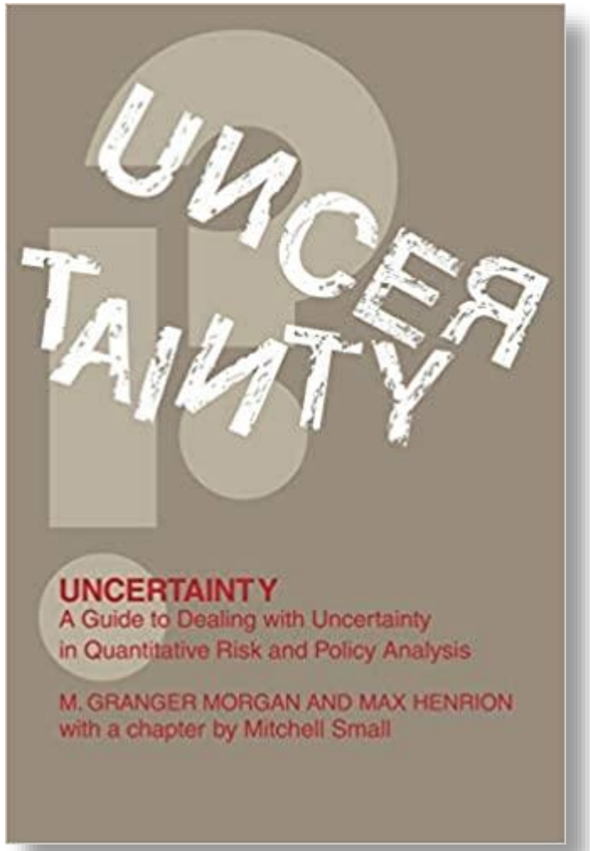
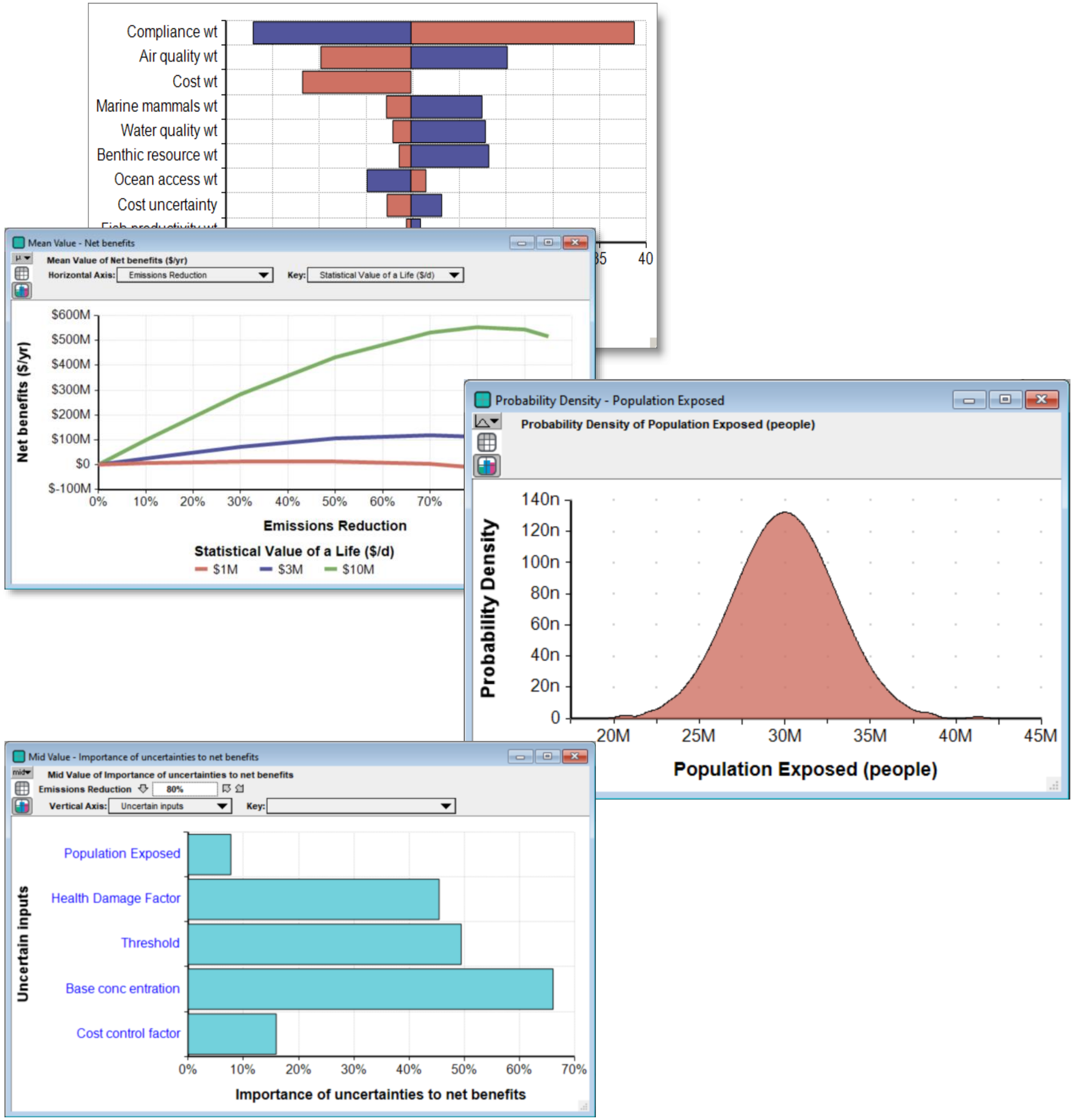
Sensitivity, uncertainty, and Monte Carlo simulation

- Approaches to express and analyze uncertainty
- Exercise: Parametric and importance analysis with TXC model
- Why treat uncertainty with probability distributions?
- Exercise: The Plane Catching Decision Problem
- How to create a Chance variable
- The expected value of including uncertainty (EVIU)
- Monte Carlo and other random simulation methods
- Statistical functions
- How to select a probability distribution
- Correlated and dependent variables
- Ways to view uncertain quantities

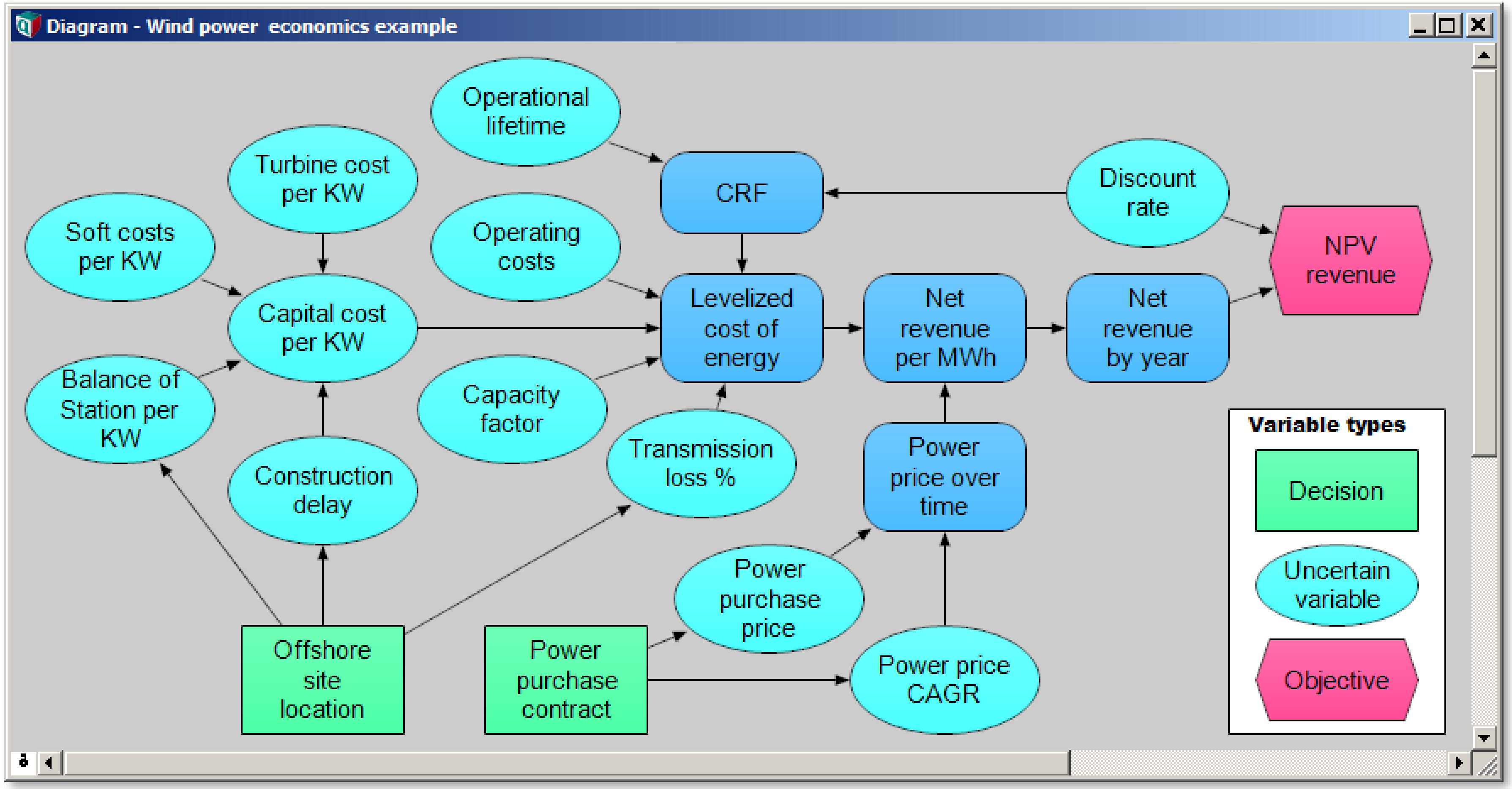


Approaches to express and analyze uncertainty

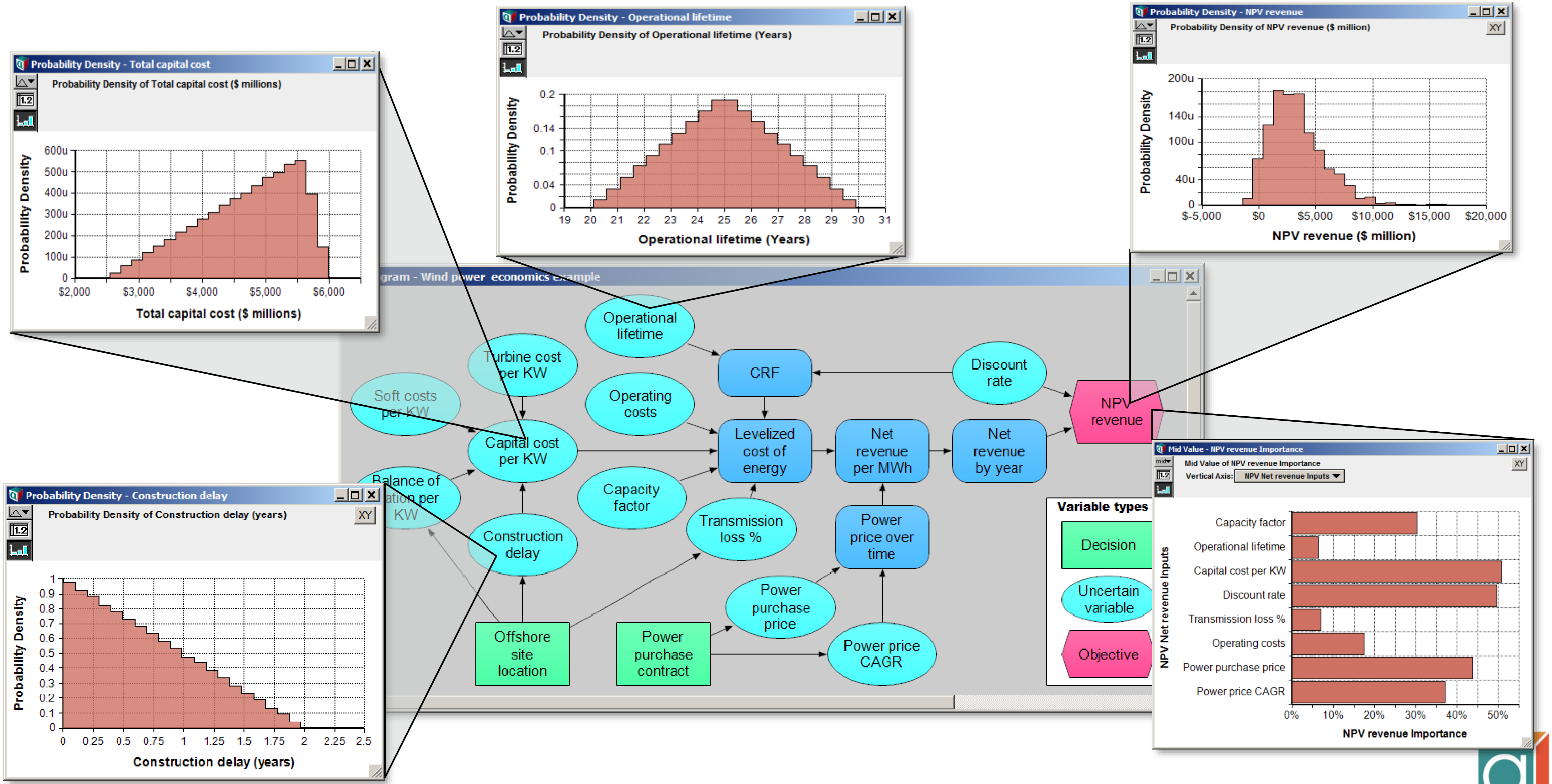
- Specify a range, low, mid, high
- Range sensitivity (tornado chart)
- Parametric analysis
- Express as probability distributions
- Importance analysis



