rapidly with the h/w ratio lower than 3. Beyond this value, the beam temperature tends to decrease gradually. The maximum temperature can be obtained in the range of h/w ratio from 3 to 4, so in this range, the ability to transfer heat from the beam to the substrate through the air gap is the lowest (i.e. the temperature of beam is the highest). Therefore, to avoid beam-overheating when designing V-shaped actuator, the ratio h/w should not be chosen between 3 and 4. It is clear that the beam dimension ratio (h/w), as well as the air gap g_a, significantly affects the heat transfer of the beam cross-section to the substrate. In other words, the shape factor S depends strongly on these parameters.

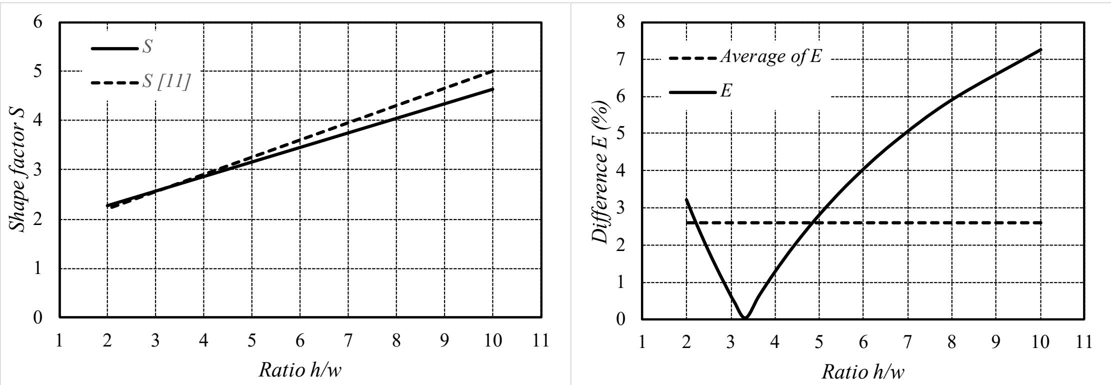

Fig. 6. The comparison of shape factor values (a); The relative deviation E of shape factors (b)

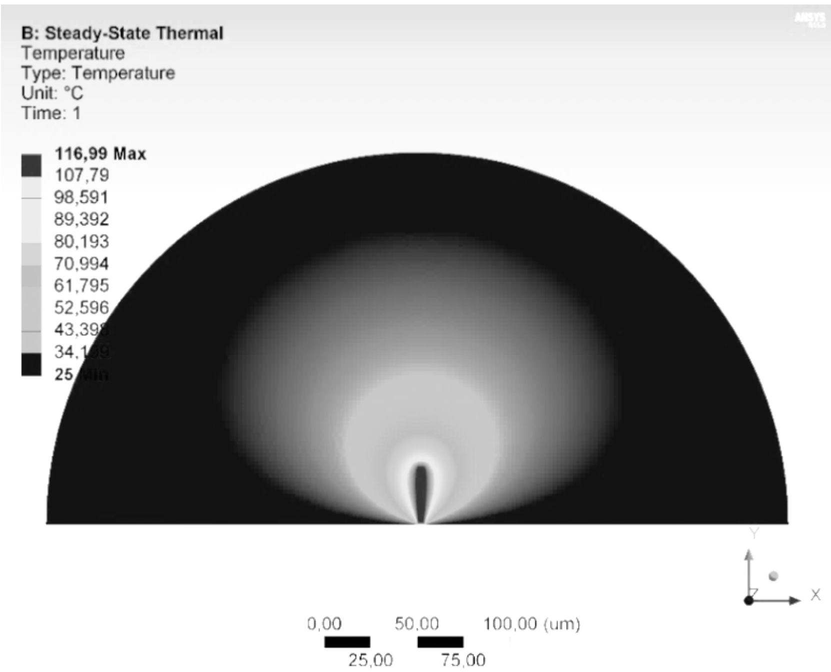

Fig. 7. Temperature distribution around the beam cross-section.

