MAA AMC
American Mathematics Competitions

American Mathematics Competitions

19th Annual

AMC 10A

American Mathematics Competition 10A
Wednesday, February 7, 2018

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR COMPETITION MANAGER TELLS YOU.
2. This is a 25-question multiple-choice exam. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Sheet with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer sheet will be graded. **You must use and submit the original answer sheets provided by the MAA AMC. Photocopies will not be scored.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only scratch paper, graph paper, rulers, compasses, protractors, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the exam will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the exam, your competition manager will ask you to record certain information on the answer form.
8. When your competition manager gives the signal, begin working on the problems. You will have **75 minutes** to complete the exam.
9. When you finish the exam, *sign your name* in the space provided at the top of the Answer Sheet.

The Committee on the American Mathematics Competitions reserves the right to disqualify scores from a school if it determines that the required security procedures were not followed.

Students who score well on this AMC 10 will be invited to take the 36th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 6, 2018 or Wednesday, March 21, 2018. More details about the AIME are on the back page of this test booklet.

1. What is the value of
$$\left(\left((2+1)^{-1}+1\right)^{-1}+1\right)^{-1}+1\,?$$

(A) $\frac{5}{8}$ **(B)** $\frac{11}{7}$ **(C)** $\frac{8}{5}$ **(D)** $\frac{18}{11}$ **(E)** $\frac{15}{8}$

2. Liliane has 50% more soda than Jacqueline, and Alice has 25% more soda than Jacqueline. What is the relationship between the amounts of soda that Liliane and Alice have?

(A) Liliane has 20% more soda than Alice.

(B) Liliane has 25% more soda than Alice.

(C) Liliane has 45% more soda than Alice.

(D) Liliane has 75% more soda than Alice.

(E) Liliane has 100% more soda than Alice.

3. A unit of blood expires after $10! = 10\cdot 9\cdot 8\cdots 1$ seconds. Yasin donates a unit of blood at noon on January 1. On what day does his unit of blood expire?

(A) January 2 **(B)** January 12 **(C)** January 22

(D) February 11 **(E)** February 12

4. How many ways can a student schedule 3 mathematics courses—algebra, geometry, and number theory—in a 6-period day if no two mathematics courses can be taken in consecutive periods? (What courses the student takes during the other 3 periods is of no concern here.)

(A) 3 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** 24

5. Alice, Bob, and Charlie were on a hike and were wondering how far away the nearest town was. When Alice said, “We are at least 6 miles away,” Bob replied, “We are at most 5 miles away.” Charlie then remarked, “Actually the nearest town is at most 4 miles away.” It turned out that none of the three statements was true. Let d be the distance in miles to the nearest town. Which of the following intervals is the set of all possible values of d?

(A) $(0,4)$ **(B)** $(4,5)$ **(C)** $(4,6)$ **(D)** $(5,6)$ **(E)** $(5,\infty)$

6. Sangho uploaded a video to a website where viewers can vote that they like or dislike a video. Each video begins with a score of 0, and the score increases by 1 for each like vote and decreases by 1 for each dislike vote. At one point Sangho saw that his video had a score of 90, and that 65% of the votes cast on his video were like votes. How many votes had been cast on Sangho's video at that point?

   (A) 200 **(B)** 300 **(C)** 400 **(D)** 500 **(E)** 600

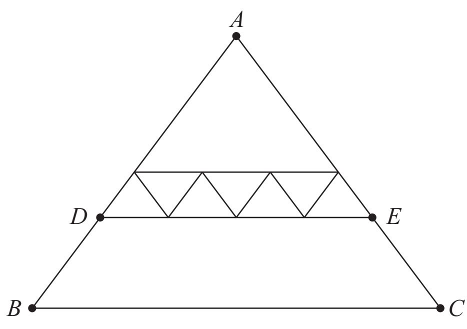
7. For how many (not necessarily positive) integer values of n is the value of $4000 \cdot \left(\frac{2}{5}\right)^n$ an integer?

   (A) 3 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 9

8. Joe has a collection of 23 coins, consisting of 5-cent coins, 10-cent coins, and 25-cent coins. He has 3 more 10-cent coins than 5-cent coins, and the total value of his collection is 320 cents. How many more 25-cent coins does Joe have than 5-cent coins?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

9. All of the triangles in the diagram below are similar to isosceles triangle ABC, in which $AB = AC$. Each of the 7 smallest triangles has area 1, and $\triangle ABC$ has area 40. What is the area of trapezoid $DBCE$?