Influence diagram for risk analysis of off-shore wind power

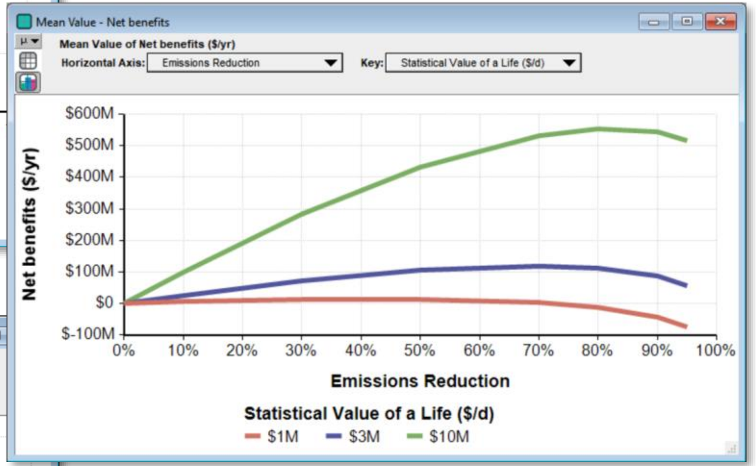


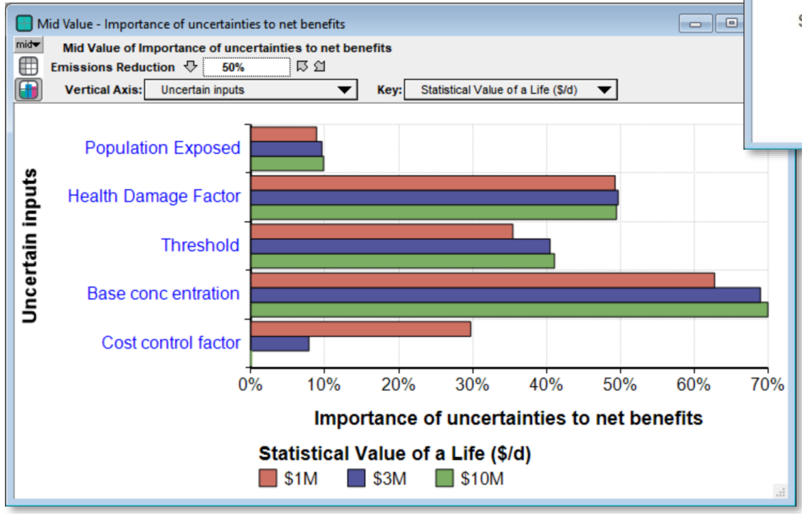
Risk analysis of off-shore wind power



U1Exercise: Parametric and importance analysis with TXC model

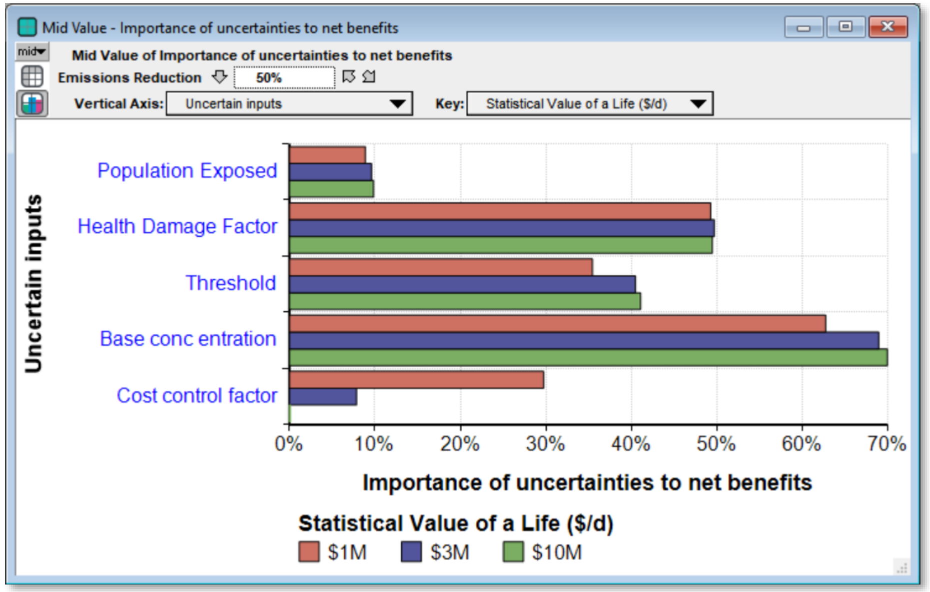
- Fire up TXC 2020.ANA
- Look at Benefits and costs as a function of Emissions Reduction
- Mean Net benefits for different Statistical values of a life
- Look at Importance of uncertainties to Net benefits as a function of Statistical value of a life



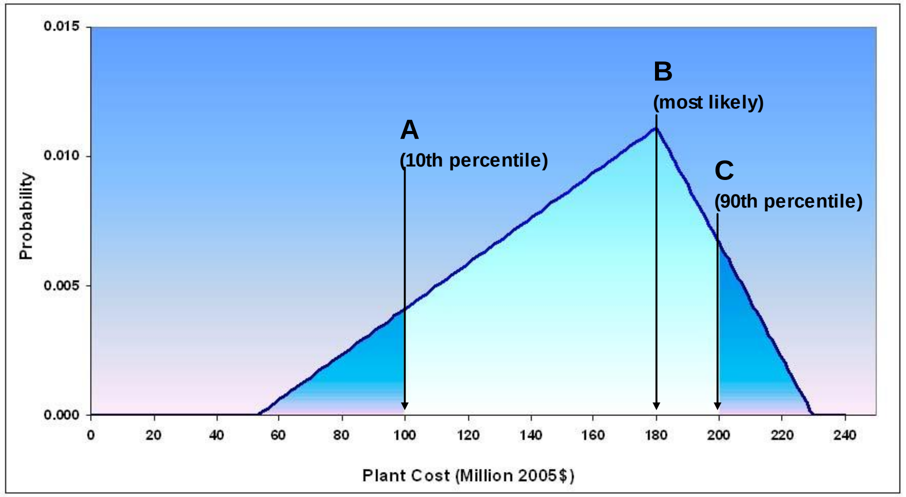


Importance Analysis

- Indicates which are the most important uncertainties, and therefore which might be worth understanding better.
- The built-in importance analysis examines the rank correlation between each uncertainty in the model with the outcome of interest.
- It provides guidance on which quantities really matter to the result – and may be worth further investigation, modeling, or data gathering.



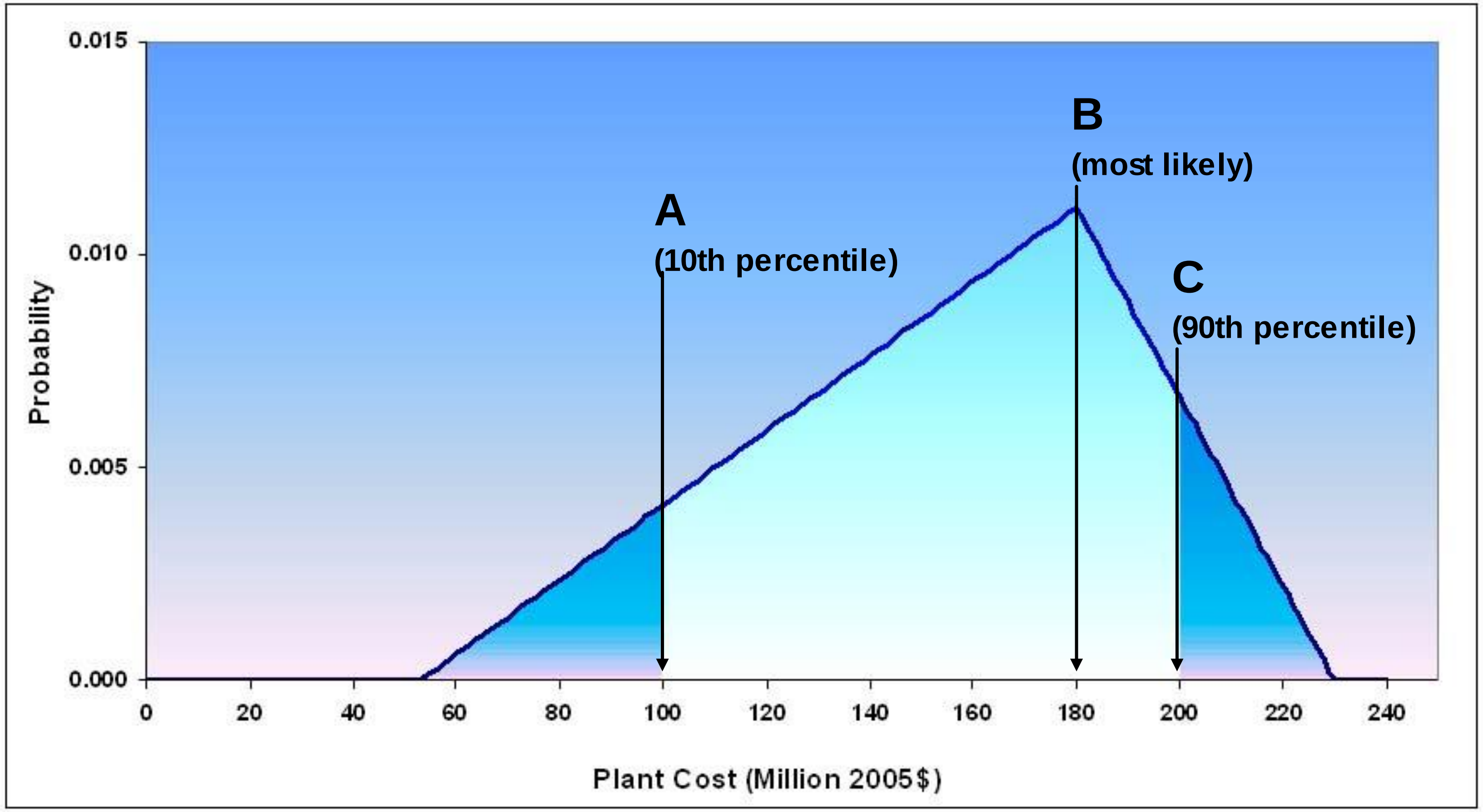
Why express uncertainty as probability distributions?