Table 2. The relation between the beam temperature and its geometric dimensions

w (*µm*)	*h* (*µm*)	*h/w*	*T* (^0C)			
			$g_a = 1\ \mu m$	$g_a = 2\ \mu m$	$g_a = 3\ \mu m$	$g_a = 4\ \mu m$
3	30	10	117	148	169	184
4	27	6.75	126	165	191	210
5	24	4.8	128	173	203	226
6	21	3.5	125	173	206	231
7	18	2.57	119	167	201	227
7,5	15	2	107	150	181	206
8	12	1.5	93	132	159	181
9	9	1	80	112	137	156
10	6	0.6	64	88	107	122

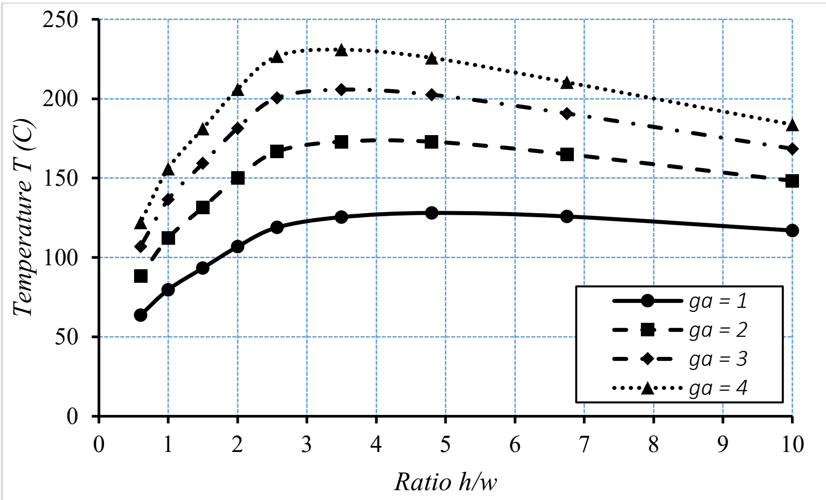

Fig. 8. The relation between temperature and ratio *h/w* at various air gaps

Table 3. The dependent of the shape factor (S) on the ratio (*h/w*)

h/w	*S*			
	$g_a = 1\ \mu m$	$g_a = 2\ \mu m$	$g_a = 3\ \mu m$	$g_a = 4\ \mu m$
10,0	3,11	4,66	6,02	7,25
7,0	2,57	3,72	4,72	5,64
5,0	2,23	3,13	3,92	4,63
3,5	2,00	2,72	3,35	3,92
2,6	1,83	2,43	2,95	3,43
2,0	1,75	2,29	2,76	3,18
1,5	1,67	2,15	2,57	2,95
1,0	1,58	1,98	2,33	2,64
0,5	1,47	1,80	2,08	2,33

From (8) combined with the data in Table 1 and the results in Table 2, the values of the shape factor are completely determined as in Table 3. The relation between the shape factor *S* and *h/w* ratio is given in Fig. 9. As shown in Fig. 9, the shape factor *S* is almost proportional to the height-to-width ratio (*h/w*) at the examined air gaps. For each value of the air gap, there is a linear relationship between the shape factor *S* and the *h/w* ratio. It is also clear that with the larger air gap, we get a larger shape factor value. It shows that the shape factor depends not only on the geometry of the beam cross-section, but also on the gap between the device layer and the substrate. In other words, the heat transfer process depends strongly on the beam shape when the air gap is larger.

In order to demonstrate the influence of the shape factor on the temperature distribution of beam, the temperature distributions of two V-beam structures with the material and geometrical dimension parameters are listed in Tables 1 and 4 and shown in Fig. 10.