   (A) 16 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

10. Suppose that real number x satisfies

$$\sqrt{49 - x^2} - \sqrt{25 - x^2} = 3.$$

   What is the value of $\sqrt{49 - x^2} + \sqrt{25 - x^2}$?

   (A) 8 **(B)** $\sqrt{33} + 3$ **(C)** 9 **(D)** $2\sqrt{10} + 4$ **(E)** 12

11. When 7 fair standard 6-sided dice are thrown, the probability that the sum of the numbers on the top faces is 10 can be written as

$$\frac{n}{6^7},$$

where n is a positive integer. What is n?

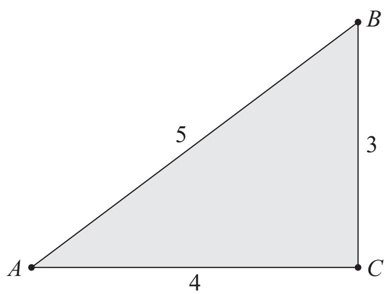
(A) 42 **(B)** 49 **(C)** 56 **(D)** 63 **(E)** 84

12. How many ordered pairs of real numbers (x, y) satisfy the following system of equations?

$$x + 3y = 3$$
$$\big|\,|x| - |y|\,\big| = 1$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 8

13. A paper triangle with sides of lengths 3, 4, and 5 inches, as shown, is folded so that point A falls on point B. What is the length in inches of the crease?



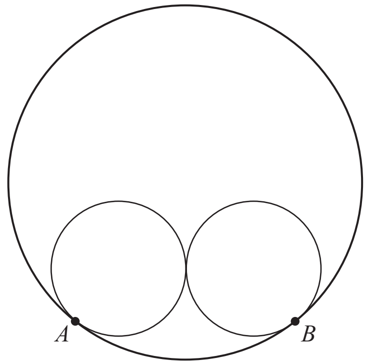
(A) $1 + \frac{1}{2}\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** $\frac{7}{4}$ **(D)** $\frac{15}{8}$ **(E)** 2

14. What is the greatest integer less than or equal to

$$\frac{3^{100} + 2^{100}}{3^{96} + 2^{96}}?$$

(A) 80 **(B)** 81 **(C)** 96 **(D)** 97 **(E)** 625

15. Two circles of radius 5 are externally tangent to each other and are internally tangent to a circle of radius 13 at points A and B, as shown in the diagram. The distance AB can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?



(A) 21 **(B)** 29 **(C)** 58 **(D)** 69 **(E)** 93

16. Right triangle ABC has leg lengths $AB = 20$ and $BC = 21$. Including $\overline{AB}$ and $\overline{BC}$, how many line segments with integer length can be drawn from vertex B to a point on hypotenuse $\overline{AC}$?

(A) 5 **(B)** 8 **(C)** 12 **(D)** 13 **(E)** 15

17. Let S be a set of 6 integers taken from $\{1, 2, \ldots, 12\}$ with the property that if a and b are elements of S with $a < b$, then b is not a multiple of a. What is the least possible value of an element of S?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 7

18. How many nonnegative integers can be written in the form

$$a_7 \cdot 3^7 + a_6 \cdot 3^6 + a_5 \cdot 3^5 + a_4 \cdot 3^4 + a_3 \cdot 3^3 + a_2 \cdot 3^2 + a_1 \cdot 3^1 + a_0 \cdot 3^0,$$

where $a_i \in \{-1, 0, 1\}$ for $0 \le i \le 7$?

(A) 512 **(B)** 729 **(C)** 1094 **(D)** 3281 **(E)** 59,048

19. A number m is randomly selected from the set $\{11, 13, 15, 17, 19\}$, and a number n is randomly selected from $\{1999, 2000, 2001, \ldots, 2018\}$. What is the probability that m^n has a units digit of 1?

(A) $\frac{1}{5}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{3}{10}$ **(D)** $\frac{7}{20}$ **(E)** $\frac{2}{5}$

20. A scanning code consists of a 7×7 grid of squares, with some of its squares colored black and the rest colored white. There must be at least one square of each color in this grid of 49 squares. A scanning code is called *symmetric* if its look does not change when the entire square is rotated by a multiple of 90° counterclockwise around its center, nor when it is reflected across a line joining opposite corners or a line joining midpoints of opposite sides. What is the total number of possible symmetric scanning codes?