- Judgment is unavoidable in extrapolating from what we know to what we need to make decisions about. Let’s be explicit about it
- Probability is the clearest, most widely used language for expressing uncertainty.
- Obtaining probability distributions from a range of experts is the best way to quantify the current state of knowledge (and lack thereof)
- There are well-developed methods for obtaining expert judgment as probability distributions
- Careful assessment methods can minimize cognitive biases

Uncertainty: A Guide to Dealing with Uncertainty in Risk and Policy Analysis. M Granger Morgan & Max Henrion, Cambridge UP, 1990

Assessing points on a triangular distribution



- A. Low value -- 10th percentile
- B. Most likely -- mode or peak
- C. High value -- 90th percentile)

Exercise U2: The Plane Catching Decision:
Draw an influence diagram and add definitions

- Your flight is scheduled to depart at 7.00am. You plan to drive or get a cab from your home to the airport.
- **Time to drive to airport** (mins)
- **Time from parking** (or drop-off) to travel to the gate (mins)
- **Time to be at the gate** before departure (mins)
- Define each of these quantities as a probability distribution, for example:

  = Triangular(low, mid, high)

  = UncertainLMG(low, mid, high)

  = Normal(mid, sd)

  = Lognormal(mid, gsd)

- **Total time to gate** (mins)

  Sum of the above three uncertain quantities.
- Decision **Time to leave** (mins before departure)
- **Loss if I miss the plane** (mins)

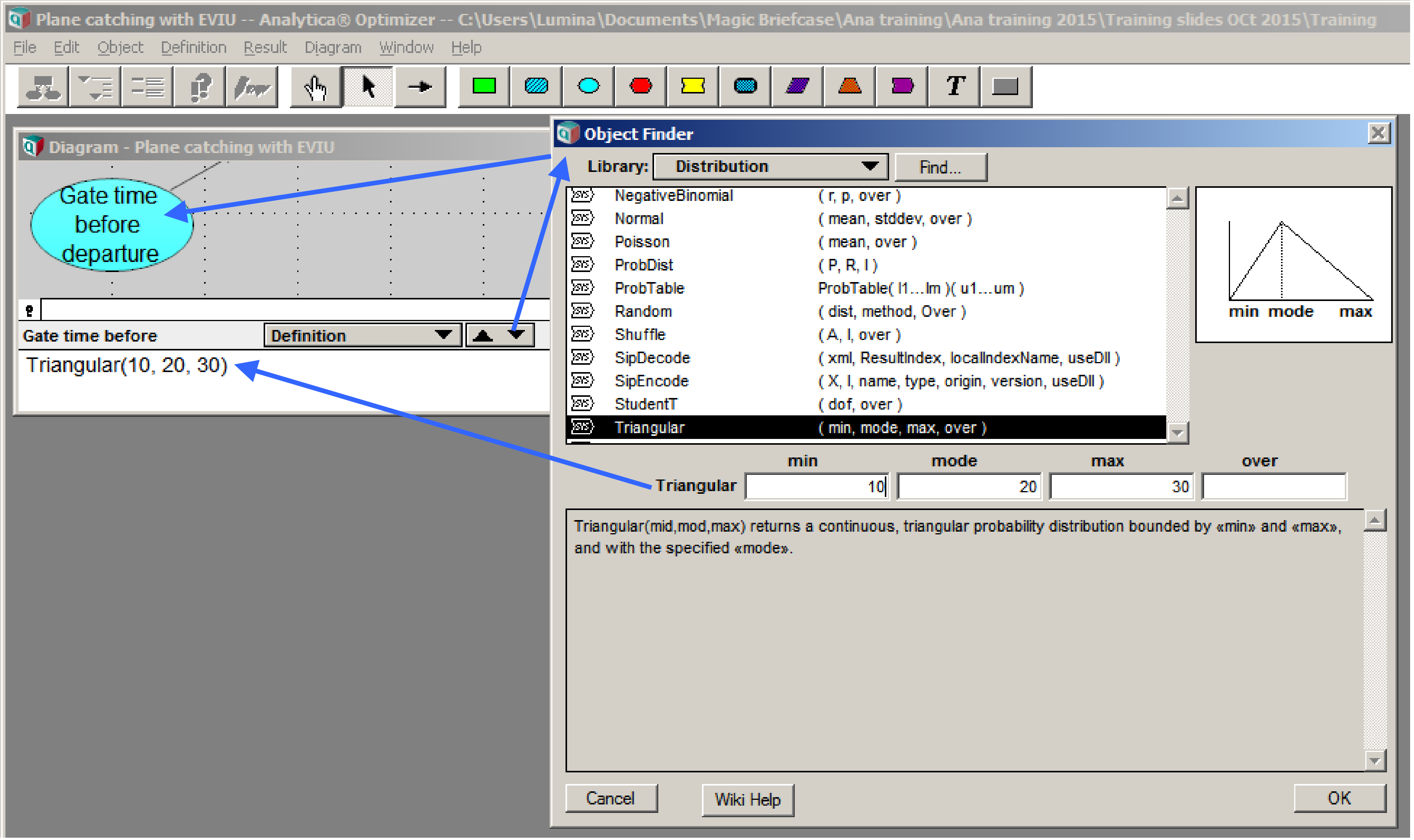
  = ~ 400 mins
- Objective to minimize **Time lost** (mins)

  = Time_to_leave – Travel_time

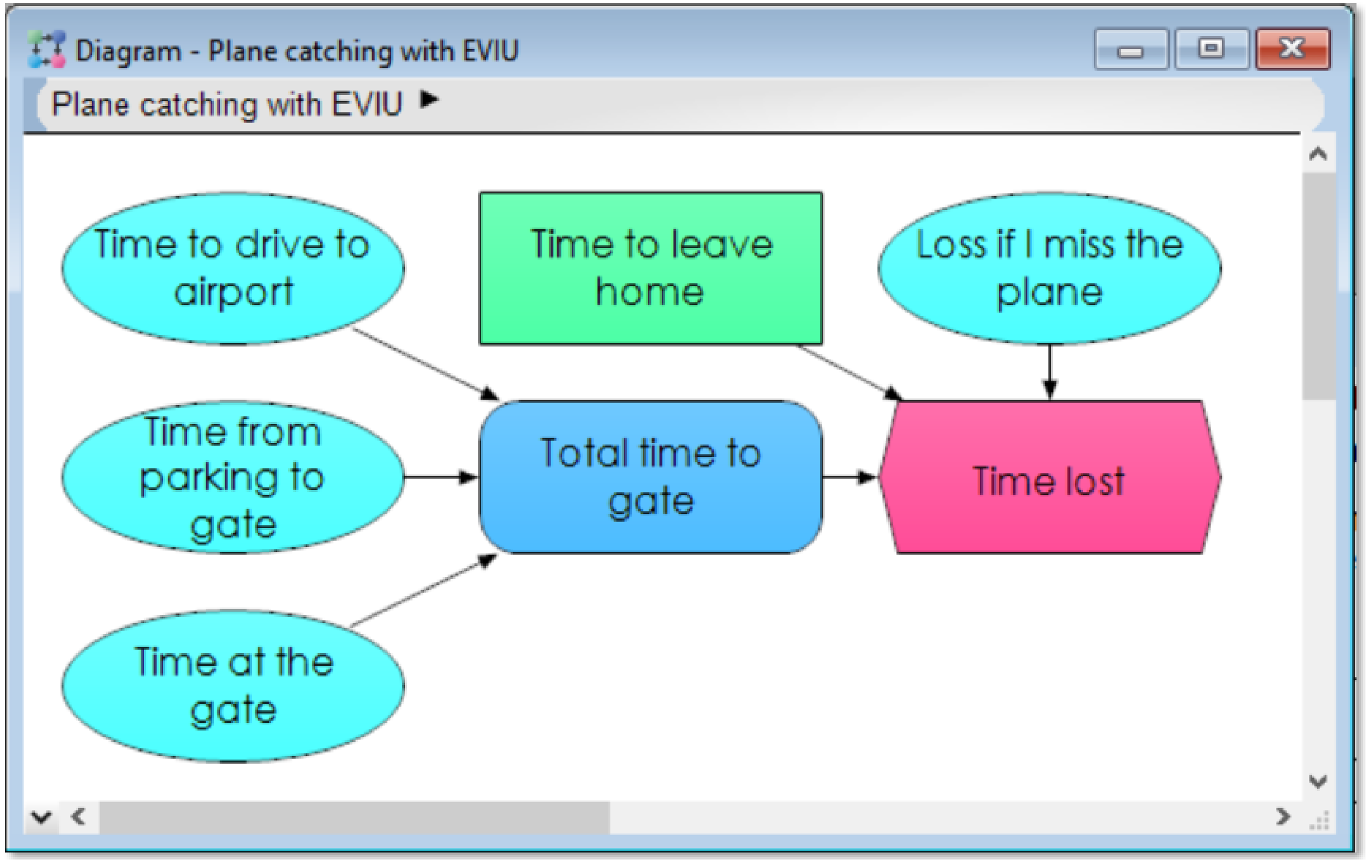
  + (IF Travel_time > Time_to_leave

  THEN Loss_if_miss ELSE 0)

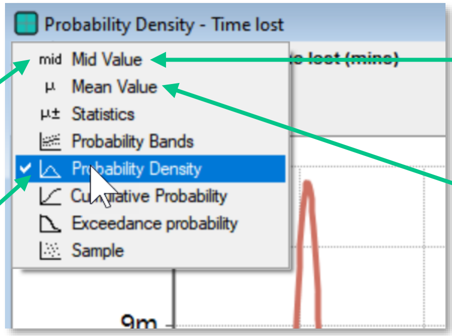
Create a chance variable and select a distribution



Plane catching problem: Example influence diagram



Exercise U3: Analyze the Plane Catching Decision

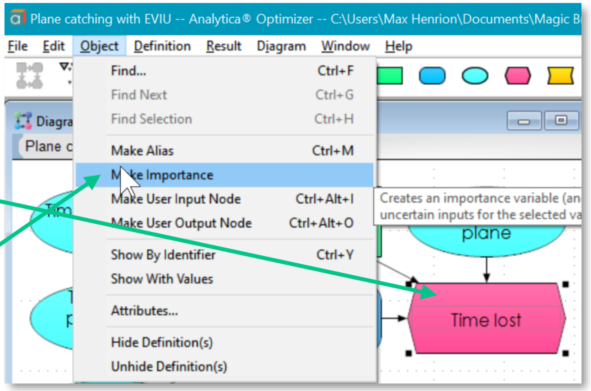


- **Result view** for objective **Time lost**
  - Look at the **Mid** value
  - Select **Probability density** from *Uncertainty view*: Does it have more than one mode?
  - Select **Cumulative Probability** from *Uncertainty view*: What is the probability that you will miss the plane if **Leave time** = mid value?
  - Select **Sample** from Uncertainty view: Look at dots. Change to Table view.
- Redefine Decision **Leave time** to be a sequence of values

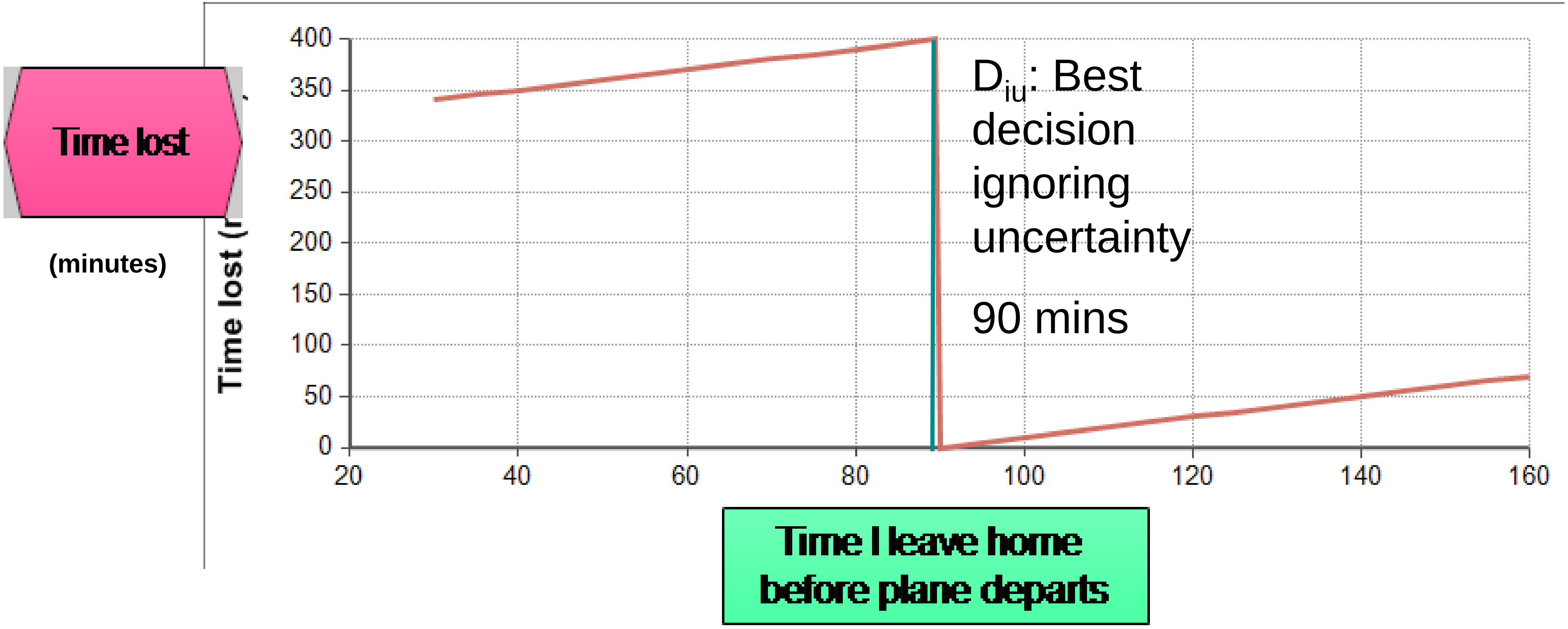
  = Sequence(<low>, <high>, <step>)

