MATHEMATICAL ASSOCIATION OF AMERICA

# Solutions Pamphlet

MAA American Mathematics Competitions

34th Annual

# AMC 8

American Mathematics Contest 8

**Tuesday, November 13, 2018**

This Solutions Pamphlet gives at least one solution for each problem on this year's exam and shows that all the problems can be solved using material normally associated with the mathematics curriculum for students in eighth grade or below. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

We hope that teachers will share these solutions with their students. However, the publication, reproduction, or communication of the problems or solutions of the AMC 8 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. *Dissemination beyond the classroom at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

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1. An amusement park has a collection of scale models, with ratio 1:20, of buildings and other sights from around the country. The height of the United States Capitol is 289 feet. What is the height in feet of its replica at this park, rounded to the nearest whole number?

**(A)** 14 **(B)** 15 **(C)** 16 **(D)** 18 **(E)** 20

**Answer (A):** The height of the replica is $\frac{289}{20} = 14.45$ feet, so the answer is 14 feet when rounded to the nearest whole number.

2. What is the value of the product

$$\left(1+\frac{1}{1}\right)\cdot\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{3}\right)\cdot\left(1+\frac{1}{4}\right)\cdot\left(1+\frac{1}{5}\right)\cdot\left(1+\frac{1}{6}\right)?$$

**(A)** $\frac{7}{6}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{7}{2}$ **(D)** 7 **(E)** 8

**Answer (D):** The product may be written as

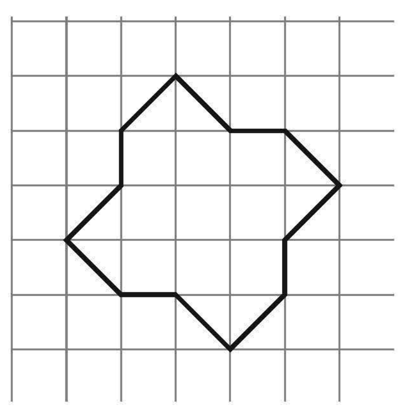
$$2\cdot\frac{3}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}\cdot\frac{6}{5}\cdot\frac{7}{6} = 7.$$

3. Students Arn, Bob, Cyd, Dan, Eve, and Fon are arranged in that order in a circle. They start counting: Arn first, then Bob, and so forth. When the number contains a 7 as a digit (such as 47) or is a multiple of 7 that person leaves the circle and the counting continues. Who is the last one present in the circle?

**(A)** Arn **(B)** Bob **(C)** Cyd **(D)** Dan **(E)** Eve

**Answer (D):** Dan is the last one present because Arn is out on 7, Cyd on 14, Fon on 17, Bob on 21 and Eve on 27.

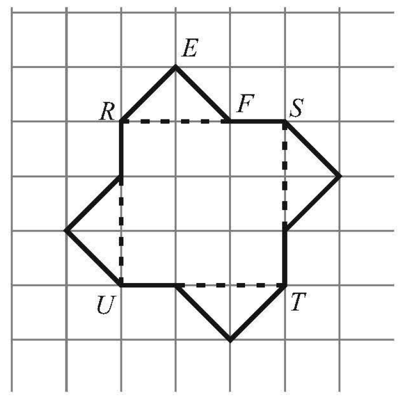
4. The twelve-sided figure shown has been drawn on 1 cm $\times$ 1 cm graph paper. What is the area of the figure in cm$^2$?



**(A)** 12 **(B)** 12.5 **(C)** 13 **(D)** 13.5 **(E)** 14

**Answer (C):**

In the figure below, the square $RSTU$ has area $3 \cdot 3 = 9$, and $\triangle REF$ has area $\frac{1}{2} \cdot 2 \cdot 1 = 1$. The other three triangles are congruent to $\triangle REF$, so the total area is $9 + 4 \cdot 1 = 13$.




5. What is the value of $1 + 3 + 5 + \cdots + 2017 + 2019 - 2 - 4 - 6 - \cdots - 2016 - 2018$?

**(A)** $-1010$ **(B)** $-1009$ **(C)** $1008$ **(D)** $1009$ **(E)** $1010$

**Answer (E):** The expression can be written as

$$1 + (3 - 2) + (5 - 4) + (7 - 6) + \cdots + (2019 - 2018) = 1 + \frac{2018}{2} = 1010.$$

6. On a trip to the beach, Anh traveled 50 miles on the highway and 10 miles on a coastal access road. He drove three times as fast on the highway as on the coastal road. If Anh spent 30 minutes driving on the coastal road, how many minutes did his entire trip take?

