An Anisotropic Failure Criterion for Masonry Suitable for Numerical Implementation

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The analysis of masonry structures built from a large number of units and joints can only be carried out with *macro-models*, in which a relation between average stresses and strains in the composite material is established. The effective constitutive behavior of masonry features anisotropy arising from the geometrical arrangement of units and mortar, even if the properties of these constituents are isotropic. Due to the difficulties of modeling anisotropic behavior, attempts to develop specific macro-models for the analysis of masonry structures have been few. To the knowledge of the authors, only Dhanasekar *et al.* (1985) and Seim (1994) - based on the work of Ganz (1989) - dealt with the implementation of a specific numerical model for masonry. The cited authors proposed rather complex yield surfaces which almost precludes robust numerical algorithms and, simultaneously, an accurate representation of inelastic behavior (hardening and softening). The absence of well established models hinders innovative applications of structural masonry and bounds the existing knowledge of masonry behavior.

The composite failure criterion proposed in this article is suitable for the modeling of anisotropic materials under plane stress conditions even if it comprehends only two individual failure criteria, respectively for tension and compression according to two different failure mechanisms. The former is associated with a localized fracture process, denoted by cracking of the material, and, the latter, is associated with a more distributed fracture process which is usually termed as crushing of the material.

THE COMPOSITE FAILURE CRITERION

A representation of an orthotropic yield surface in terms of principal stresses only is not possible. For plane stress situations a graphical representation in terms of the full stress vector (σ_x, σ_y, τ_{xy}) is necessary. The material axes are assumed to be defined by the bed joints direction (*x* direction) and the head joints direction (*y* direction). Other possible representation can be obtained in terms of principal stresses and an angle θ which measures the rotation between the principal stress axes and the material axes.

Basically, two different approaches for the macro-modeling of masonry can be used. The first approach is to describe the material behavior with a single failure criterion. The Hoffman criterion is quite flexible and attractive to use but yields a non-acceptable fit of the masonry experimental values as shown in Figure 1. A least squares fit of the experimental results from Page (1981,1983) with a Hoffman type[3] criterion turns out to show no tensile strength in uniaxial behavior. A manual fit through the different uniaxial strengths and the compressive failure obtained upon loading with $\sigma_1 = \sigma_2$ and $\theta = 0$ gives a very poor representation of the diagrams for the other θ values and a critical overestimation of strength in the tension-compression regime. A single surface fit of the experimental values would lead to a complex yield surface and extreme difficulties to describe the inelastic behavior. A second approach, which consists of an extension of conventional formulations for isotropic quasi-brittle materials to describe orthotropic behavior, will therefore be adopted for this study. Formulations of isotropic quasi-brittle materials behavior consider, generally, different inelastic criteria for tension and compression. In the present study, an extension of the work of Feenstra and de Borst (1996), who utilized this approach for concrete with a Rankine and a Drucker-Prager criterion, will be presented. In order to model orthotropic material behavior, a Hill type criterion for compression and a Rankine type[4] criterion for tension, see Figure 2, will be proposed.

Tension Mode

The Rankine criterion can be written as

$$f_1 = \frac{(\sigma_x - f_t) + (\sigma_y - f_t)}{2} + \sqrt{\left(\frac{(\sigma_x - f_t) - (\sigma_y - f_t)}{2}\right)^2 + \tau_{xy}^2} = 0 \qquad (1)$$

where f_t is the (isotropic) tensile strength. Setting forth a Rankine type yield surface for an orthotropic material, with

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[3] The word *type* is used here because the original authors assumed a 3-dimensional formulation. The plane stress yield surface adopted should be considered a case of the Tsai-Wu formulation.

[4] The word *type* is used here because the Rankine criterion represents the strength along the maximum principal stress. For an anisotropic material such definition is clearly not possible.