  Where <low> and <high> are the min and max plausible times to leave, and <step> = 5

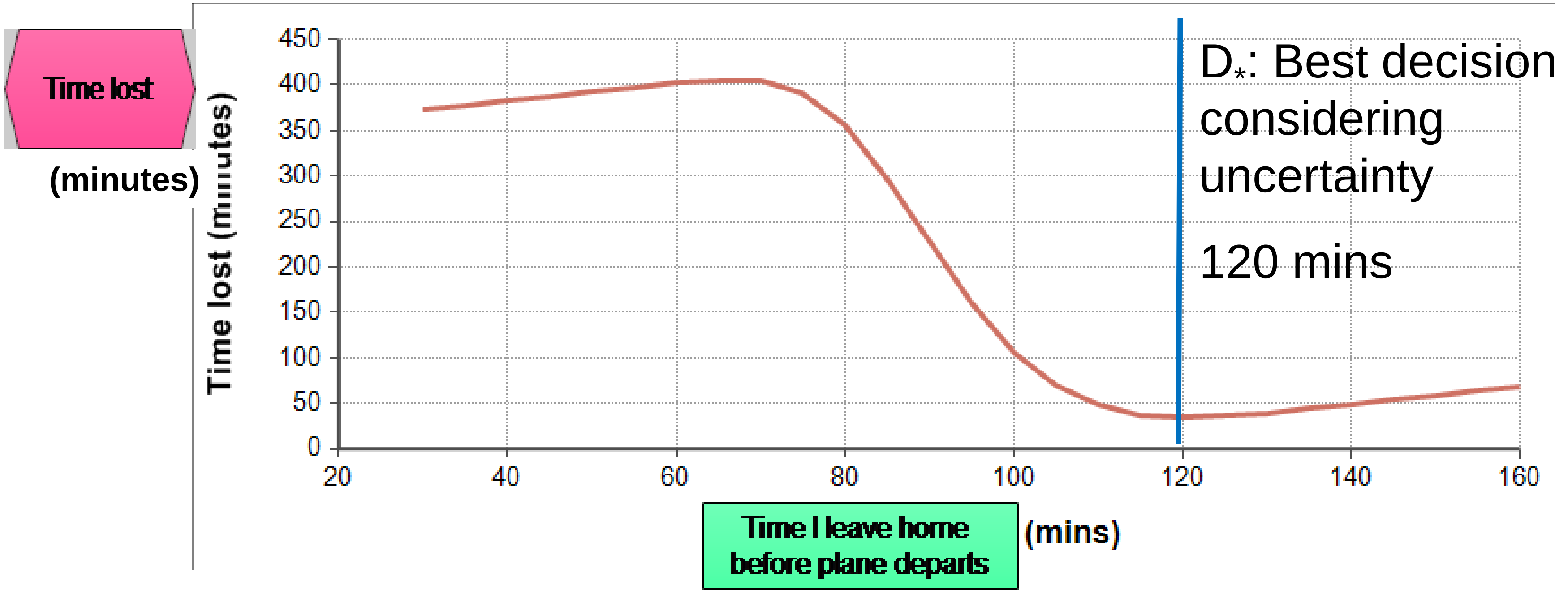
- Reselect **Mid** from the uncertainty view for **Time lost**
  - What **Leave time** minimizes the **Time lost** (ignoring uncertainty)?
- Select **Mean** from the uncertainty view for **Time lost**
  - What **Leave time** minimizes *mean* **Time lost** (considering uncertainty)?
- **Importance analysis**
  - Select the objective **Time lost**
  - From the **Object** menu select **Make Importance**
  - It makes **Time lost inputs Time lost Importance.**
  - Which uncertain input contributes the most uncertainty?

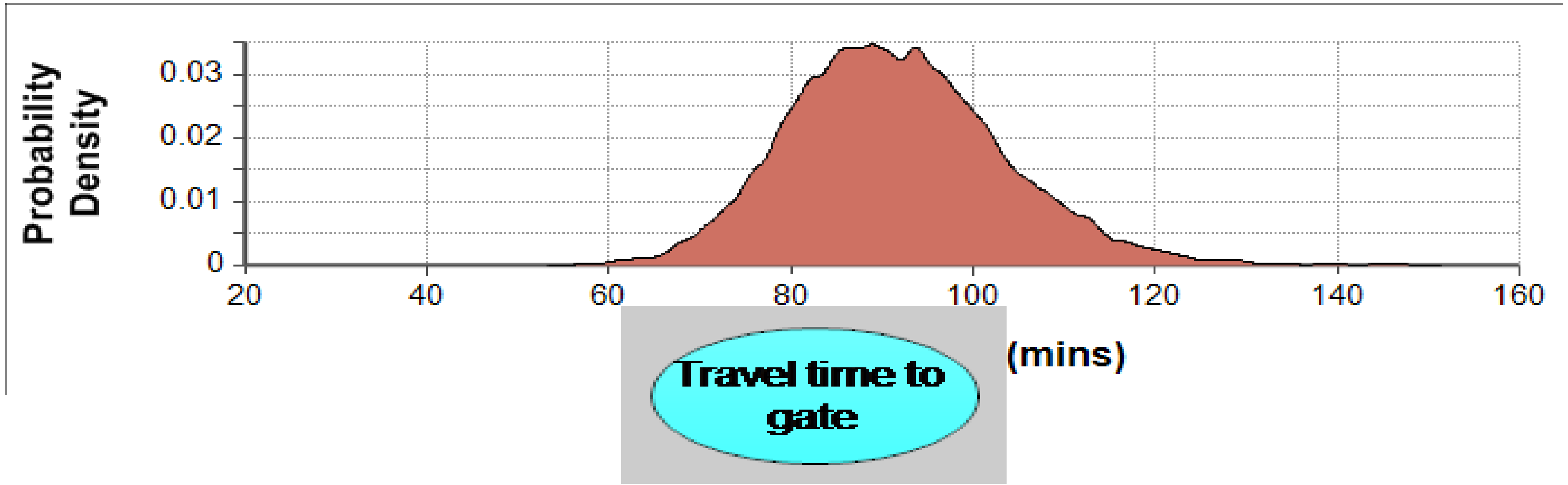


Loss function ignoring uncertainty



Loss function ignoring uncertainty





Exercise U4: Tornado chart – range sensitivity analysis

Add these variables:

- Index LMH_percentiles

  = [10%, 50%, 90%]

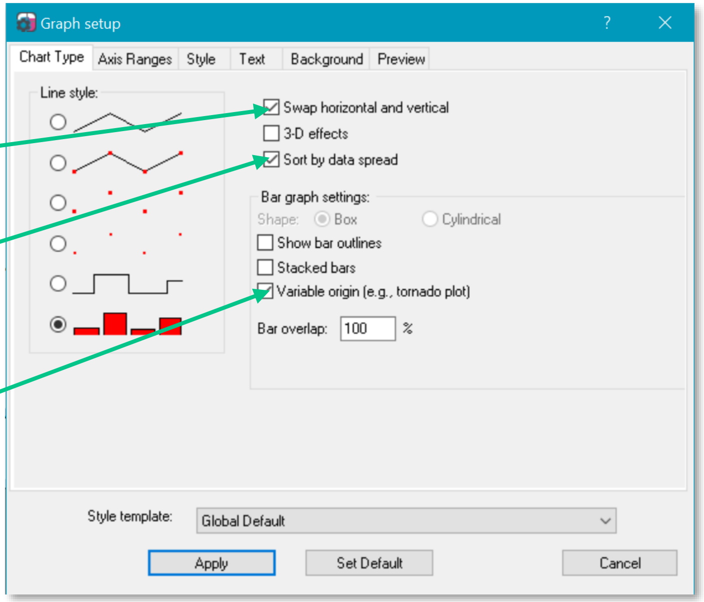
- Variable PCiles_of_inputs

  = Getfract(Time_lost_Inputs,
  LMH_percentiles)

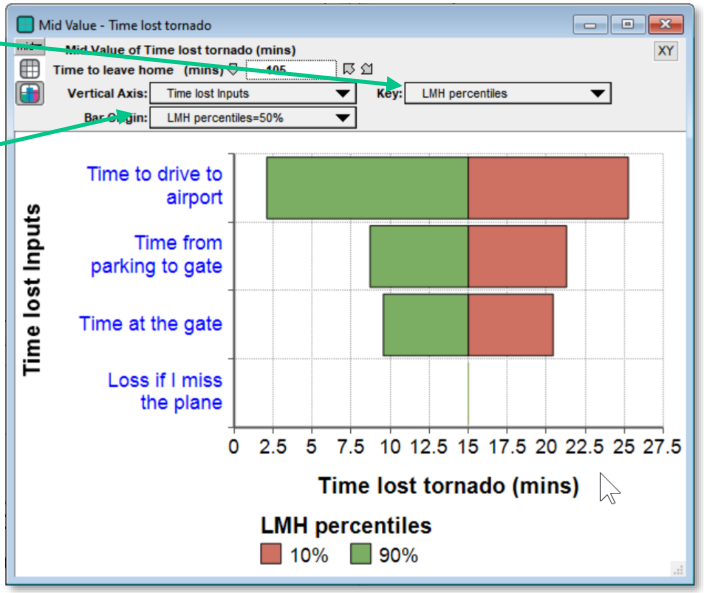
- Variable Time_lost_range

  = WhatIfAll(Time_lost,
  Time_lost_Inputs,
  PCiles_of_inputs)

- In Graph setup …
  - Check Swap horizontal and vertical
  - Check Sort by data spread
  - Check Variable origin (e.g. tornado plot)