**(A)** 50 **(B)** 70 **(C)** 80 **(D)** 90 **(E)** 100

**Answer (C):** Anh's speed on the coastal road was 20 miles per hour because he drove 10 miles in half an hour. This implies that Anh traveled 60 miles per hour on the highway, and so it took Anh 50 minutes to drive the 50 miles. Anh's entire trip took $30 + 50 = 80$ minutes.

OR

Because Anh drives three times as fast on the highway, he needs only $\frac{30}{3} = 10$ minutes to drive 10 miles on the highway and five times 10 minutes to drive 50 miles on the highway. Thus his total time is $30 + 5(10) = 80$ minutes.

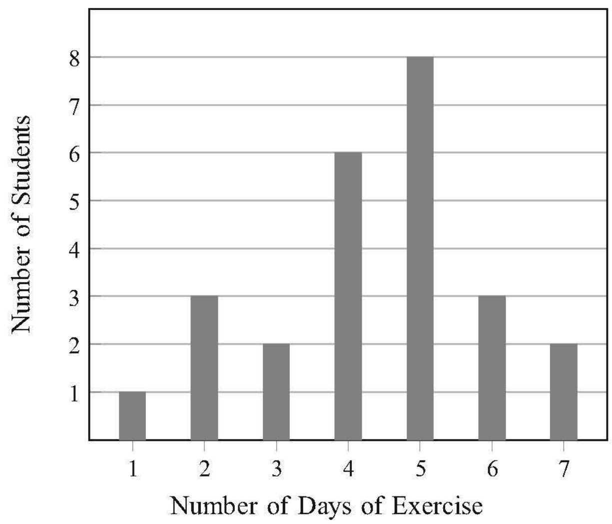
7. The 5-digit number $\underline{2}\ \underline{0}\ \underline{1}\ \underline{8}\ \underline{U}$ is divisible by 9. What is the remainder when this number is divided by 8?

   **(A)** 1 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 7

   **Answer (B):**

   To be divisible by 9, the sum of the digits must be divisible by 9, so $2+0+1+8+U = 11+U$ is divisible by 9. Thus $U = 7$, and the 5-digit number is 20187. Then because $20187 = 8\cdot 2523+3$, the remainder is 3.

8. Mr. Garcia asked the members of his health class how many days last week they exercised for at least 30 minutes. The results are summarized in the following bar graph, where the heights of the bars represent the number of students.




   What was the mean number of days of exercise last week, rounded to the nearest hundredth, reported by the students in Mr. Garcia's class?

   **(A)** 3.50 **(B)** 3.57 **(C)** 4.36 **(D)** 4.50 **(E)** 5.00

   **Answer (C):** The number of students surveyed was $1+3+2+6+8+3+2 = 25$, and the number of days of exercise reported was $1\cdot 1+3\cdot 2+2\cdot 3+6\cdot 4+8\cdot 5+3\cdot 6+2\cdot 7 = 109$. So the mean number of days of exercise was $109 \div 25 = 4.36$.

9. Tyler is tiling the floor of his 12 foot by 16 foot living room. He plans to place one-foot by one-foot square tiles to form a border along the edges of the room and to fill in the rest of the floor with two-foot by two-foot square tiles. How many tiles will he use?

   **(A)** 48 **(B)** 87 **(C)** 91 **(D)** 96 **(E)** 120

   **Answer (B):** The perimeter of the room is $2(12+16) = 56$ feet. The 4 corner tiles each occupy 2 feet of the perimeter, so Tyler needs those 4 plus another $56 - 4\cdot 2 = 48$ one-foot square tiles to make the border. The remaining space in the room is a 10 foot by 14 foot rectangle, so Tyler needs $\frac{10}{2} \cdot \frac{14}{2} = 35$ two-foot square tiles to cover it. Therefore he needs a total of $4 + 48 + 35 = 87$ tiles.

