Are you ready to start ME25?

Self-Assessment Test — Answer Key

1. (a) Simplify each expression:
   i. $2^6 \cdot 2^{-4} = 2^2 = 4$
   ii. $\dfrac{3^3}{3^0} = 27$
   iii. $\dfrac{x^8 y^{-4}}{x^{-2} y^6} = x^{10} y^{-10} = \dfrac{x^{10}}{y^{10}}$

2. (a) Line through $A(2, -1)$ and $B(-4, 5)$:
   i. $m = \dfrac{5 - (-1)}{-4 - 2} = -1$
   ii. $y = -x + 1$
   iii. $y = -x + 5$
   iv. $y = x - 3$

3. (a) Savings options for \$5 000:
   i. Simple interest: $I = 5000 \times 0.03 \times 4 = \$600$
   ii. Compound: $A = 5000(1.028)^4 \approx \$5\,583.96$
   iii. Option B total $\approx$ \$5583.96 vs Option A total $=$ \$5 600. Option A gives more by $\approx$ \$16.04.

4. (a) Right triangle with legs 7 cm and 24 cm:
   i. $c = \sqrt{7^2 + 24^2} = \sqrt{625} = 25$ cm
   ii. For smallest angle θ (opposite 7 cm): $\sin\theta = \dfrac{7}{25}$, $\cos\theta = \dfrac{24}{25}$, $\tan\theta = \dfrac{7}{24}$
   iii. $\theta = \arctan\left(\dfrac{7}{24}\right) \approx 16°$

   (b) Solve each triangle:
   i. Adjacent $= 14\cos 38° \approx 11.0$ cm; Opposite $= 14\sin 38° \approx 8.6$ cm
   ii. $\dfrac{\sin B}{12} = \dfrac{\sin 43°}{9} \implies \sin B \approx 0.9093 \implies \angle B \approx 66°$
   iii. $\cos C = \dfrac{7^2 + 10^2 - 8^2}{2(7)(10)} = \dfrac{85}{140} \implies \angle C \approx 53°$

   (c) Rectangular prism (5 cm × 3 cm × 8 cm):
   i. $SA = 2(5 \cdot 3 + 5 \cdot 8 + 3 \cdot 8) = 158$ cm^2
   ii. $V = 5 \times 3 \times 8 = 120$ cm^3
   iii. Volume scales by $2^3 = 8$. New volume $= 960$ cm^3.

(d) Cone and cylinder:

i. $V_{\text{cone}} = \dfrac{1}{3}\pi(5^2)(12) = 100\pi \approx 314.2 \text{ cm}^3$

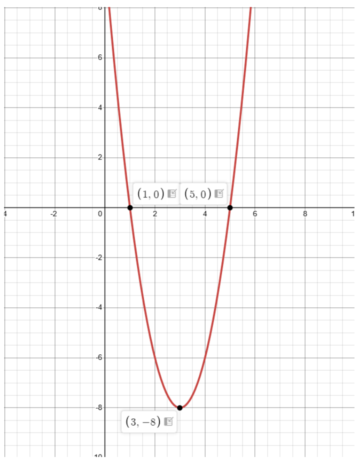
ii. $V_{\text{cyl}} = \pi(5^2)(12) = 300\pi \approx 942.5 \text{ cm}^3$

iii. $\dfrac{100\pi}{300\pi} = \dfrac{1}{3}$

5. (a) Parabola $y = 2(x-3)^2 - 8$:

i. Vertex: (3, −8); axis: $x = 3$; opens upward; y-intercept: (0, 10)

ii. $2(x-3)^2 = 8 \implies (x-3)^2 = 4 \implies x = 5$ or $x = 1$

iii. 

(b) Solve $x^2 + 4x - 1 = 0$:

i. $x = \dfrac{-4 \pm \sqrt{16+4}}{2} = -2 \pm \sqrt{5}$

(c) Projectile motion $h = -4.9t^2 + 14.7t + 2$:

i. $t = 1.5$ s; $h(1.5) = 13.025$ m

ii. Solve $-4.9t^2 + 14.7t + 2 = 0 \implies t \approx 3.13$ s

6. (a) Triangle $P(2,5), Q(-4,1), R(4,-3)$:

i. $PQ = 2\sqrt{13}$; $QR = 4\sqrt{5}$; $PR = 2\sqrt{17}$

ii. Midpoint of PQ: (−1, 3); QR: (0, −1); PR: (3, 1)

iii. $PQ^2 + PR^2 = 120 \neq 80 = QR^2$. Not a right triangle.

(b) Circle geometry:

i. $r = 13$; equation: $x^2 + y^2 = 169$

ii. $3^2 + 7^2 = 58 > 49$, so B lies outside the circle.

(c) Line segment $J(1,7)$ and $K(9,3)$:

i. Perpendicular bisector: $y = 2x - 5$

ii. Distance $d = \dfrac{13\sqrt{5}}{5}$

(d) Parallelogram proof:

i. Slope of $AB = 0$; slope of $DC = 0 \implies AB \parallel DC$.

ii. Slope of $AD = 2$; slope of $BC = 2 \implies AD \parallel BC$.

iii. Since opposite sides are parallel, $ABCD$ is a parallelogram. $\square$