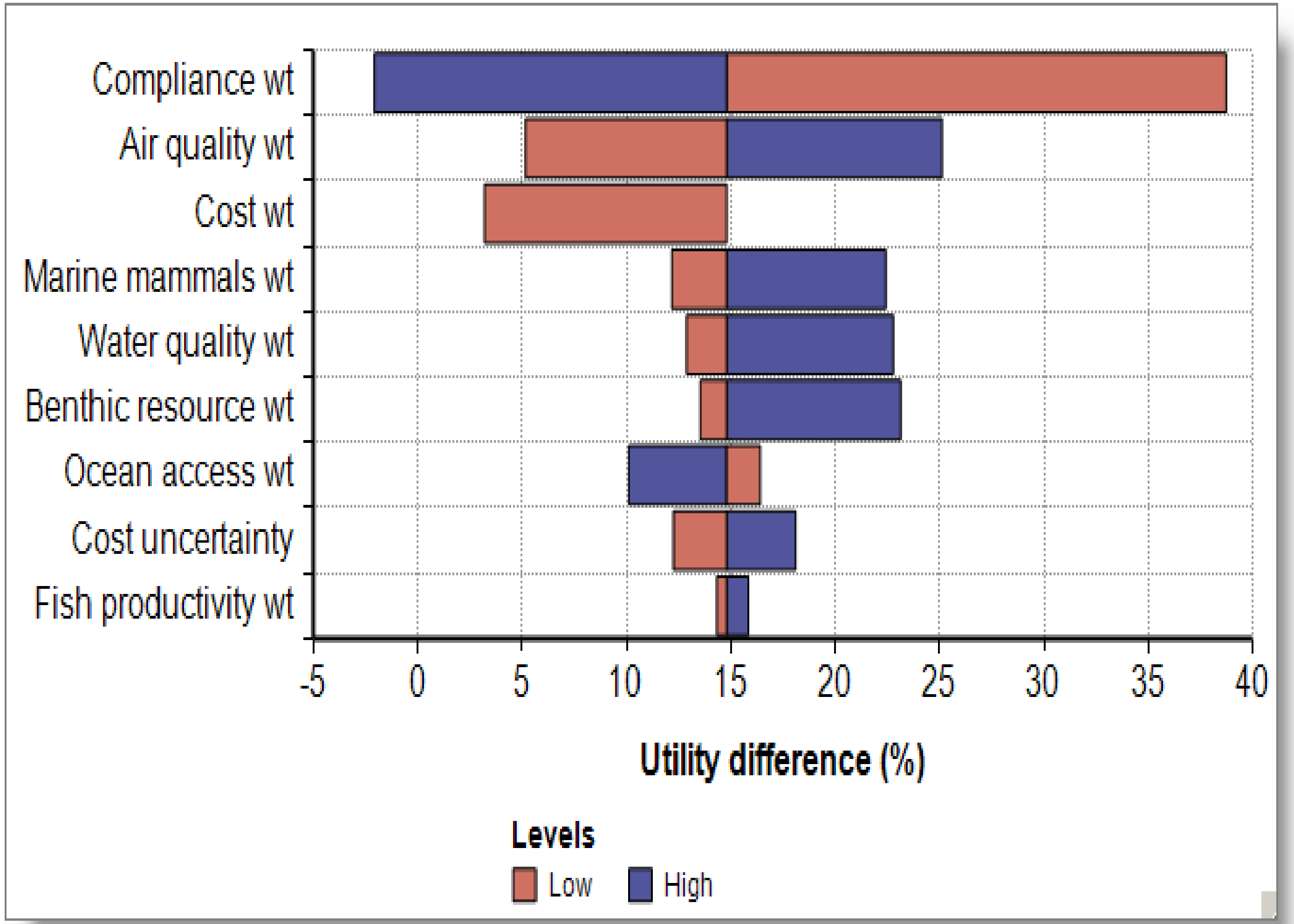


- Set Key to LMH percentiles
- Select Bar origin to LMH_pc = 50%



Tornado Diagrams

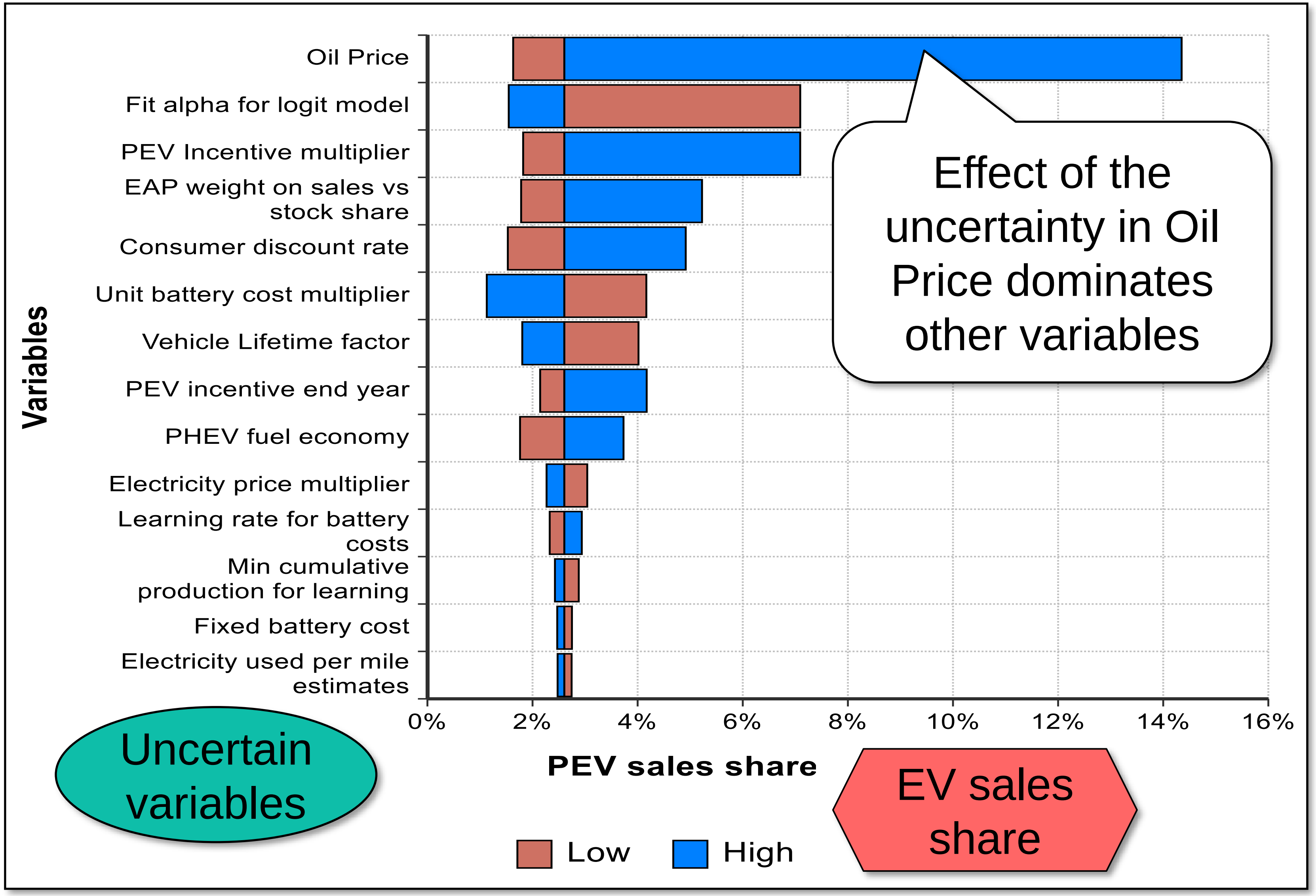
- Tornado diagrams (or range sensitivity charts) show the effect on a result of varying each uncertain variable from low to high value, while keeping the others at their base (mid) value.



Tornado Chart sensitivity analysis
for Electric Vehicle sales share in 2021 (forecast in 2013)

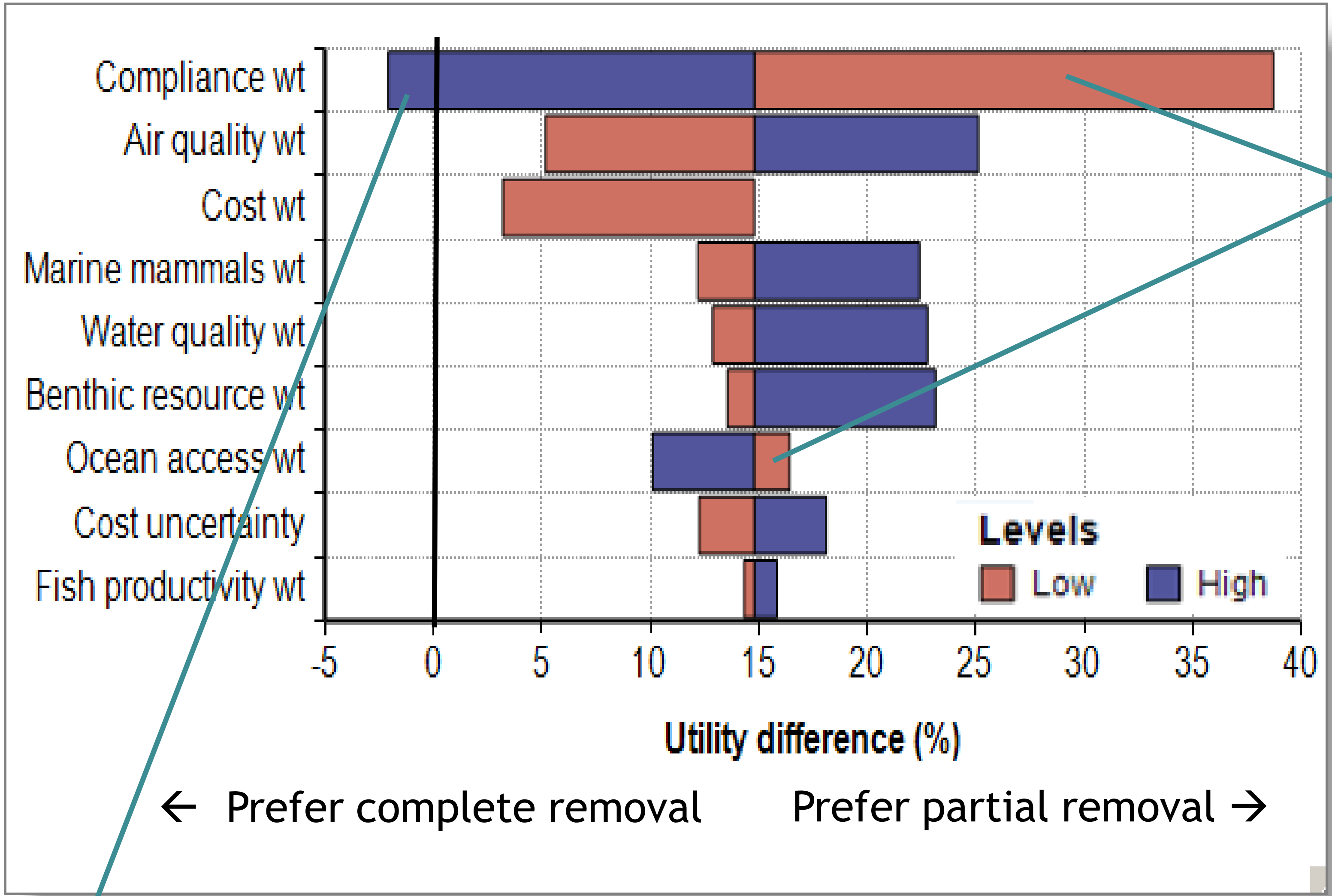
The range of EV sales share % as we change each uncertain variable from low to high, keeping the other variables at their base value.

Ordered by range.



Tornado chart sensitivity analysis

Each bar shows the effect on a variable of changing swing weight from 0 to 100



Higher level favors complete removal only for Compliance and Ocean access weight

Compliance weight is the only variable that could change preference from partial to complete removal

Monte Carlo Simulation

- Represent each input distribution by a random sample of **n** (**SampleSize**)
- **Run** is the Index of the **n** samples (or scenarios)
- Compute each uncertain result for each of the **n** samples.
- Estimate a probability distribution (density function, cumulative distribution, mean, or other statistics) for each uncertain quantity from the **n** samples
- Precision of these estimates increases with **Sqrt(n)**
- Precision of depends on **n**, but not on the number of uncertain inputs – unlike probability trees where the effort is exponential in number of uncertainties

The **Run** index

- **Run** (Title - Iteration) is the Index over random samples in Monte Carlo
- **Run** is like any other index for array abstraction.

Sample - Time lost

Sample of Time lost (mins)

Iteration (Run) ☐ Totals

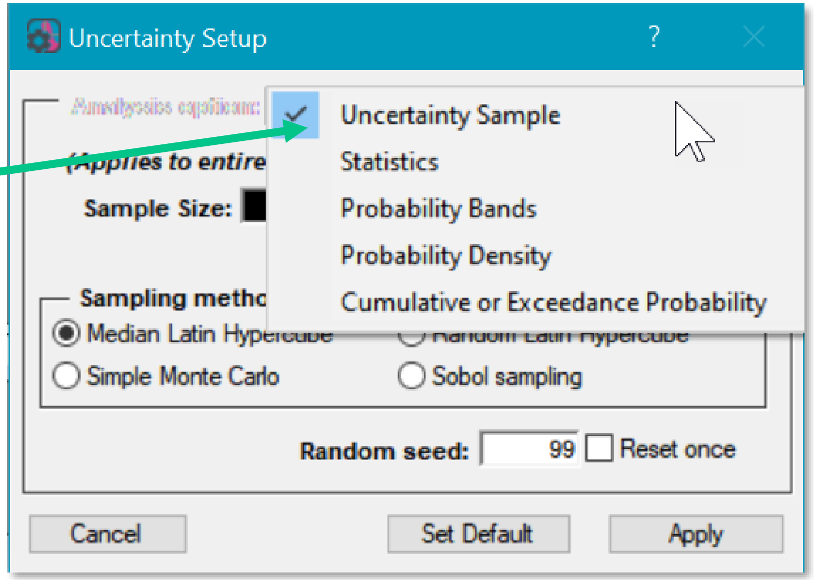
Time to leave home (mins) ☐ Totals

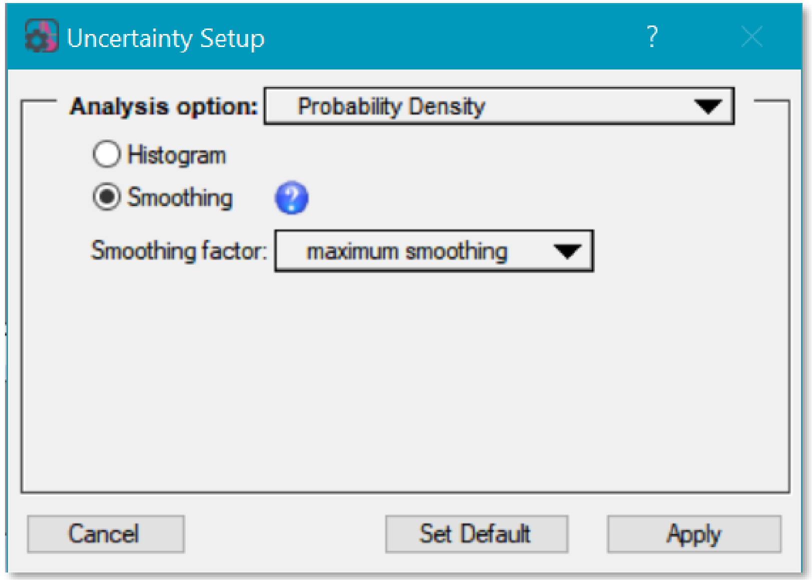
XY

	60	65	70	75	80	85
1	572.2	577.2	582.2	587.2	592.2	597.2
2	807.5	812.5	817.5	822.5	827.5	832.5
3	254.6	259.6	264.6	269.6	274.6	279.6
4	587.9	592.9	597.9	602.9	607.9	612.9
5	310.8	315.8	320.8	325.8	330.8	335.8
6	522.2	527.2	532.2	537.2	542.2	547.2
7	452.7	457.7	462.7	467.7	2.465	7.465
8	236.5	241.5	246.5	251.5	256.5	261.5
9	307.1	312.1	317.1	322.1	327.1	332.1
10	142.5	147.5	152.5	157.5	162.5	1.619
11	465.3	470.3	475.3	480.3	485.3	490.3
12	282.5	287.5	292.5	297.5	302.5	307.5
13	408.2	413.2	418.2	0.1681	5.168	10.17
14	634	639	644	649	654	659
15	112.1	117.1	122.1	127.1	132.1	137.1
16	477	482	487	492	497	502