OR

If Tyler used only one-foot square tiles, he would need a total of $12 \cdot 16 = 192$ tiles. As above, the room minus the border is a 10 foot by 14 foot rectangle to be covered with two-foot square tiles, each of which has the same area as 4 one-foot square tiles. Therefore the number of tiles needed to cover that portion of the room is not $10 \cdot 14 = 140$, but $\frac{140}{4} = 35$, and the total number of tiles needed is $(192 - 140) + 35 = 87$.

10. The *harmonic mean* of a set of non-zero numbers is the reciprocal of the average of the reciprocals of the numbers. What is the harmonic mean of 1, 2, and 4?

**(A)** $\dfrac{3}{7}$ **(B)** $\dfrac{7}{12}$ **(C)** $\dfrac{12}{7}$ **(D)** $\dfrac{7}{4}$ **(E)** $\dfrac{7}{3}$

**Answer (C):** The reciprocals of 1, 2, and 4 are 1, $\frac{1}{2}$, and $\frac{1}{4}$, and their average is $\frac{1+\frac{1}{2}+\frac{1}{4}}{3} = \frac{7}{12}$. So the harmonic mean of 1, 2, and 4 is the reciprocal of $\frac{7}{12}$, which is $\frac{12}{7}$.

11. Abby, Bridget, and four of their classmates will be seated in two rows of three for a group picture, as shown.

$$\begin{array}{ccc} X & X & X \\ X & X & X \end{array}$$

If the seating positions are assigned randomly, what is the probability that Abby and Bridget are adjacent to each other in the same row or the same column?

**(A)** $\dfrac{1}{3}$ **(B)** $\dfrac{2}{5}$ **(C)** $\dfrac{7}{15}$ **(D)** $\dfrac{1}{2}$ **(E)** $\dfrac{2}{3}$

**Answer (C):** There are 6 possible positions for Abby, and this leaves 5 possible positions for Bridget. Because their order doesn't matter, the two girls can be placed in any of $\frac{6 \cdot 5}{2} = 15$ pairs of positions. There are 2 pairs of positions that are adjacent in the top row, 2 pairs that are adjacent in the bottom row, and 3 pairs that are adjacent in the same column. So the probability that they occupy adjacent positions is $\frac{2+2+3}{15} = \frac{7}{15}$.

OR

There is a $\frac{2}{3}$ chance Abby is assigned to a corner position and a $\frac{1}{3}$ chance that Abby is assigned to a middle position. If Abby is assigned to a corner position, there is a $\frac{2}{5}$ chance that Bridget is adjacent to Abby. If Abby is assigned to a middle position, there is a $\frac{3}{5}$ chance that Bridget is adjacent to Abby. Thus, the probability that Abby and Bridget are adjacent to each other in the same row or column is $\frac{2}{3} \cdot \frac{2}{5} + \frac{1}{3} \cdot \frac{3}{5} = \frac{7}{15}$.

12. The clock in Sri's car, which is not accurate, gains time at a constant rate. One day as he begins shopping he notes that his car clock and his watch (which is accurate) both say 12:00 noon. When he is done shopping, his watch says 12:30 and his car clock says 12:35. Later that day, Sri loses his watch. He looks at his car clock and it says 7:00. What is the actual time?

**(A)** 5:50 **(B)** 6:00 **(C)** 6:30 **(D)** 6:55 **(E)** 8:10

**Answer (B):** The ratio of time as measured by Sri's car clock to actual time is $\frac{35}{30} = \frac{7}{6}$. Therefore when the clock shows that 7 hours have passed since noon, only 6 hours have actually passed, so the actual time is 6:00.

OR

Gaining 5 minutes every half hour is equivalent to gaining 1 hour every 6 hours. Hence when the car clock reads 7:00 the actual time is 6:00.

13. Laila took five math tests, each worth a maximum of 100 points. Laila's score on each test was an integer between 0 and 100, inclusive. Laila received the same score on the first four tests, and she received a higher score on the last test. Her average score on the five tests was 82. How many values are possible for Laila's score on the last test?

**(A)** 4 **(B)** 5 **(C)** 9 **(D)** 10 **(E)** 18

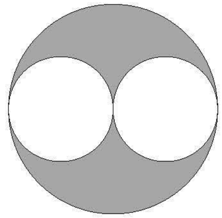
**Answer (A):** Because Laila's score on the last test was higher than her other test scores and all of her scores were integers, for each point below 82 that she scored on each of the first four tests, she had to score four points above 82 on the last test. Therefore, her possible scores on the last test were 86, 90, 94, and 98 (with scores on the first four tests of 81, 80, 79, and 78, respectively). Thus there are four possible values for her score on the last test.