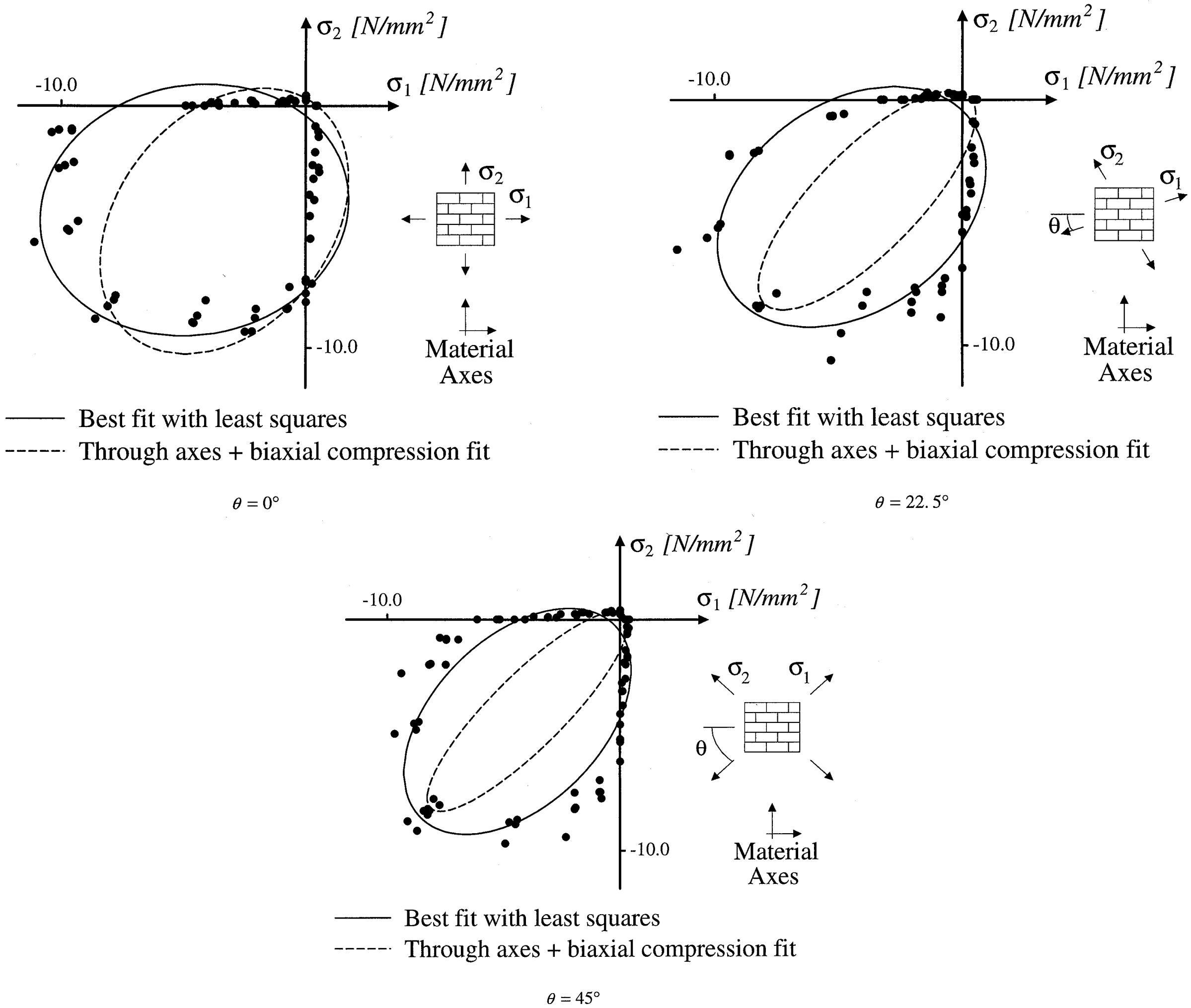

Figure 1—Comparison Between a Hoffman Type Criterion and Experimental Results from Page (1981,1983) (1 N/mm^2 = 145 lbf/in.^{2})

different tensile strengths along the *x*, *y* directions, see Figure 3, is now straightforward if Equation (1) is modified to

$$f_1 = \frac{(\sigma_x - f_{tx}) + (\sigma_y - f_{ty})}{2} + \sqrt{\left(\frac{(\sigma_x - f_{tx}) - (\sigma_y - f_{ty})}{2}\right)^2 + \alpha\tau_{xy}^2} = 0 \quad \textbf{(2)}$$