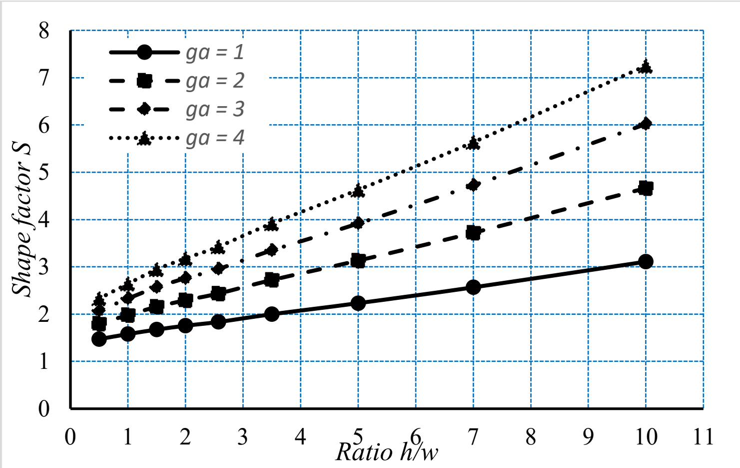

Fig. 9. The relation between the shape factor *S* and ratio *h/w*

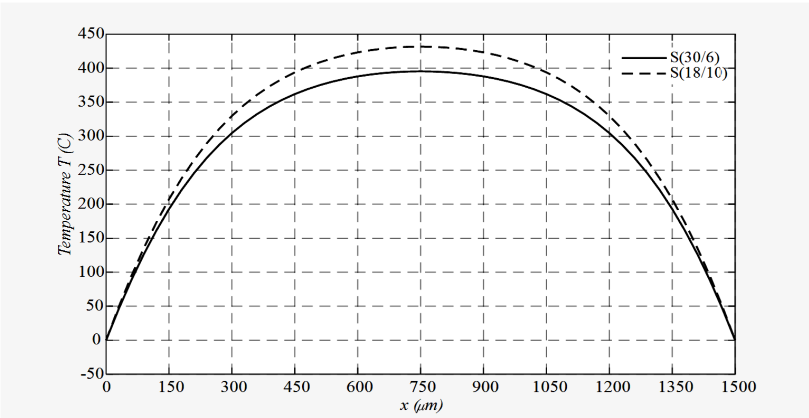

Fig. 10. The distributions of temperature along the V-beam at two values of *S*

Table 4. The parameters of the V-beam

Name	Symbol	Value
Applied voltage	U	20 (V)
Resistivity of silicon	ρ_0	2300 (Ω.μm)
The height of V-beam 1	h_1	30 (μm)
The width of V-beam 1	w_1	6 (μm)
The height of V-beam 2	h_2	18 (μm)
The width of V-beam 2	w_2	10 (μm)
The air gap	g_a	4 (μm)

According to Fig. 10, the shape factor impacts strongly on the distribution of temperature along the V-beam. It is clear that the temperature in the case of the h/w = 18/10 is larger than the temperature at the ratio of 30/6. Furthermore, the simulation results also confirm that: At the same cross-sectional area, the magnitude of the temperature distribution on the V-beam is inversely proportional to the value of the shape factor.

5. Conclusion

This paper has examined a rectangular beam cross-section model of the electrothermal V-shaped micro actuator by ANSYS WORKBENCH. This model was used to investigate the influence of beam cross-sectional dimensions as well as the air gap on heat transfer process from the beam to the substrate via the shape factor *S*. From the simulation and calculation results, some discussions and conclusions can be summarized as follows:

The heat transfer process from the V-beam to substrate through the air gap depends strongly on the ratio of the rectangular cross-section dimensions h/w and the value of air gap g_a.

If the h/w ratio ranges from 3 to 4, the heat transfer from the beam to the substrate is minimal and can lead to a beam-overheating phenomenon while the device operates at a high temperature. Therefore, when designing, this range of h/w ratio should be avoided for the safety of the device.

The shape factor is almost linearly dependent on the ratio (h/w) of the beam cross-section. The value of this coefficient is proportional to the height-to-width ratio as well as the air gap between the V-beam and the substrate.

In the future, the influence of the number of beam pairs and the distance between them on the heat transfer process through the air gap will be continuously considered and examined.