Exercise U5: Explore Uncertainty Setup dialog box

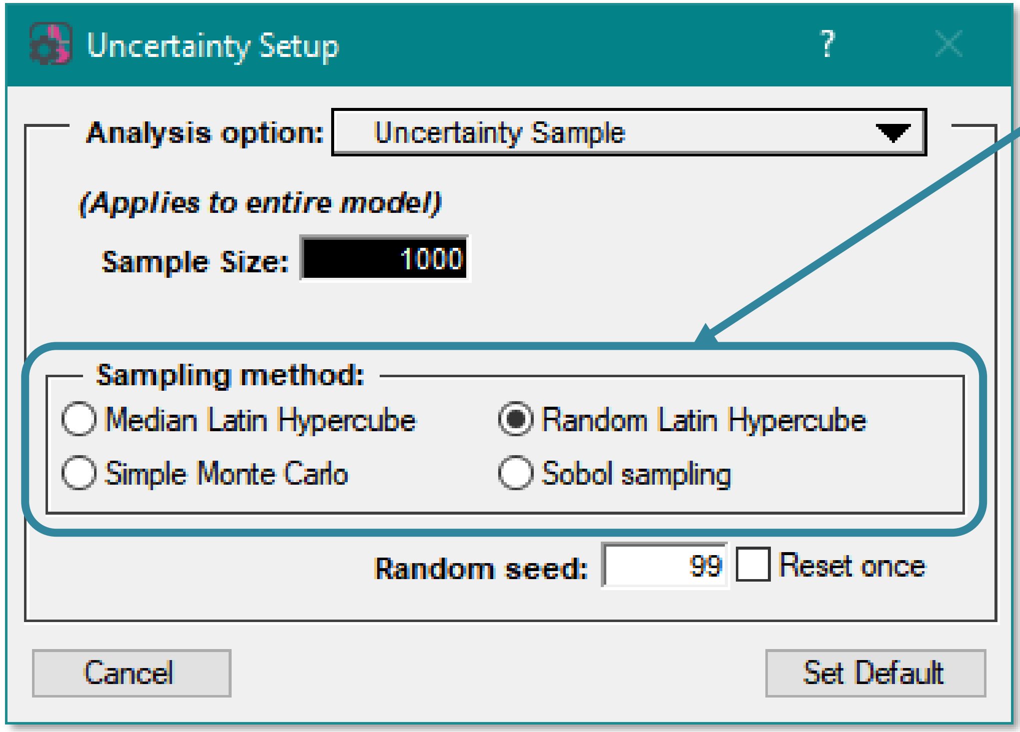
- Select **Uncertainty Options…** from the **Result** menu bar, or shortcut *ctrl+U* .
- Explore options



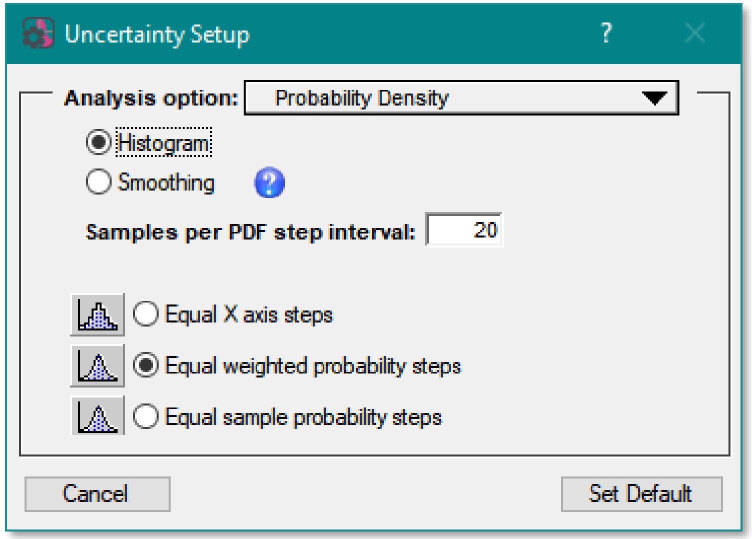


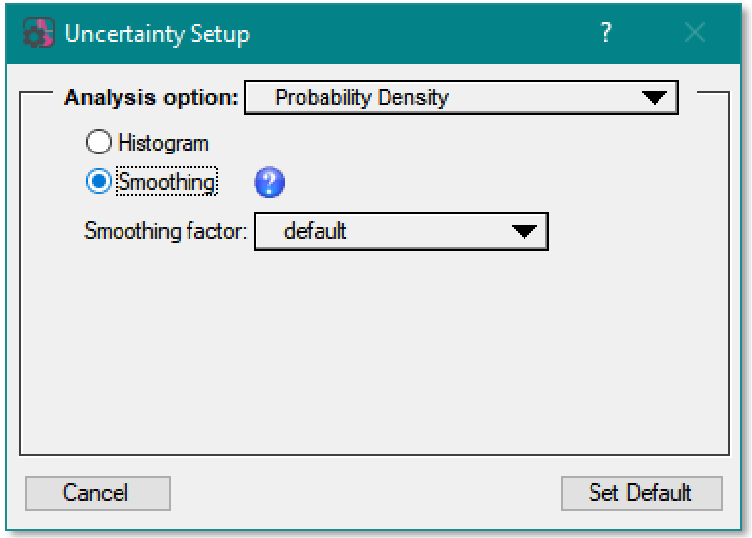
Monte Carlo and other sampling methods

- Simple **Monte Carlo simulation** samples at random from each input distribution
- **Convergence** is the rate at which a statistic from a sample converges to the exact value as a function of sample size N
- **Random Latin Hypercube sampling (RLHS)** generates a random sample from each of the N equiprobable intervals for each uncertain quantity. Converges a bit faster than simple Monte Carlo.
- **Median Latin Hypercube sampling (MLHS)** generates the **median** of each of the N equiprobable intervals. (Analytica default). Converges faster than RLHS, but can give wrong results in very rare cases.
- **Sobol sampling:** Systematic sampling over multiple dimensions. Works best with a modest number (say <=7) chance variables.



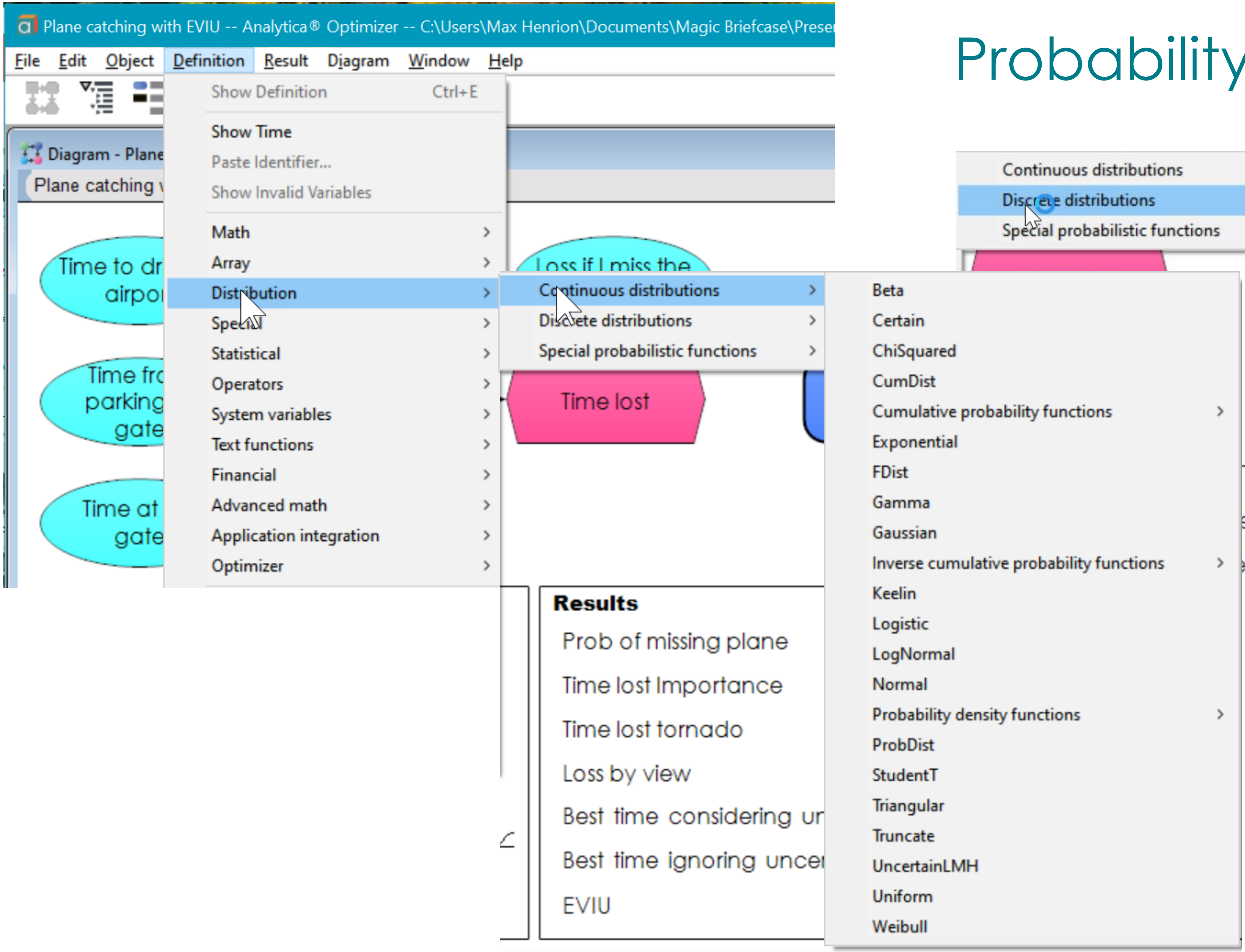
How large a sample should you use?

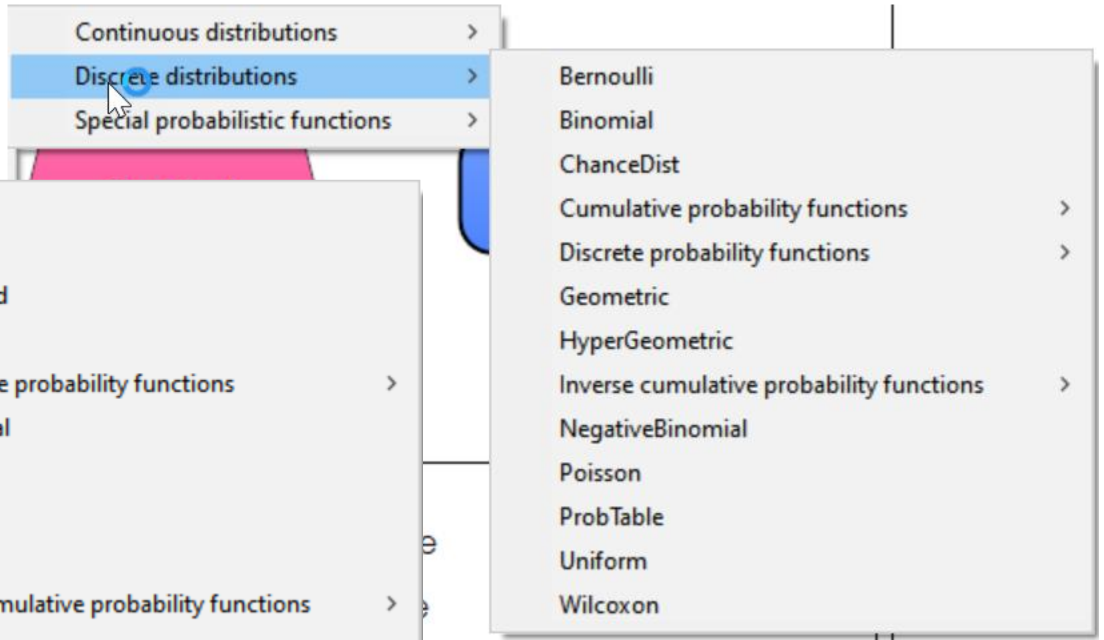




- Start small (e.g. 100) while building and testing a large model
- You can control the apparent smoothness of distributions with **Samples per PDF plot point** or **Samples per CDF plot interval** in Uncertainty dialog from Results menu
- What kind of measures of the distribution are you interested in?
  - **Cumulative distribution** (all the percentiles)
  - **Probability density function** – more sensitive to sample size
  - Median, mean, standard deviation, 10% -- 90%iles?
    - See Analytica Docs, Choosing a Sample size. https://wiki.analytica.com/index.php?title=Selecting_the_Sample_Size
- How will the uncertainty affect decisions?