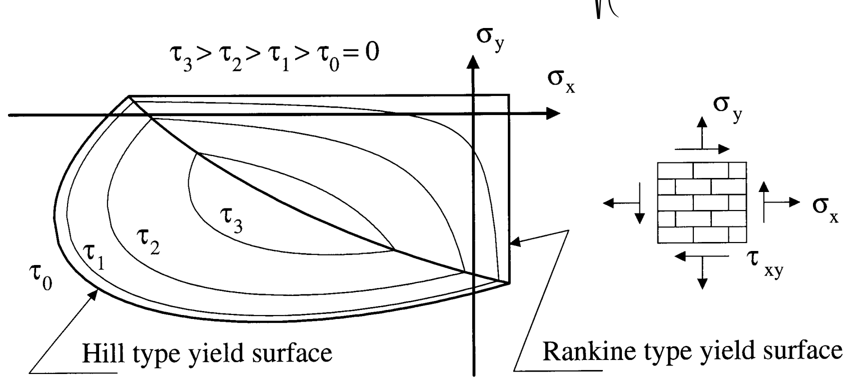

Figure 2—Proposed Composite Failure Criterion with Iso-Shear Stress Lines. Different Strength Values for Tension and Compression Along Each Material Axis (1 N/mm^2 = 145 lbf/in.^{2})

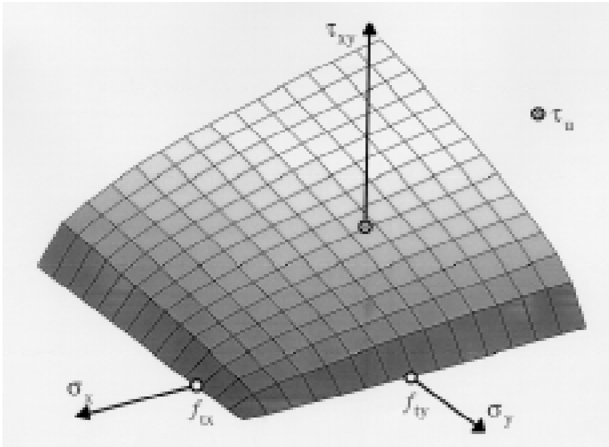

Figure 3—Orthotropic Rankine Type Yield Surface (Shown for t_{xy} ³ 0)

where f_{tx}, f_{ty} are the tensile strengths, respectively, along the x, y directions and the parameter a controls the shear stress contribution to failure. Note that the material axes are now fixed with respect to a specific frame of reference and it shall be assumed that all stresses are given in the material reference axes.

Compression Mode

The simplest yield surface that features different compressive strengths along the material axes is a rotated centered ellipsoid in the full plane stress space (s_x, s_y and t_{xy}), see Figure 4. The expression for such a quadric is

$$f_2 = A\sigma_x^2 + B\sigma_x\sigma_y + C\sigma_y^2 + D\tau_{xy}^2 - 1 = 0 \tag{3}$$

where A, B, C and D are four material parameters such that $B^2 - 4\,A\,C < 0$, in order to ensure convexity. This expression can be recast using clearer material parameters as

$$f_2 = \frac{f_{my}}{f_{mx}}\sigma_x^2 + \beta\sigma_x\sigma_y + \frac{f_{mx}}{f_{my}}\sigma_y^2 + \gamma\tau_{xy}^2 - f_{mx}\,f_{my} = 0 \tag{4}$$

where f_{mx}, f_{my} are the compressive strengths, respectively,

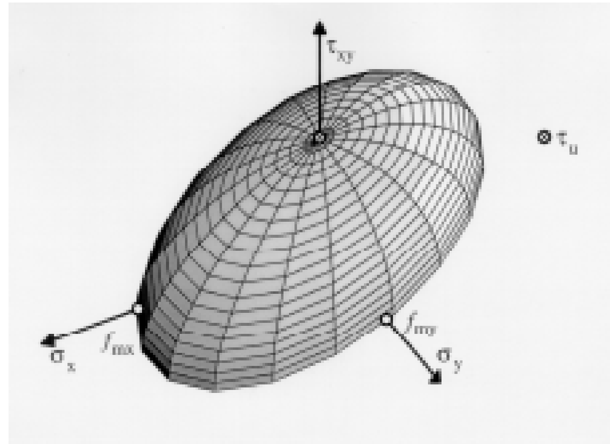

Figure 4—The Hill Type Yield Surface (Shown for t_{xy} ³ 0)

along the x, y directions, the parameter b controls the coupling between the normal stress values, i.e. rotates the yield surface around the shear stress axis, and the parameter g controls the shear stress contribution to failure.