14. Let $N$ be the greatest five-digit number whose digits have a product of 120. What is the sum of the digits of $N$?

**(A)** 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 20

**Answer (D):** The leftmost digit of $N$ must be the greatest single-digit factor of $120 = 2^3 \cdot 3 \cdot 5$. Note that $2^3 = 8$ is a factor of 120, but 9 is not, hence the leftmost digit of $N$ must be 8. Because $5 \cdot 3 = 15$ cannot be a digit, it follows that 5 must be immediately to the right of the 8, and 3 must be immediately to the right of the 5. The remaining rightmost two digits of $N$ must be 1, so that $N = 85311$. Thus the sum of the digits of $N$ is 18.

15. In the diagram below, a diameter of each of the two smaller circles is a radius of the larger circle. If the two smaller circles have a combined area of 1 square unit, then what is the area of the shaded region, in square units?



**(A)** $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** 1 **(E)** $\frac{\pi}{2}$

**Answer (D):** If one of the smaller circles has radius $r$, then its area is $\pi r^2$, and the area of the larger circle is $\pi(2r)^2 = 4\pi r^2$. Thus the area of the large circle is 4 times the area of one of the small circles. So the area of the shaded region is equal to the combined area of the two smaller circles, which is 1 square unit.

OR

The ratio of the areas of two similar figures is the square of the ratio of their corresponding lengths. Because the diameter of the larger circle is twice the diameter of each smaller circle, it follows that the area of the larger circle is four times the area of each smaller circle. Therefore the shaded area is equal to the combined area of the two smaller circles.

16. Professor Chang has nine different language books lined up on a bookshelf: two Arabic, three German, and four Spanish. How many ways are there to arrange the nine books on the shelf keeping the Arabic books together and keeping the Spanish books together?

**(A)** 1440 **(B)** 2880 **(C)** 5760 **(D)** 182,440 **(E)** 362,880

**Answer (C):** There are two ways to arrange the two Arabic books among themselves and there are $4 \cdot 3 \cdot 2 \cdot 1 = 24$ ways to arrange the four Spanish books among themselves. If the two Arabic books are considered as a unit and the four Spanish books are considered as a unit, then, including the three German books, there are five objects to arrange on the shelf. These five objects can be arranged on the shelf in $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ ways. So altogether the books can be arranged in $2 \cdot 24 \cdot 120 = 5760$ ways.

17. Bella begins to walk from her house toward her friend Ella's house. At the same time, Ella begins to ride her bicycle toward Bella's house. They each maintain a constant speed, and Ella rides 5 times as fast as Bella walks. The distance between their houses is 2 miles, which is 10,560 feet, and Bella covers $2\frac{1}{2}$ feet with each step. How many steps will Bella take by the time she meets Ella?

**(A)** 704 **(B)** 845 **(C)** 1056 **(D)** 1760 **(E)** 3520

**Answer (A):** By the time that the two girls meet, Ella will ride 5 times the distance that Bella walks, so Ella will ride $\frac{5}{6}$ of the total distance, and Bella will walk $\frac{1}{6}$ of the total distance, or $\frac{1}{6}(10,560) = 1760$ feet. Each of her steps covers $2\frac{1}{2} = \frac{5}{2}$ feet, so the number of steps she will take is $1760 \div \frac{5}{2} = 1760 \cdot \frac{2}{5} = 704$.

OR

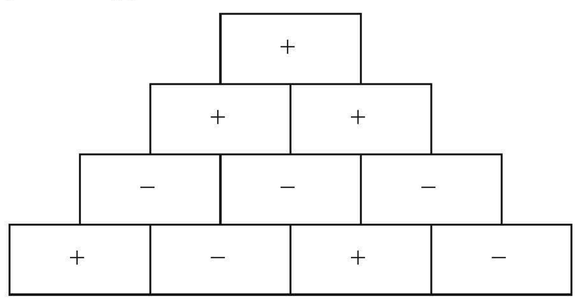
Bella's rate is 2.5 feet per step whereas Ella travels 12.5 feet for each of Bella's steps. So together they travel 15 feet for each of Bella's steps. Hence the number of steps taken by Bella when she meets Ella is $\frac{10{,}560}{15} = 704$.

