

Are you ready to start ME45?

Self-Assessment Test — Answer Key

1. (a) Applications of Logarithms:

(i) $\text{pH} = -\log(2.5 \times 10^{-4}) \approx 3.6$

(ii) $10^{9.0-6.9} = 10^{2.1} \approx 126$ times more intense

(iii) $I = 10^{10} I_0$

2. (a) Prove each identity:

(i) $\frac{\sin(2x)}{1 + \cos(2x)} = \frac{2 \sin x \cos x}{1 + (2 \cos^2 x - 1)} = \frac{2 \sin x \cos x}{2 \cos^2 x} = \frac{\sin x}{\cos x} = \tan x \quad \square$

(ii) $\frac{\cos(2x) + 1}{\sin(2x)} = \frac{2 \cos^2 x}{2 \sin x \cos x} = \frac{\cos x}{\sin x} = \cot x \quad \square$

(iii) $\cos\left(x + \frac{\pi}{6}\right) + \cos\left(x - \frac{\pi}{6}\right) = 2 \cos x \cos\left(\frac{\pi}{6}\right) = 2 \cos x \cdot \frac{\sqrt{3}}{2} = \sqrt{3} \cos x \quad \square$

(b) Solve each equation:

(i) $2 \cos^2 x - \cos x - 1 = 0 \Rightarrow (2 \cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}: x = \frac{2\pi}{3}, \frac{4\pi}{3}; \quad \cos x = 1: x = 0$

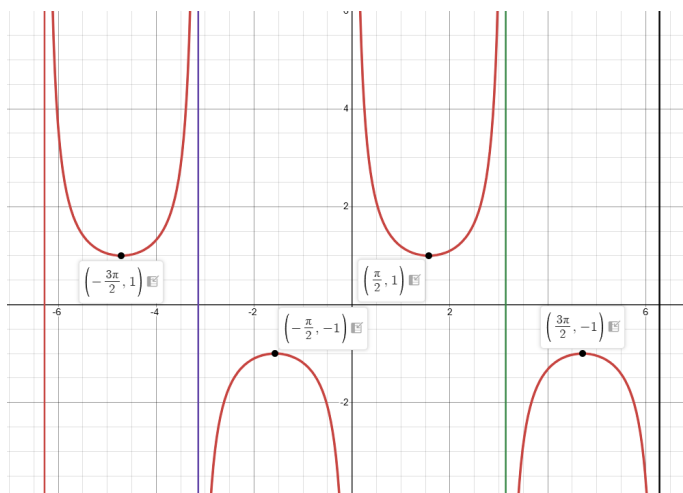
$x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

(ii) $(2 \sin x - 1)(\sin x - 1) = 0$

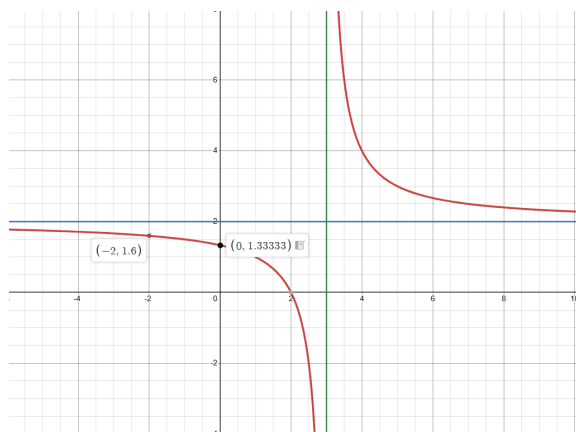
$\sin x = \frac{1}{2}: x = \frac{\pi}{6}, \frac{5\pi}{6}; \quad \sin x = 1: x = \frac{\pi}{2}$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

(c) Graph of $y = \csc x$:



3. (a) For $f(x) = -2x^4 + 3x^3 - x + 5$:
- (i) As $x \rightarrow +\infty$, $f(x) \rightarrow -\infty$; as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$
 - (ii) Maximum 4 x -intercepts; maximum 3 turning points
- (b) Given $p(x) = 2x^3 - 3x^2 - 11x + 6$:
- (i) $p(x) = (x - 3)(2x - 1)(x + 2)$
 - (ii) $p(-1) = 2(-1) - 3(1) - 11(-1) + 6 = -2 - 3 + 11 + 6 = 12$
- (c) Asymptotes and holes of $g(x) = \frac{3x^2 + 1}{x^2 - 9}$: No holes; VA: $x = 3$ and $x = -3$; HA: $y = 3$
- (d) Solution to $\frac{2x - 1}{x + 3} \leq 3$: $\frac{-x - 10}{x + 3} \leq 0 \Rightarrow (-\infty, -10] \cup (-3, \infty)$
- (e) For $f(x) = \frac{2x^2 - 8}{x^2 - x - 6}$: $f(x) = \frac{2(x - 2)(x + 2)}{(x - 3)(x + 2)}$: x -intercept $(2, 0)$; y -intercept $(0, \frac{4}{3})$;
 VA: $x = 3$; HA: $y = 2$; hole at $(-2, \frac{8}{5})$



4. (a) Let $f(x) = x^3 - 2x$:
- (i) $\frac{f(3) - f(1)}{3 - 1} = \frac{21 - (-1)}{2} = 11$
 - (ii) $\frac{f(1.001) - f(1)}{0.001} \approx \frac{-0.998997 - (-1)}{0.001} \approx 1.003$
 - (iii) $\frac{f(1 + h) - f(1)}{h} = \frac{(1 + h)^3 - 2(1 + h) + 1}{h} = \frac{h + 3h^2 + h^3}{h} = 1 + 3h + h^2 \xrightarrow{h \rightarrow 0} 1$
- (b) Analysis of the given graph:
- (i) $\frac{f(3) - f(0)}{3 - 0} = \frac{0 - 3}{3} = -1$
 - (ii) Increasing on $(-1.5, -0.2)$ and $(2.2, 4)$; decreasing on $(-0.2, 2.2)$
 - (iii) Local maximum at $x \approx -0.2$; local minimum at $x \approx 2.2$