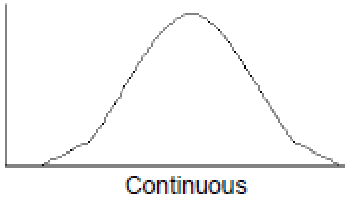
Probability distributions



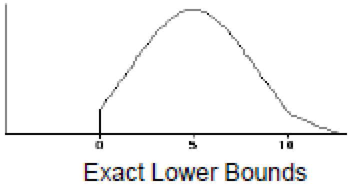


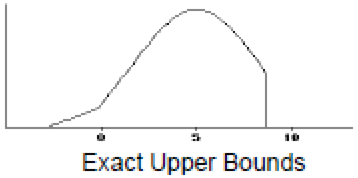
How to select a distribution to express uncertainty?

- Is it discrete or continuous?

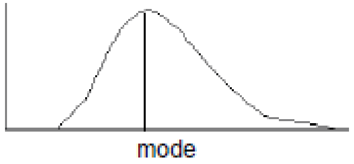


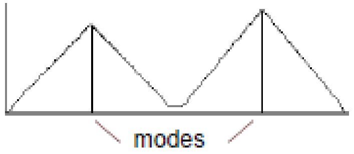
- Does it have upper and/or lower bounds?



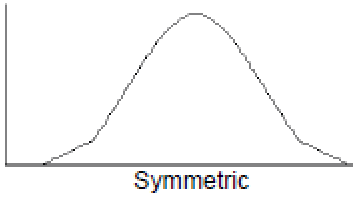


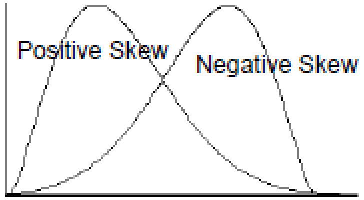
- How many modes?





- Symmetric or skewed?



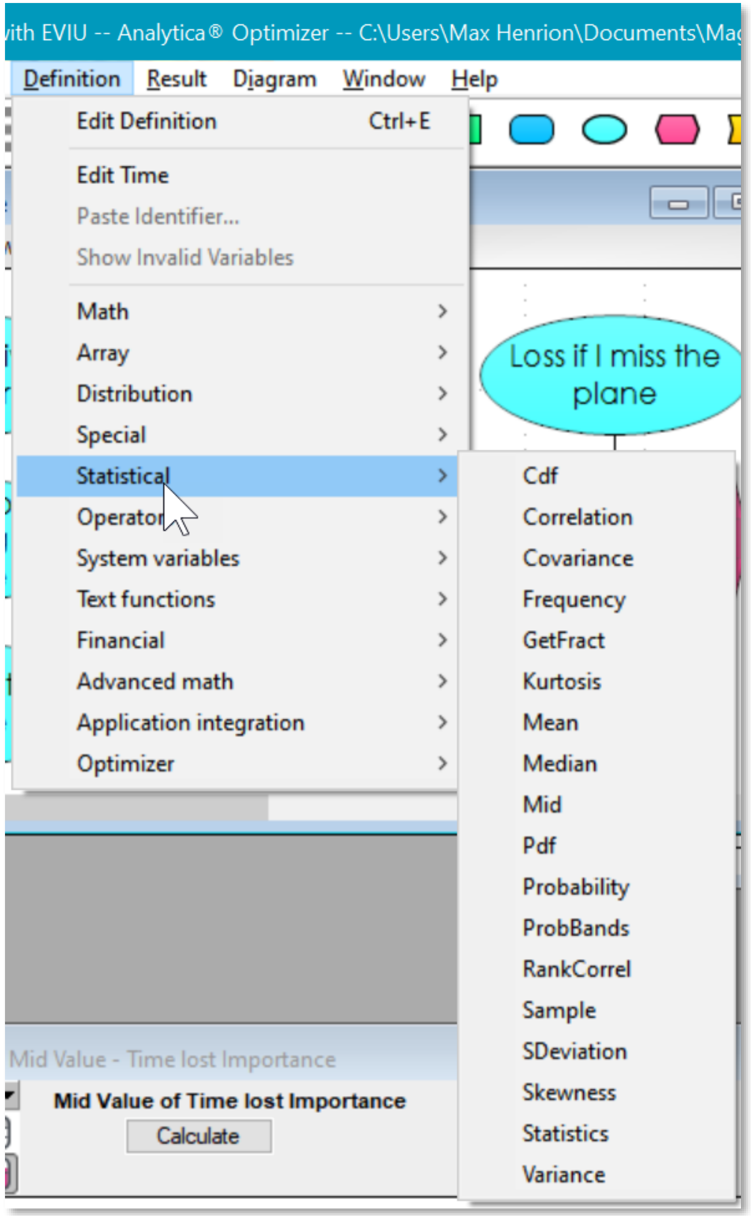


- Recommend **UncertaintyLMH(**low, mid, high, prob**)**
- prob is Pr(x<low) = Pr(x>high)
- Can add lower or upper bound.

Truncate(X, *xmin, xmax*) generates a distribution the same as X, removing tails below *xmin* or above *xmax*

- Try Truncate(X, -2)
- How would you truncate "My Chance Node" such that values higher than 2 are thrown out? Try -Truncate(-X, -2)
- If you don't want to resample but want to replace rejected values with a boundary value, use Max([min, *expr*]) or Min([max, *expr*])

Statistical functions: Array-reducing functions



- Mean(x, *i*)
- SDeviation(x, *i*)
- Variance(x, *i*)
- Skewness(x, *i*)
- Kurtosis(x, *i*)
- CoVariance(x, y, *i*)
- Correlation(x, y, *i*)
- RankCorrel(x, y, *i*)
- Regression(y, b, i, k)

If you omit index ***i***, by default each assumes the **Run** Index from Monte Carlo simulation.

Sensitivity analysis shows the quantitative relationship of one variable to another

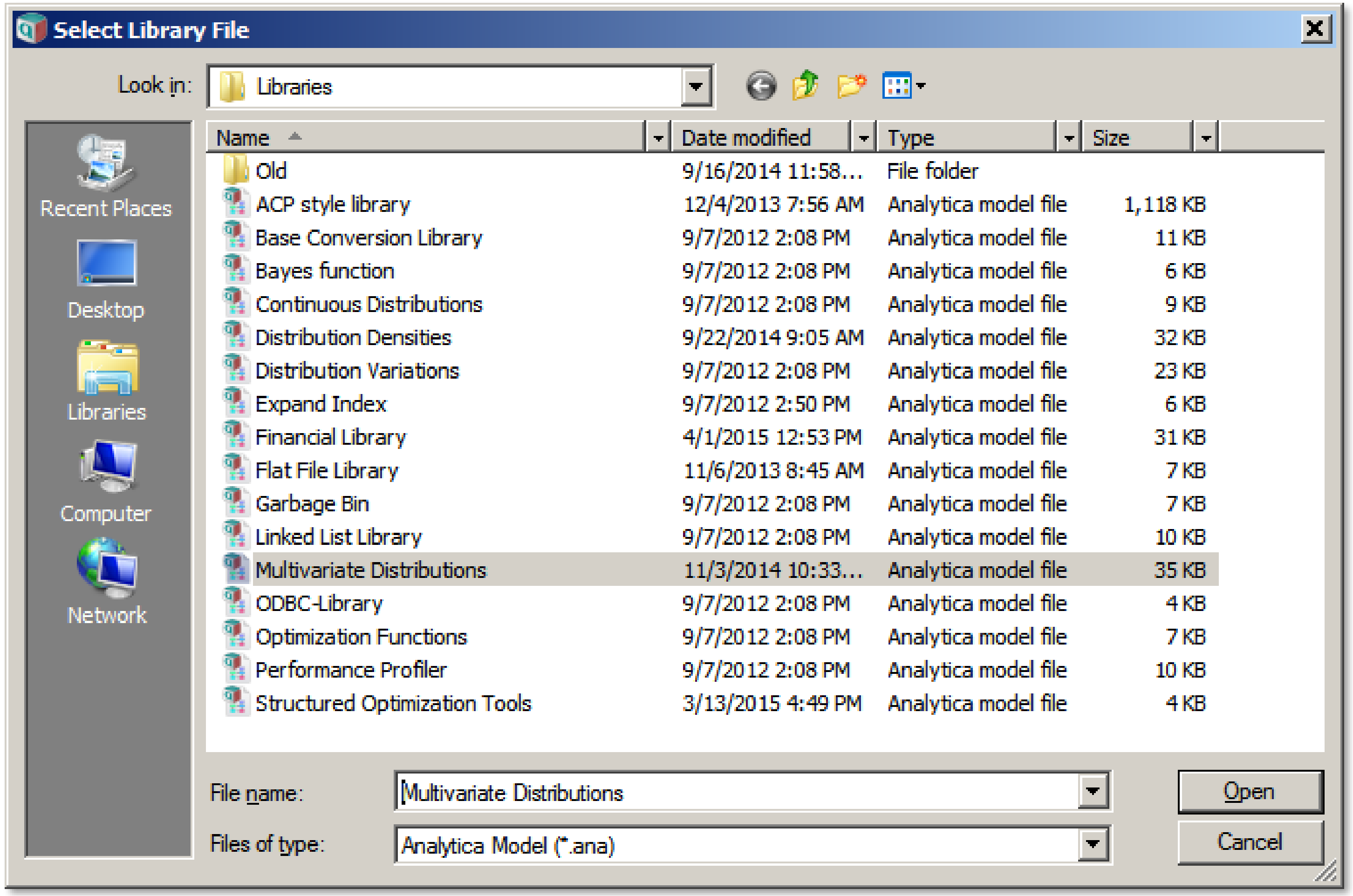
- **Importance** – calculates the absolute rank ordered correlation of an objective variable to its predecessor variables.
- **Tornado plots** – shows how an objective variable changes in response to a predecessor variable that moves from a low to a high position while all the other variables follow their natural variation.
- **Scatter plots** – shows the shape of the relationship of the probability sample of y against a comparison sample of x.
  - We'll look at these with our last example.

Other methods for studying sensitivity

- **Dydx(y, x)** – what is the slope of **y** with respect to **x** – i.e., the ratio of the change in **y** to change in **x**, for a small change in **x**?
- **Elasticity(y, x)** – what is the percent change in **y** due to a 1% change in **x**?