DEFINITION OF THE PARAMETERS FROM EXPERIMENTS

Required Information to Define the Failure Criterion

The proposed failure criterion has seven strength parameters (f_{tx}, f_{ty}, f_{mx}, f_{my}, a, b and g). The first group of four strength parameters are the uniaxial tensile and compressive strengths along the material axes, which are natural due to the orthotropic behavior of masonry. This means that a complete characterization of the material model requires the tests shown in Figure 5. It suffices that these tests are performed under displacement control conditions to obtain extra parameters necessary to define the inelastic behavior. Additional tests are necessary to fully calibrate the failure criterion. Figure 6 illustrates three possible tests to determine the parameter a, which weights the shear stress

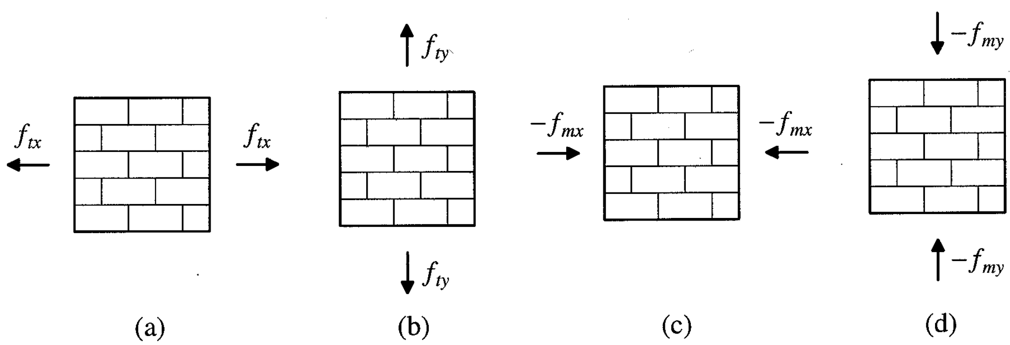

Figure 5—Natural tests to calibrate the failure criterion: uniaxial tension (a) parallel to the bed joints and (b) normal to the bed joints; uniaxial compression (c) parallel to the bed joints and (d) normal to the bed joints

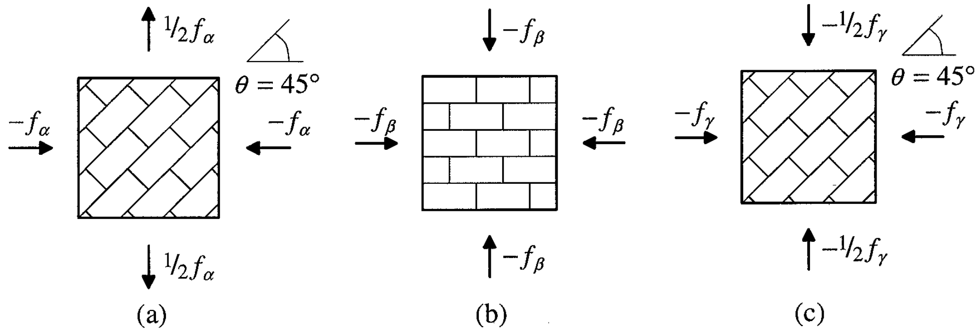

Figure 6—Possible non-standard tests to calibrate the composite model and calculate (a) parameter α, (b) parameter β and (c) parameter γ

contribution to tensile failure, parameter β, which controls the coupling between normal stress values in the case of compressive failure, and parameter γ which weights the shear stress contribution to compressive failure.