References

[1] P. Shivhare, G. Uma, M. Umapathy, Design enhancement of a chevron electrothermally actuated microgripper for improved gripping performance, Microsyst. Technol., vol. 22, no. 11, pp. 2623–2631, 2016. https://doi.org/10.1007/s00542-015-2561-0

[2] H. D. Espinosa, Y. Zhu, and N. Moldovan, Design and operation of a MEMS-based material testing system for nanomechanical characterization, Journal of Microelectromechanical Systems, vol. 16, no. 5. pp. 1219–1231, 2007. https://doi.org/10.1109/JMEMS.2007.905739

[3] T. Hu, Y. Zhao, X. Li, Y. Zhao, and Y. Bai, Integration design of MEMS electro-thermal safety-and-arming devices, Microsyst. Technol., vol. 23, no. 4, pp. 953–958, 2017. https://doi.org/10.1007/s00542-016-2901-8

[4] T. Hu, Y. Zhao, X. Li, Y. Zhao, and Y. Bai, Design and fabrication of an electro-thermal linear motor with large output force and displacement, in Proceedings of IEEE Sensors, 2016, pp. 1–3. https://doi.org/10.1109/ICSENS.2016.7808548

[5] Z. Zhang, W. Zhang, Q. Wu, Y. Yu, X. Liu, and X. Zhang, Closed-form modelling and design analysis of V- and Z-shaped electrothermal microactuators, J. Micromechanics and Microengineering, vol. 27, no. 1, Jan. 2017. https://doi.org/10.1088/1361-6439/27/1/015023

[6] Z. Zhang, Y. Yu, X. Liu, and X. Zhang, Dynamic modelling and analysis of V- and Z-shaped electrothermal microactuators, Microsyst. Technol., vol. 23, no. 8, pp. 3775–3789, 2017. https://doi.org/10.1007/s00542-016-3180-0

[7] T. Shan, X. Qi, L. Cui, and X. Zhou, Thermal behavior modeling and characteristics analysis of electrothermal microactuators, Microsyst. Technol., vol. 23, no. 7, pp. 2629–2640, 2017. https://doi.org/10.1007/s00542-016-3070-5

[8] C. D. Lott, T. W. McLain, J. N. Harb, and L. L. Howell, Modeling the thermal behavior of a surface-micromachined linear-displacement thermomechanical microactuator, Sensors Actuators, A Phys., vol. 101, no. 1–2, pp. 239–250, 2002. https://doi.org/10.1016/S0924-4247(02)00202-9

[9] R. G. Li, Q. A. Huang, and W. H. Li, A nodal analysis method for simulating the behavior of electrothermal microactuators, Microsyst. Technol., vol. 14, no. 1, pp. 119–129, 2008. https://doi.org/10.1007/s00542-007-0406-1

[10] B. López-Walle, M. Gauthier, and N. Chaillet, Dynamic modelling for thermal micro-actuators using thermal networks, Int. J. Therm. Sci., vol. 49, no. 11, pp. 2108–2116, 2010. https://doi.org/10.1016/j.ijthermalsci.2010.06.012

[11] J. M. Maloney, D. L. Devoe, and D. S. Schreiber, Analysis and Design of Electrothermal Actuators Fabricated from Single Crystal Silicon, Micro-Electro-Mechanical-Systems - ASME 2000, vol. Vol. 2, pp. 233–240, 2000. https://doi.org/10.1115/IMECE2000-1097

[12] Y.-L. Zhao, T.-J. Hu, X.-Y. Li, Z.-D. Jiang, W. Ren, and Y.-W. Bai, Design and Characterization of a Large Displacement Electro-thermal Actuator for a New Kind of Safety-and-Arming Device, Energy Harvest. Syst., vol. 2, no. 3–4, pp. 143–148, 2015. https://doi.org/10.1515/ehs-2014-0037

[13] S. C. Chen and M. L. Culpepper, Design of contoured microscale thermomechanical actuators, Journal of Microelectromechanical Systems, vol. 15, no. 5. pp. 1226–1234, 2006. https://doi.org/10.1109/JMEMS.2006.879692