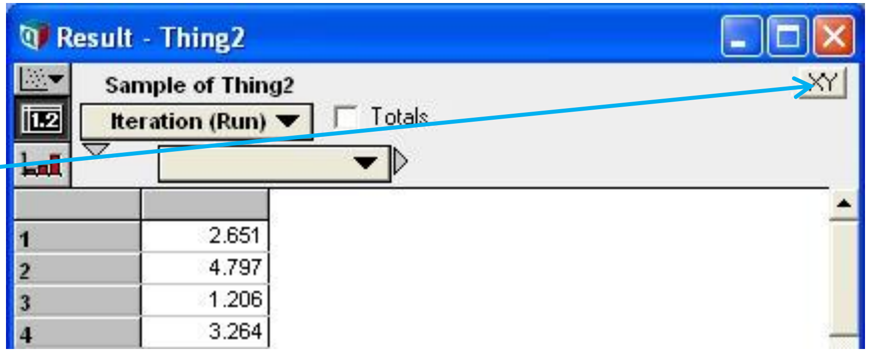
Multivariate Distributions library to set correlations among variables

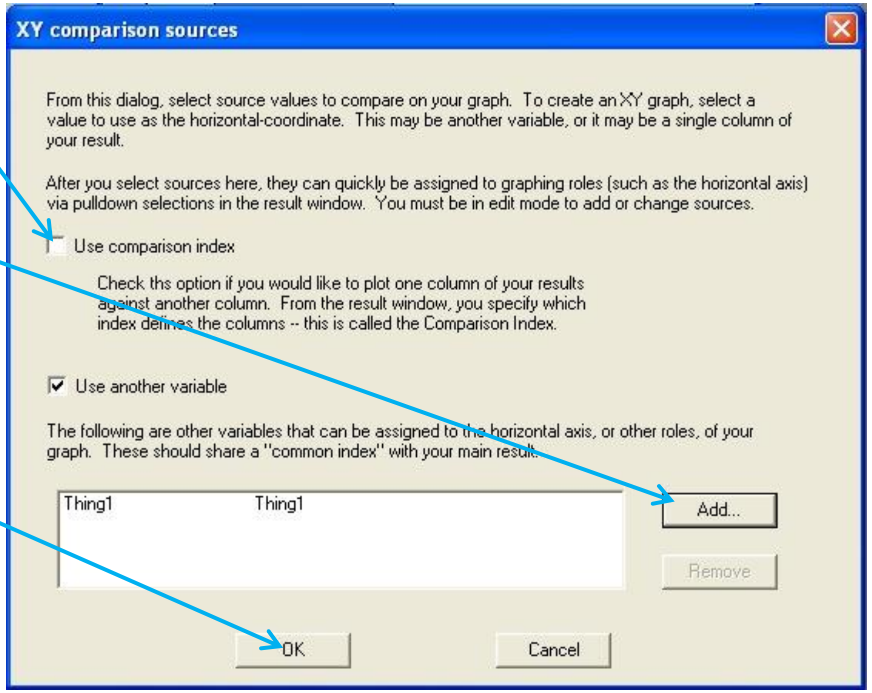
- Select **Add Library** from **File** menu



Scatter plots help visualize dependencies

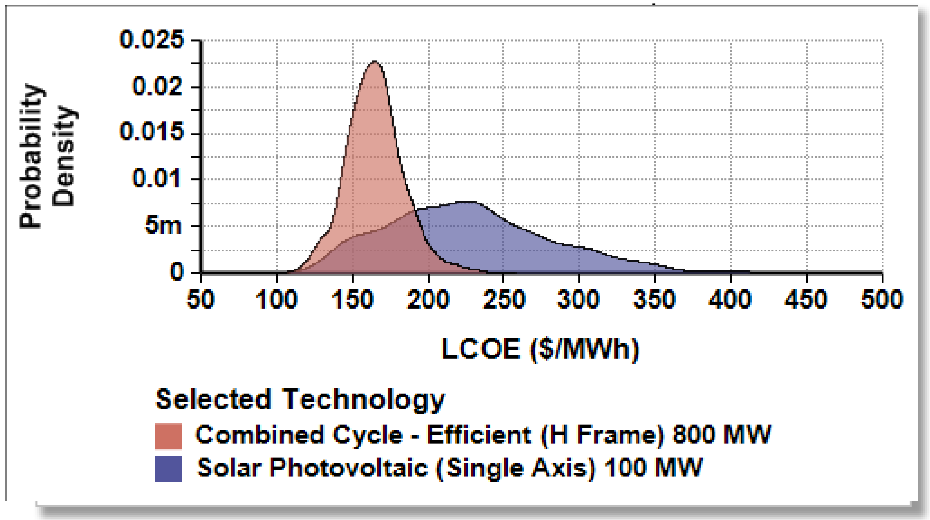
- Set up a **scatter plot** in the result window of a variable as dependent on another.
- Click the **XY** button while in edit mode.
- Check Use another variable.
- Click **Add…** and select the variable you want to compare. As the “X” or independent variable.
- Click **OK**.
- Use **Result/Graph Setup…** to show individual points (not Lines or Bars).



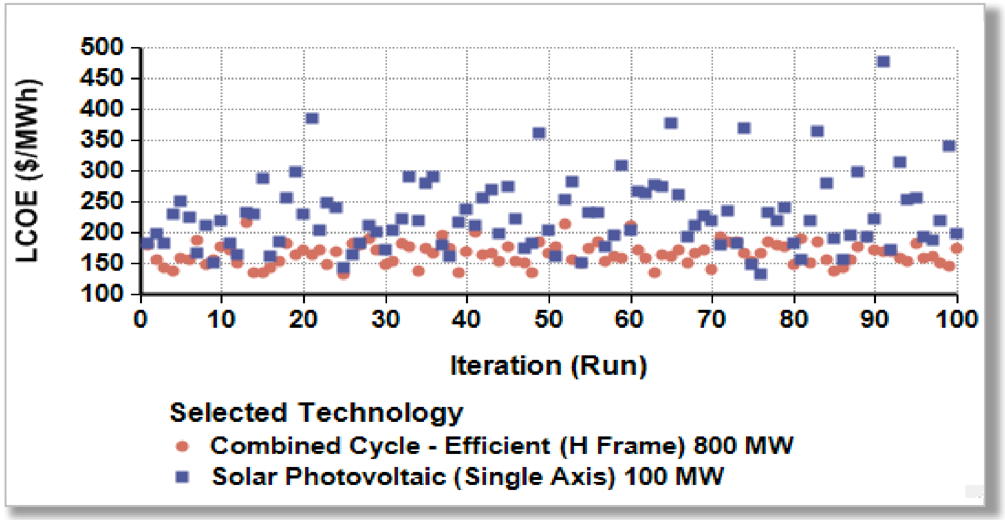


Ways to visualize uncertainty with probabilities

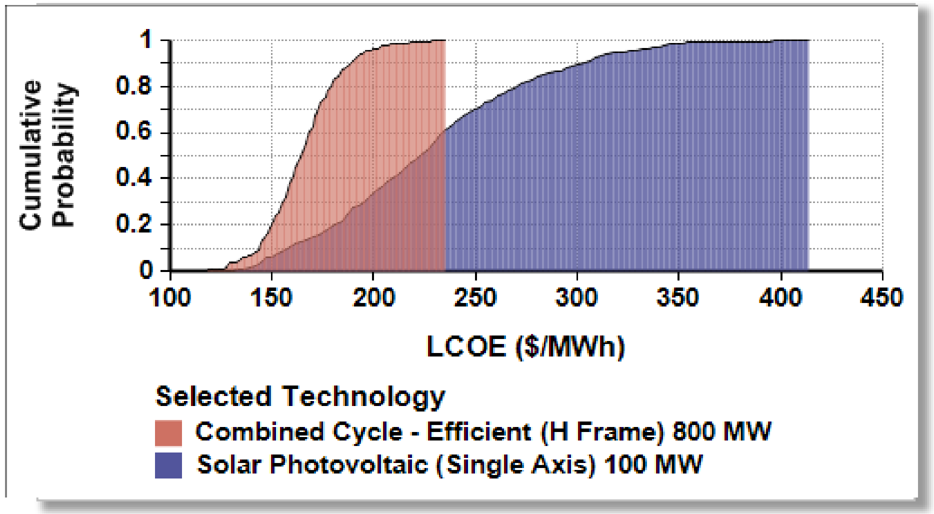
Probability density functions



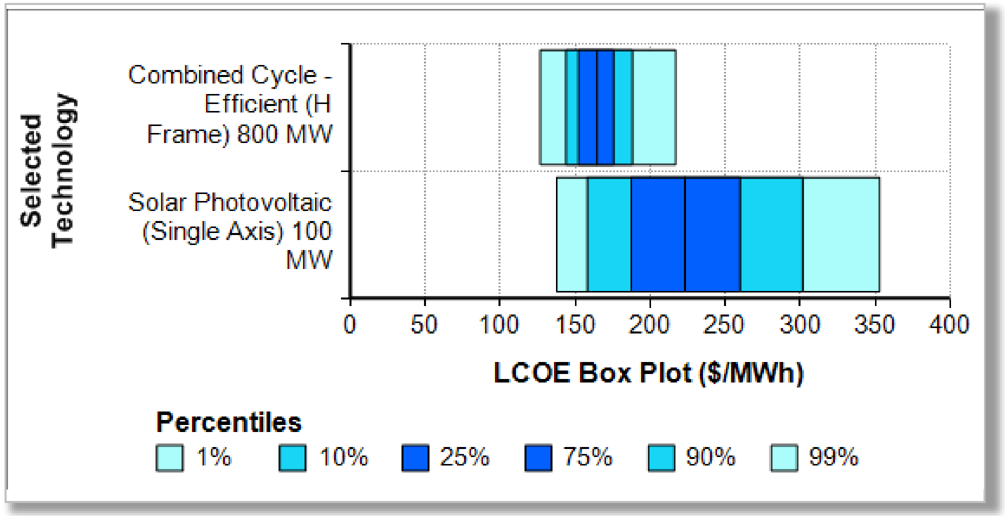
Random sample



Cumulative probability distributions



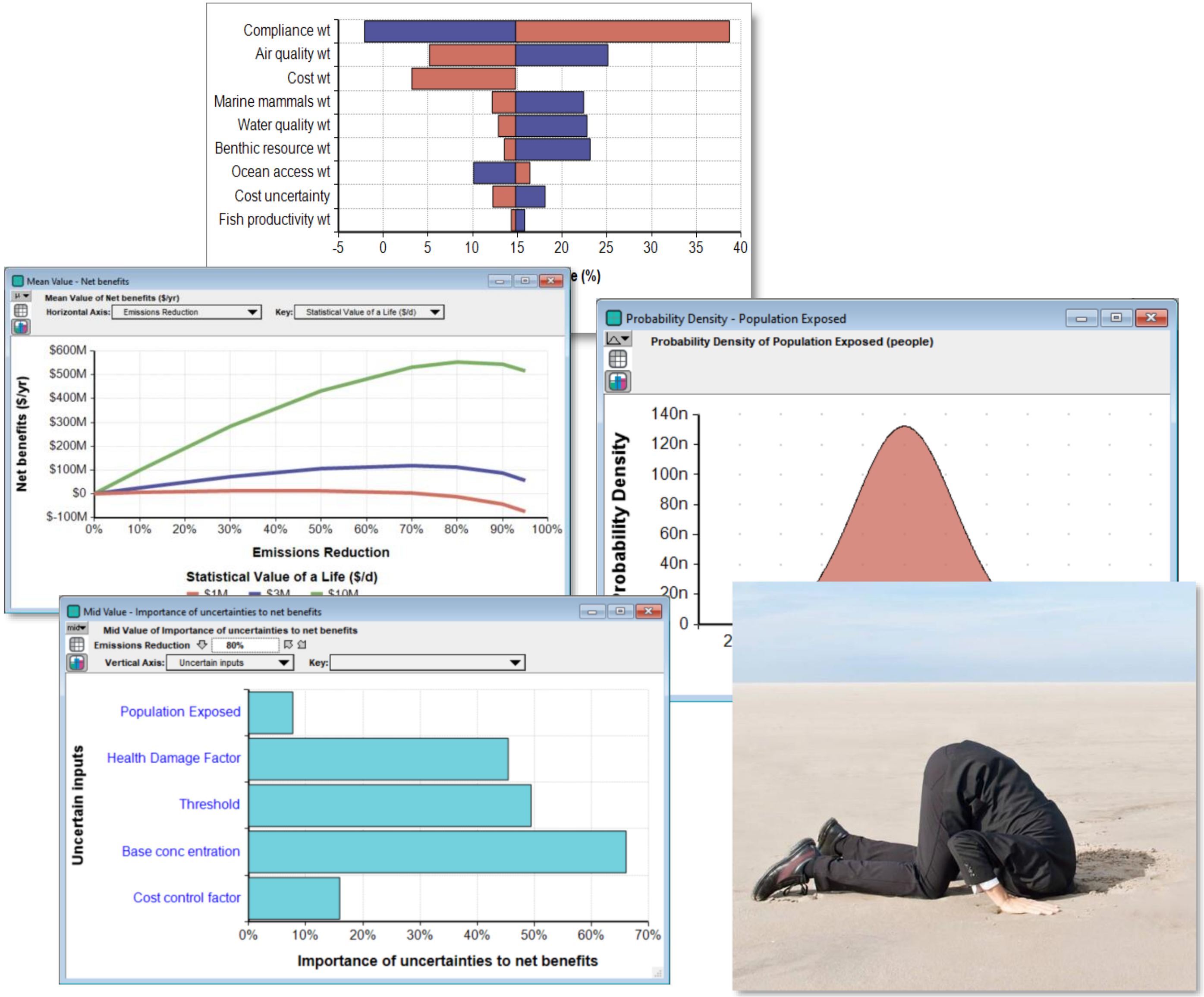
Probability bands or percentiles



Expert estimates in 2010 for generators built in 2020,

Approaches to express and analyze uncertainty

- Specify a range, low, mid, high
- Range sensitivity (tornado chart)
- Parametric analysis
- Express as probability distributions
- Importance analysis
- Or just ignore it …



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Q & A