18. How many positive factors does 23,232 have?

**(A)** 9 **(B)** 12 **(C)** 28 **(D)** 36 **(E)** 42

**Answer (E):** The prime factorization of 23,232 is $2^6 \cdot 3 \cdot 11^2$. Each factor of 23,232 must be of the form $2^a \cdot 3^b \cdot 11^c$, where $a = 0, 1, 2, 3, 4, 5,$ or $6$, $b = 0$ or $1$, and $c = 0, 1,$ or $2$. Therefore, the number of factors of 23,232 is $7 \cdot 2 \cdot 3 = 42$.

19. In a sign pyramid a cell gets a "+" if the two cells below it have the same sign, and it gets a "–" if the two cells below it have different signs. The diagram below illustrates a sign pyramid with four levels. How many possible ways are there to fill the four cells in the bottom row to produce a "+" at the top of the pyramid?




**(A)** 2 **(B)** 4 **(C)** 8 **(D)** 12 **(E)** 16

**Answer (C):** Think of the + sign as +1, and the − sign as −1. Let $a$, $b$, $c$, and $d$ denote the values of the four cells at the bottom of the pyramid, in that order. Then the cells in the second row from the bottom have values $a \cdot b$, $b \cdot c$ and $c \cdot d$, and the cells in the row above this are $a \cdot b \cdot b \cdot c = a \cdot c$ and $b \cdot c \cdot c \cdot d = b \cdot d$ (because both 1 and −1 squared are 1.) Finally, the top cell has value $a \cdot b \cdot c \cdot d$. This value is +1 if all four variables are +1 or all four are −1, giving two ways; or, if two of the variables are +1 and two are −1, giving 6 additional ways (++--, +-+-, +--+, -++-, -+-+, and --++). Thus there are a total of 8 ways to fill the fourth row.

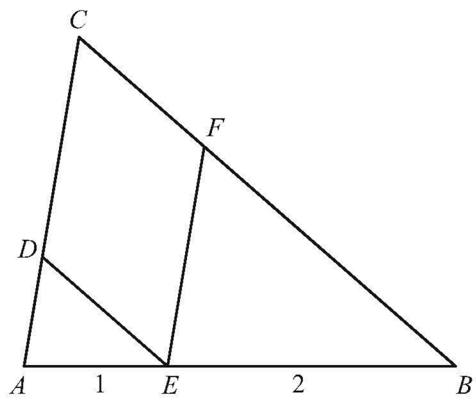
OR

In order to produce a + at the top of the pyramid, the second row must contain either ++ or −−. Each leads to two possible arrangements for the third row. Consider the following cases.

| Second row | Third row | Fourth row |
|---|---|---|
| $++$ | $+\ +\ +$ | $+\ +\ ++$ or $-\ -\ --$ |
| $++$ | $-\ -\ -$ | $+\ -\ +-$ or $-\ +\ -+$ |
| $--$ | $+\ -\ +$ | $+\ +\ --$ or $-\ -\ ++$ |
| $--$ | $-\ +\ -$ | $+\ -\ -+$ or $-\ +\ +-$ |

Thus, there are eight possible ways to fill the fourth row.

20. In $\Delta ABC$, point $E$ is on $\overline{AB}$ with $AE = 1$ and $EB = 2$. Point $D$ is on $\overline{AC}$ so that $\overline{DE} \parallel \overline{BC}$ and point $F$ is on $\overline{BC}$ so that $\overline{EF} \parallel \overline{AC}$. What is the ratio of the area of $CDEF$ to the area of $\Delta ABC$?

