

Are you ready to start ME35?
Self-Assessment Test

1. (a) For the function $h(x) = \frac{1}{x-2} + 3$:
 - (i) State the domain and range of $h(x)$.
 - (ii) Find $h^{-1}(x)$ and state its domain and range.
 - (iii) Sketch $h(x)$ and $h^{-1}(x)$.
2. (a) Describe all transformations applied to $f(x) = 2^x$ to obtain $g(x) = -\frac{1}{2}(2)^{x-3} + 4$. Then state the equation of the horizontal asymptote and the y -intercept of $g(x)$.
 - (b) Solve for x where $25^{2x-1} = 125^x$.
 - (c) A new car is purchased for \$28 000 and depreciates at a rate of 15% per year.
 - (i) Write an equation for the value V (in dollars) of the car after t years.
 - (ii) Determine the value of the car after 5 years, to the nearest dollar.
 - (iii) In which complete year will the car first be worth less than \$10 000?
 - (d) How much should be deposited today into an account earning 5% per year, compounded annually, to accumulate \$20 000 in 8 years?
3. (a) A point $P(-5, 12)$ lies on the terminal arm of angle θ in standard position.
 - (i) Find $r = \sqrt{x^2 + y^2}$.
 - (ii) State the exact values of $\sin \theta$, $\cos \theta$, and $\tan \theta$.
 - (iii) Find θ to the nearest degree, for $0^\circ \leq \theta < 360^\circ$.
 - (b) Use the CAST rule to answer each of the following.
 - (i) In which quadrant(s) are $\sin \theta > 0$ and $\cos \theta < 0$ simultaneously?
 - (ii) For $\theta \in [0^\circ, 360^\circ)$, in which interval(s) is $\tan \theta < 0$?
4. (a) For the function $y = -3 \sin\left(2\left(x - \frac{\pi}{4}\right)\right) + 1$, state each of the following.
 - (i) Amplitude
 - (ii) Period
 - (iii) Phase shift (direction and magnitude)
 - (iv) Vertical shift
 - (v) Maximum and minimum values of y
 - (b) Write a sinusoidal equation of the form $y = a \sin(k(x - d)) + c$ for a function with: amplitude 4, period π , a phase shift of $\frac{\pi}{6}$ to the right, and a vertical shift of 2 units downward.
 - (c) Describe all transformations that map $y = \cos x$ onto $y = 2 \cos(x + \pi) - 3$. Then sketch one full period of $y = 2 \cos(x + \pi) - 3$ on a clearly labelled set of axes, marking the maximum, minimum, and zeros.

- (d) Solve each equation for $x \in [0, 2\pi)$.
- (i) $2 \cos x - 1 = 0$
 - (ii) $\sin^2 x - \sin x = 0$
 - (iii) $2 \cos^2 x + \cos x - 1 = 0$
5. (a) Determine whether each sequence is arithmetic, geometric, or neither. If arithmetic or geometric, state the common difference or ratio and find the general term t_n .
- (i) 7, 11, 15, 19, ...
 - (ii) 3, 6, 12, 24, ...
 - (iii) 1, 4, 9, 16, ...
- (b) A sequence is defined recursively by $t_1 = 3$ and $t_n = 2t_{n-1} - 1$ for $n \geq 2$.
- (i) List the first five terms of the sequence.
 - (ii) Find t_7 .
 - (iii) Is this sequence arithmetic, geometric, or neither? Explain your reasoning.