With these tests, the model parameters α, β and γ read

$$\alpha = \frac{4}{9}\left(1 + 4\frac{f_{tx}}{f_\alpha}\right)\left(1 + 4\frac{f_{ty}}{f_\alpha}\right)$$

$$\beta = \left[\frac{1}{f_\beta^2} - \frac{1}{f_{mx}^2} - \frac{1}{f_{my}^2}\right] f_{mx}\, f_{my} \quad \textbf{(5)}$$

$$\gamma = \left[\frac{16}{f_\gamma^2} - 9\left(\frac{1}{f_{mx}^2} + \frac{\beta}{f_{mx}\, f_{my}} + \frac{1}{f_{my}^2}\right)\right] f_{mx}\, f_{my}$$

Other loading paths can be adopted for the calculation of the parameters α, β and γ but a nice feature of the tests shown in Figure 6 is that they are equidistant from both material axes ($\sigma_x = \sigma_y$). The most important characteristic of the proposed non-standard tests is, however, their ability to trigger the correct failure regime. Figure 7 shows the typical position of all tests with respect to the composite failure criterion. The picture demonstrates the importance of selecting adequate tests to calculate parameters α and γ. The intersection of the tension and compression regimes is not known in advance and quite different shapes can be obtained for this intersection, see also the next section. For this reason, the tests suggested to calculate parameters α and γ include a lateral load capable of, in principle, leading to a failure mode distant from the corner regime. This has been checked for all the experimental results presented in the next section.

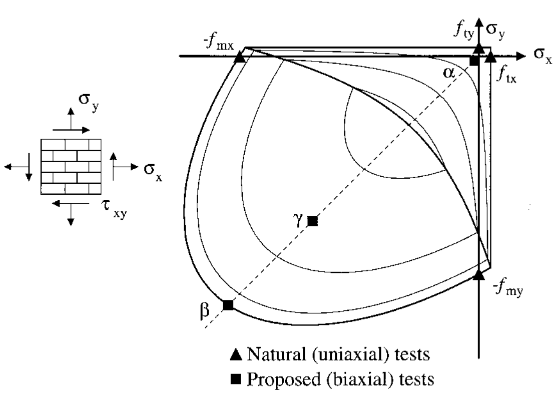

Figure 7—Typical position of the natural tests and proposed non-standard tests with respect to the composite model

It is stressed that, ideally, a large number of tests should be carried out for different loading paths so that the failure criterion can be determined resorting to the least squares method.

Comparison with Experimental Data of Masonry Strength

The ability of the proposed model to reproduce the strength of different masonry types is assessed next. A comparison with different experimental data is carried out, in which the material parameters are calculated with the least squares method. For the tension regime, the tensile and shear strength values are adopted, where the data are available.

The most complete strength data of biaxially loaded masonry is given by Page (1981,1983) who tested 102 panels of half-scale solid clay brick masonry, with dimensions 360 mm × 360 mm × 50 mm (14.2in.× 14.2in.× 2in.). The panels were loaded proportionally in the principal stress directions σ_1 and σ_2 along different orientations θ with respect to the material axes. The comparison between the experimental values and the model is given in Figure 8. Globally, good agreement is found. The uniaxial compressive strength parallel to the bed joints seems to be overpredicted by the model, see Figure 8 for $\theta = 0$, which is due to a debatable definition of failure in the experiments for these loading conditions (early splitting of the bed joints in tension), see Dhanasekar *et al.* (1985).

Figure 9 illustrates the shape of the composite failure criterion. The failure criterion features a low degree of anisotropy (f_{tx} / f_{ty} = 1.34 and f_{mx} / f_{my} = 1.09). The parameter α equals 1.26 which is close to the unitary Rankine value and the parameter β is about -1.0, which will be shown to be