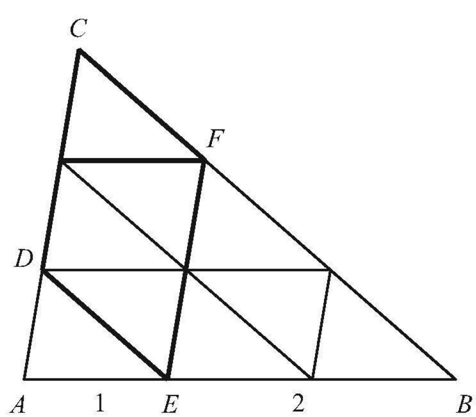



**(A)** $\frac{4}{9}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{9}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{2}{3}$

**Answer (A):** Triangles $AED$, $EBF$, and $ABC$ are similar with their sides in the ratio of 1:2:3. Therefore their areas are in the ratio of 1:4:9. The combined areas of $\Delta AED$ and $\Delta EBF$ constitute $\frac{5}{9}$ of the area of $\Delta ABC$, so the area of $CDEF$ is $\frac{4}{9}$ of the area of $\Delta ABC$.

OR

Connecting corresponding points of trisection of the three sides of the triangle, as shown in the figure below, results in nine congruent triangles, four of which constitute $CDEF$.

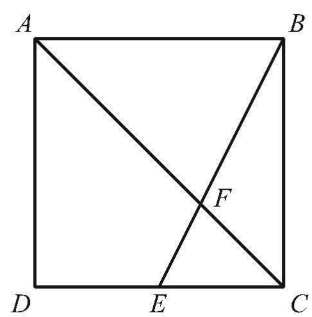


21. How many positive three-digit integers have a remainder of 2 when divided by 6, a remainder of 5 when divided by 9, and a remainder of 7 when divided by 11?

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

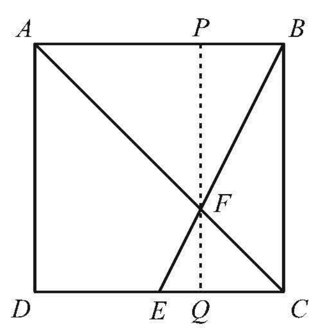
**Answer (E):** Let $n$ be a positive three-digit integer that satisfies the conditions stated in the problem. Note that because $n$ has a remainder of 2 when divided by 6, $n+4$ is divisible by 6. Similarly, $n+4$ is divisible by 9 and 11. Hence $n+4$ is divisible by the least common multiple of 6, 9, and 11, which is 198. Thus $n+4=198k$ where $k=1$, 2, 3, 4, or 5. So $n=194$, 392, 590, 788, or 986, and there are therefore five positive three-digit integers satisfying the given conditions.

22. Point $E$ is the midpoint of side $\overline{CD}$ in square $ABCD$, and $\overline{BE}$ meets diagonal $\overline{AC}$ at $F$. The area of quadrilateral $AFED$ is 45. What is the area of $ABCD$?



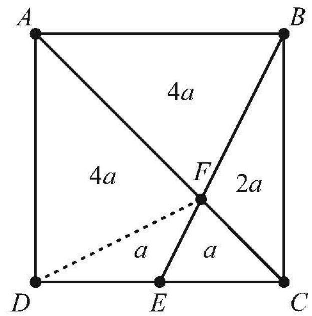

**(A)** 100 **(B)** 108 **(C)** 120 **(D)** 135 **(E)** 144

**Answer (B):** To see what fraction of the square is occupied by quadrilateral $AFED$, first note that $\Delta ADC$ occupies half of the area. Next note that because $\overline{AB}$ and $\overline{CD}$ are parallel, it follows that $\Delta ABF$ and $\Delta CEF$ are similar, and $CE=\frac{1}{2}AB$. Draw $\overline{PQ}$ through $F$ such that $\overline{PF}$ and $\overline{FQ}$ are altitudes of $\Delta ABF$ and $\Delta CEF$, respectively. Then $FQ=\frac{1}{2}PF$, so $FQ=\frac{1}{3}PQ=\frac{1}{3}BC$. Therefore the area of $\Delta CEF$ is $\frac{1}{2}(CE)(FQ)=\frac{1}{2}\left(\frac{1}{2}AB\right)\left(\frac{1}{3}BC\right)=\frac{1}{12}(AB)(BC)$, which is $\frac{1}{12}$ of the area of the square. Because the area of quadrilateral $AFED$ equals the area of $\Delta ADC$ minus the area of $\Delta CEF$, the fraction of the square occupied by quadrilateral $AFED$ is $\frac{1}{2}-\frac{1}{12}=\frac{5}{12}$. Because the area of $AFED$ is 45, the area of $ABCD$ is $\left(\frac{12}{5}\right)(45)=108$.

