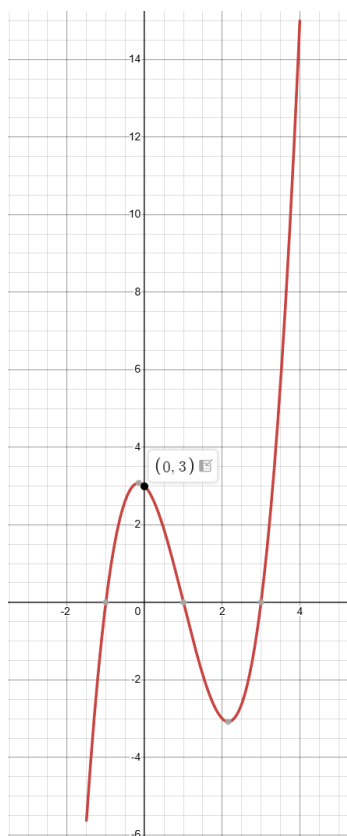


Are you ready to start ME45?

Self-Assessment Test

1. (a) Applications of Logarithms:
 - (i) The pH of a solution is defined as $\text{pH} = -\log[\text{H}^+]$. If $[\text{H}^+] = 2.5 \times 10^{-4}$, find the pH to the nearest tenth.
 - (ii) The Richter scale magnitude is $M = \log(I/I_0)$. The 2011 Tōhoku earthquake measured $M = 9.0$ and the 1989 Loma Prieta earthquake measured $M = 6.9$. How many times more intense was the Tōhoku earthquake? Express your answer to the nearest whole number.
 - (iii) The decibel level of a sound is $\beta = 10 \log(I/I_0)$. A jackhammer produces a sound level of $\beta = 100$ dB. Express its intensity I in terms of I_0 .
2. (a) Prove each identity:
 - (i) $\frac{\sin(2x)}{1 + \cos(2x)} = \tan x$
 - (ii) $\frac{\cos(2x) + 1}{\sin(2x)} = \cot x$
 - (iii) $\cos\left(x + \frac{\pi}{6}\right) + \cos\left(x - \frac{\pi}{6}\right) = \sqrt{3} \cos x$
 (b) Solve each equation:
 - (i) $\cos(2x) = \cos x, \quad x \in [0, 2\pi)$
 - (ii) $2 \sin^2 x - 3 \sin x + 1 = 0, \quad x \in [0, 2\pi)$
 (c) Sketch $y = \csc x$ for $x \in [-2\pi, 2\pi]$ on a clearly labelled set of axes. Mark all vertical asymptotes and the coordinates of local maxima and minima.
3. (a) For $f(x) = -2x^4 + 3x^3 - x + 5$:
 - (i) Describe the end behaviour of $f(x)$.
 - (ii) State the maximum number of x -intercepts and turning points.
 (b) Given $p(x) = 2x^3 - 3x^2 - 11x + 6$:
 - (i) Fully factor $p(x)$.
 - (ii) Find the remainder when $p(x)$ is divided by $x + 1$ without performing long division.
 (c) State all vertical asymptotes, horizontal asymptotes, and holes of $g(x) = \frac{3x^2 + 1}{x^2 - 9}$.
 (d) Solve the inequality. Express your answer in interval notation: $\frac{2x - 1}{x + 3} \leq 3$
 (e) For the rational function $f(x) = \frac{2x^2 - 8}{x^2 - x - 6}$: Sketch the graph of $f(x)$ on a clearly labelled set of axes.
4. (a) Let $f(x) = x^3 - 2x$:

- (i) Find the average rate of change of f on the interval $[1, 3]$.
 - (ii) Estimate the instantaneous rate of change at $x = 1$ using the difference quotient with $h = 0.001$. Round to three decimal places.
 - (iii) Find the exact instantaneous rate of change at $x = 1$ by fully simplifying the difference quotient $\frac{f(1+h) - f(1)}{h}$ and evaluating as $h \rightarrow 0$.
- (b) The graph of $y = f(x)$ is shown below:



- (i) Estimate the average rate of change of f from $x = 0$ to $x = 3$.
- (ii) State all intervals on which f is increasing and all intervals on which f is decreasing (Estimate the values).
- (iii) Estimate the x -coordinates of any local maxima and local minima.