(A) 510 **(B)** 1022 **(C)** 8190 **(D)** 8192 **(E)** 65,534

21. Which of the following describes the set of values of a for which the curves $x^2 + y^2 = a^2$ and $y = x^2 - a$ in the real xy-plane intersect at exactly 3 points?

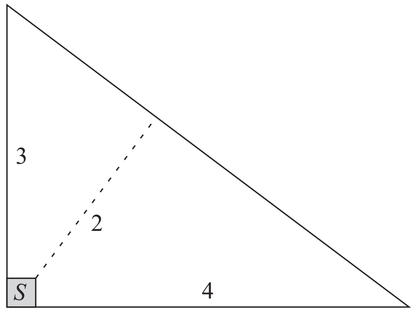
(A) $a = \frac{1}{4}$ **(B)** $\frac{1}{4} < a < \frac{1}{2}$ **(C)** $a > \frac{1}{4}$ **(D)** $a = \frac{1}{2}$

(E) $a > \frac{1}{2}$

22. Let a, b, c, and d be positive integers such that $\gcd(a, b) = 24$, $\gcd(b, c) = 36$, $\gcd(c, d) = 54$, and $70 < \gcd(d, a) < 100$. Which of the following must be a divisor of a?

(A) 5 **(B)** 7 **(C)** 11 **(D)** 13 **(E)** 17

23. Farmer Pythagoras has a field in the shape of a right triangle. The right triangle's legs have lengths of 3 and 4 units. In the corner where those sides meet at a right angle, he leaves a small unplanted square S so that from the air it looks like the right angle symbol. The rest of the field is planted. The shortest distance from S to the hypotenuse is 2 units. What fraction of the field is planted?



(A) $\frac{25}{27}$ **(B)** $\frac{26}{27}$ **(C)** $\frac{73}{75}$ **(D)** $\frac{145}{147}$ **(E)** $\frac{74}{75}$

24. Triangle ABC with $AB = 50$ and $AC = 10$ has area 120. Let D be the midpoint of $\overline{AB}$, and let E be the midpoint of $\overline{AC}$. The angle bisector of $\angle BAC$ intersects $\overline{DE}$ and $\overline{BC}$ at F and G, respectively. What is the area of quadrilateral $FDBG$?

(A) 60 **(B)** 65 **(C)** 70 **(D)** 75 **(E)** 80

25. For a positive integer n and nonzero digits a, b, and c, let A_n be the n-digit integer each of whose digits is equal to a; let B_n be the n-digit integer each of whose digits is equal to b; and let C_n be the $2n$-digit (not n-digit) integer each of whose digits is equal to c. What is the greatest possible value of $a + b + c$ for which there are at least two values of n such that $C_n - B_n = A_n^2$?

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

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Questions and comments about problems
and solutions for this exam should be sent to:

amchq@maa.org

Send questions and comments about administrative arrangements to:

amcinfo@maa.org

or

MAA American Mathematics Competitions
P.O. Box 471
Annapolis Junction, MD 20701

The problems and solutions for this AMC 10 were prepared by MAA's Subcommittee on the AMC10/AMC12 Exams, under the direction of the co-chairs Jerrold W. Grossman and Carl Yerger.

2018 AIME

The 36th annual AIME will be held on Tuesday, March 6, 2018 with the alternate on Wednesday, March 21, 2018. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate if you achieve a high score on this competition. Top-scoring students on the AMC 10/12/AIME will be selected to take the 47th Annual USA Mathematical Olympiad (USAMO) on April 18–19, 2018.

2018

AMC 10A

DO NOT OPEN UNTIL WEDNESDAY, February 7, 2018

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All the information needed to administer this exam is contained in the AMC 10/12 Teacher's Manual. PLEASE READ THE MANUAL BEFORE FEBRUARY 7, 2018.
2. Your PRINCIPAL or VICE PRINCIPAL must verify on the AMC 10/12 COMPETITION CERTIFICATION FORM (found on maa.org/amc under 'AMC 10/12') that you followed all rules associated with the administration of the exam.
3. Answer sheets must be returned to the MAA AMC office the day after the competition. Ship with appropriate postage using a trackable method. FedEx or UPS is strongly recommended.
4. The publication, reproduction, or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

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