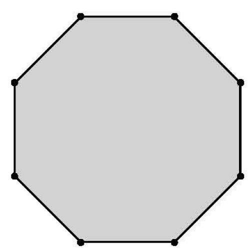



OR

Let $a$ denote the area of $\Delta CEF$. Note that $\Delta ABF$ is similar to $\Delta CEF$ with ratio of similarity $AB : CE = 2 : 1$. Therefore the area of $\Delta ABF$ is $4a$. Because $\Delta DEF$ and $\Delta CEF$ have equal bases and have a common altitude from $F$, the area of $\Delta DEF$ is $a$. Because $BF = 2FE$ and because $\Delta CBF$ and $\Delta CEF$ have a common altitude from $C$, it follows that the area of $\Delta CBF$ is $2a$. Similarly, $\Delta DAF$ has an area twice that of $\Delta DCF$, so the area of $\Delta DAF$ is 4a. Finally, the area of quadrilateral $AFED$ is $45 = 5a$, so $a = 9$, and therefore the area of square $ABCD$ is $12a = 108$.



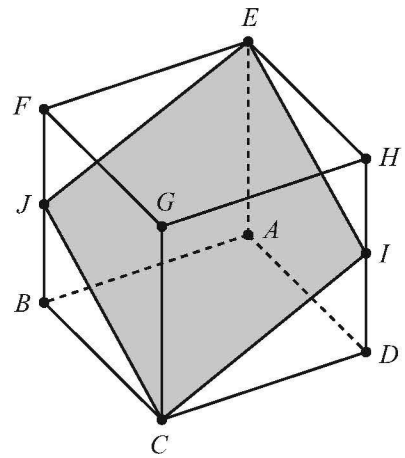

23. From a regular octagon, a triangle is formed by connecting three randomly chosen vertices of the octagon. What is the probability that at least one of the sides of the triangle is also a side of the octagon?



**(A)** $\frac{2}{7}$ **(B)** $\frac{5}{42}$ **(C)** $\frac{11}{14}$ **(D)** $\frac{5}{7}$ **(E)** $\frac{6}{7}$

**Answer (D):** For each side of the octagon, there are 6 triangles containing that side. Because the 8 triangles containing two adjacent sides of the octagon are counted twice, there are a total of $8 \cdot 6 - 8 = 40$ triangles sharing a side with the octagon. The total number of triangles that can be formed from the eight vertices is $\frac{8 \cdot 7 \cdot 6}{3!} = 56$, so the probability is $\frac{40}{56} = \frac{5}{7}$.

24. In the cube $ABCDEFGH$ with opposite vertices $C$ and $E$, $J$ and $I$ are the midpoints of edges $\overline{FB}$ and $\overline{HD}$, respectively. Let $R$ be the ratio of the area of the cross-section $EJCI$ to the area of one of the faces of the cube. What is $R^2$?




**(A)** $\frac{5}{4}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{25}{16}$ **(E)** $\frac{9}{4}$

**Answer (C):** Let $s$ denote the length of an edge of the cube. Now $EJCI$ is a non-square rhombus whose area is $\frac{1}{2}EC{\cdot}JI$, because the area of a rhombus is half the product of the lengths of its diagonals. By the Pythagorean Theorem, $JI = FH = s\sqrt{2}$, and using the Pythagorean Theorem twice, $EC = s\sqrt{3}$. Thus $R = \frac{\frac{1}{2}EC\cdot JI}{s^2} = \frac{\frac{1}{2}(s\sqrt{3})(s\sqrt{2})}{s^2} = \frac{\sqrt{6}}{2}$ and $R^2 = \frac{6}{4} = \frac{3}{2}$.

25. How many perfect cubes lie between $2^8 + 1$ and $2^{18} + 1$, inclusive?

**(A)** 4 **(B)** 9 **(C)** 10 **(D)** 57 **(E)** 58

**Answer (E):** Note that $2^8 + 1 = 257$ and that $216 = 6^3 < 257 < 7^3 = 343$. Also note that $2^{18} = (2^6)^3 = 64^3$, so the perfect cube that is closest to and less than $2^{18} + 1$ is $64^3$. Thus the numbers $7^3, 8^3, \ldots, 63^3, 64^3$ are precisely the perfect cubes that lie between the two given numbers, and so there are $64 - 6 = 58$ of these perfect cubes.

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