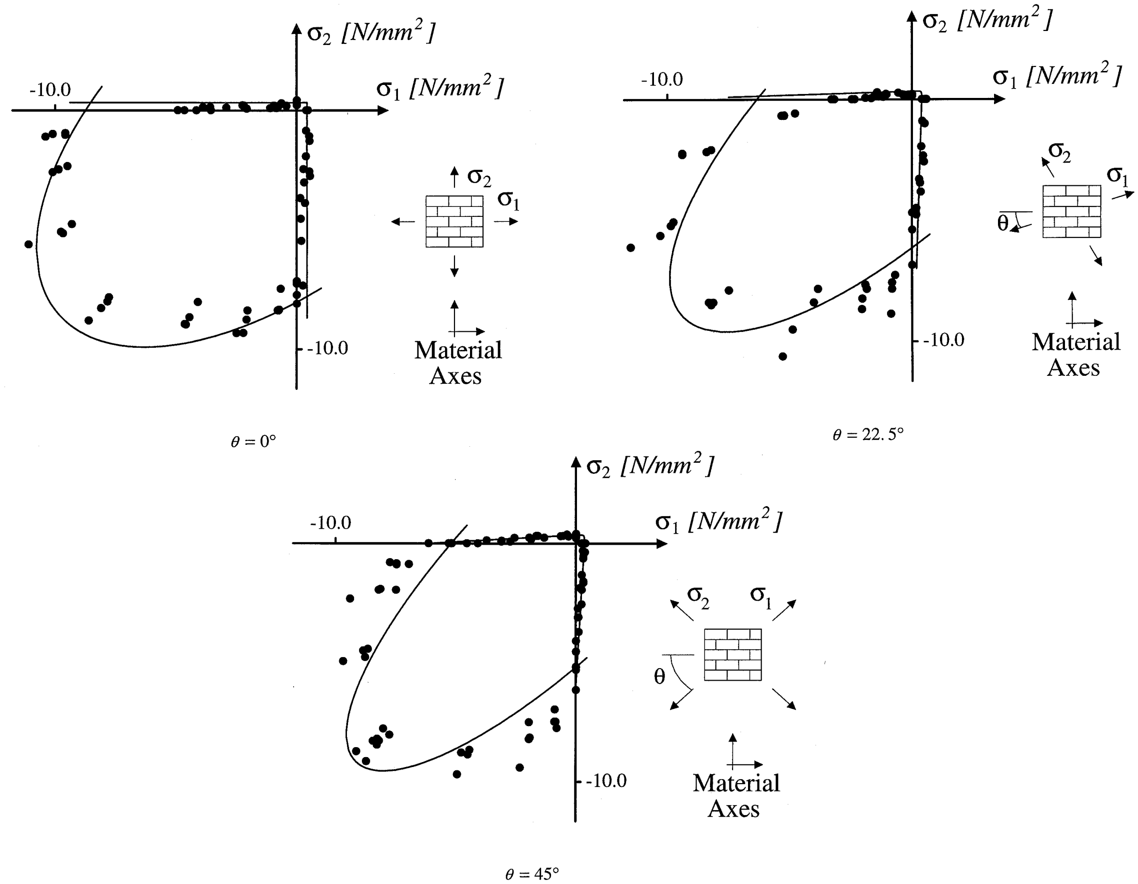

Figure 8—Comparison between the failure criterion and experimental results from Page (1981,1983). Material parameters: f_{tx} = 0.43 N/mm² (40.6 lbf/in²); f_{ty} = 0.32 N/mm² (46.4 lbf/in²); f_{mx} = 8.74 N/mm² (1,267.3 lbf/in²); f_{my} = 8.03 N/mm² (1,164.4 lbf/in²); α = 1.26; β = -1.17; γ = 9.59 (1 N/mm² = 145 lbf/in.²)

typical of masonry structures. The parameter γ is, however, very large (9.59) and unexpected, if compared to a (isotropic) von Mises model with a value of 3.0. This parameter seems to indicate a relatively small shear resistance for this particular type of masonry.

A smaller testing program of biaxially loaded masonry panels was carried out at ETH Zurich. The panels, with dimensions 1200 mm × 1200 mm × 150 mm (48 in. × 48 in. × 6 in.), were loaded proportionally in the principal stress directions σ_1 and σ_2 along different orientations θ with respect to the material axes as defined previously. The twelve panels of hollow clay brick masonry, denoted by panels K1 to K12 and reported by Ganz and Thürlimann (1982), are first considered. The test results, the proposed failure criterion results and the ratio between experimental and predicted failure are given in Table 1. Note that this ratio is obtained by calculating the intersection of the failure criterion with the given loading path. It is therefore a measure of the norm of the stress vector in the (σ_x, σ_y, τ_{xy})-space which equals $(\sigma_x + \sigma_y + \tau_{xy}^2)^{1/2}$. Panels K5 and K9 are not included because the boundary conditions affected the failure mode of panel K5 and panel K9 includes reinforcement.

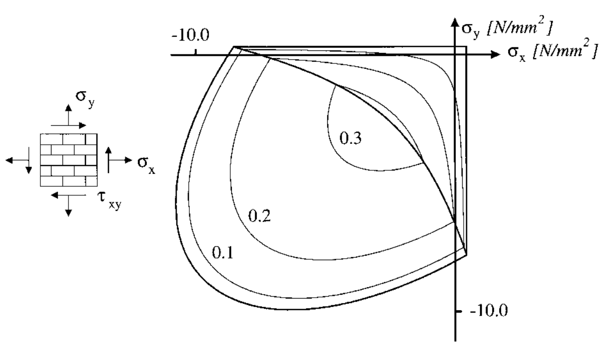

Figure 9—Calculated composite failure criterion for solid clay brick masonry of Page (1981,1983), with iso-shear stress lines. Contour spacing: 0.1 f_{my} (1 N/mm² = 145 lbf/in.²)

The failure criterion seems to be able to reproduce the strength behavior of this type of masonry with good accuracy. The large difference encountered for panel K8 is probably due to the critical loading path, with very low confining pressure in a practically no-tension material for the direction normal to the bed joints.

Table 1. Comparison between the failure criterion and experimental results from Ganz and Thürlimann (1982). Material parameters: f_{tx} = 0.28; f_{ty} = 0; f_{mx} = 1.87; f_{my} = 7.61, all in N/mm^2; a = 1.73; b = -1.05; g = 1.20 (1 N/mm^2 = 145 lbf/in.^{2})

Panel	Regime	s_1 / s_2	q(°)	Experimental Results			Failure Criterion			Ratio
				s_x N/mm^2	s_x N/mm^2	t_{xy} N/mm^2	s_x N/mm^2	s_x N/mm^2	t_{xy} N/mm^2	
K1	Tens.	-0.088	22.5	-0.08	-0.92	0.42	-0.09	-1.00	0.46	0.92
K2	Tens.	-0.050	22.5	-0.17	-1.42	0.62	-0.16	-1.32	0.58	1.07
K3	Comp.	0.000	0.0	0.00	-7.63	0.00	0.00	-7.61	0.00	1.00
K4	Comp.	0.000	90.0	-1.83	0.00	0.00	-1.87	0.00	0.00	0.98
K6	Tens.	0.000	45.0	-0.32	-0.32	0.32	-0.38	-0.38	0.38	0.84
K7	Tens.	0.000	22.5	-0.39	-2.25	0.93	-0.38	-2.22	0.92	1.01
K8	Tens.	0.000	67.5	-0.22	-0.04	0.09	-0.38	-0.07	0.16	0.58
K10	Comp.	0.328	0.0	-2.11	-6.44	0.00	-2.12	-6.47	0.00	1.00
K11	Comp.	0.306	22.5	-2.04	-4.49	1.23	-2.05	-4.51	1.23	1.00
K12	Comp.	0.304	45.0	-2.03	-2.03	1.08	-1.98	-1.98	1.06	0.98

Figure 10 illustrates the shape of the failure criterion and the position of the tests. The failure criterion features an extreme degree of anisotropy with zero tensile strength in the direction normal to the bed joints and a ratio f_{mx} / f_{my} equal to 4.06. This high ratio is due to the high perforation of the clay bricks. The parameter a equals 1.73 which is not quite close to the unitary Rankine value but the parameter b is still about -1.0. The parameter g is quite small (1.20) which is probably explained by the high degree of anisotropy, i.e. relatively high shear resistance due to the beneficial contribution of the strength f_{my}. It is noted that the "yield value" in Equation (4) represents an "average" value equal to f_{mx} ´ f_{my}. In opposition with solid clay brick masonry, see Figure 9, it appears that the tension regime represents the majority of the composite yield surface domain.

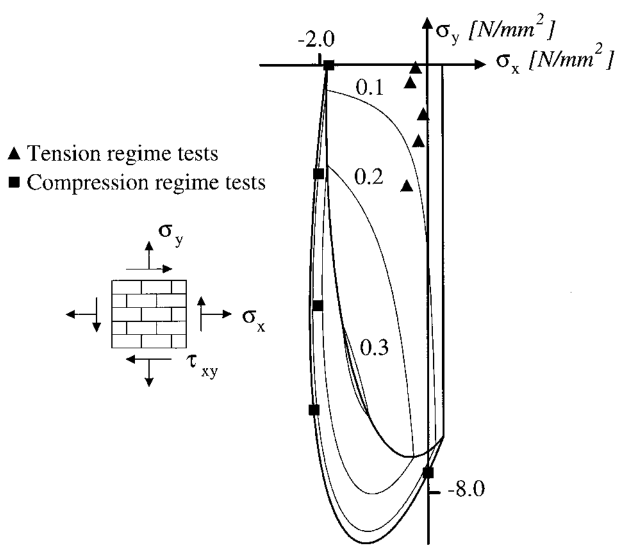

Figure 10—Calculated composite failure criterion for hollow clay brick masonry of Ganz and Thürlimann (1982), with iso-shear stress lines. Contour spacing: 0.1 f_{my} (1 N/mm^2 = 145 lbf/in.^{2})

The nine panels of hollow concrete block masonry, denoted by panels ZSW1 to ZSW9 and tested by Lurati *et al.* (1990) as a part of the ETH Zurich program, are considered next. The test results, the proposed failure criterion results and the ratio between experimental and predicted values are given in Table 2. Panel ZSW3 is not considered because the head joints were not filled. Again, the model is able to capture the strength behavior of this type of masonry. For the tensile regime only three tests are available and, therefore, the failure criterion fit is exact. It is noted that a basically no-tension material is obtained and the parameter a equals the unit value, i.e. (isotropic) Rankine is recovered. The failure criterion features a reasonable degree of anisotropy in compression with f_{mx} / f_{my} = 1.59. The parameter b is again about -1.0 and the parameter g which equals 3.36, is approximately equal to the von Mises value. Figure 11 illustrates the shape of the composite failure criterion and the position of the tests.

The no-tension material was obtained from only three tests and is, of course, physically debatable. In particular, if a material does not possess any tensile strength, a proportional loading path from the origin to calibrate the tension regime is a neutral path and, experimentally, cannot be stable.

CONCLUSIONS

A novel failure criterion to represent the behavior of masonry under plane stress conditions has been proposed. It has been shown that the model can represent equally well different masonry types, namely solid and hollow brick masonry or hollow concrete masonry.

Table 2. Comparison between failure criterion and experimental results from Lurati *et al.* (1990). Material parameters: f_{tx} = 0.01; f_{ty} = 0.00; f_{mx} = 5.78; f_{my} = 9.21, all in N/mm² *a* = 1.00; *b* = -0.97; *g* = 3.36 (1 N/mm² = 145 lbf/in.²)

Panel	Regime	s_1 / s_2	q(°)	Experimental Results			Failure Criterion			Ratio
				s_x	s_x	t_{xy}	s_x	s_x	t_{xy}	
				N/mm²	N/mm²	N/mm²	N/mm²	N/mm²	N/mm²	
ZSW1	Comp.	0.000	0.0	0.00	-9.12	0.00	0.00	-9.21	0.00	0.99
ZSW2	Comp.	0.136	0.0	-6.12	-0.83	0.00	-6.01	-0.82	0.00	1.02
ZSW4	Comp.	1.527	0.0	-5.98	-9.13	0.00	-5.81	-8.86	0.00	1.03
ZSW5	Tens.	0.000	45.0	-3.06	-3.06	3.06	-3.06	-3.06	3.06	1.00
ZSW6	Comp.	0.222	45.0	-4.60	-4.60	2.93	-4.51	-4.51	2.87	1.02
ZSW7	Comp.	1.000	45.0	-6.12	-6.12	0.00	-6.51	-6.51	0.00	0.94
ZSW8	Tens.	0.000	67.5	-2.34	-0.40	0.97	-2.34	-0.40	0.97	1.00
ZSW9	Tens.	0.000	22.5	-0.97	-5.66	2.35	-0.97	-5.66	2.35	1.00

Results of the application of the proposed failure criterion for the nonlinear analysis of masonry structures can be found in Lourenço (1996). There, the model is implemented according to the modern plasticity framework and successfully validated against experimental results in masonry shear walls.

ACKNOWLEDGEMENTS

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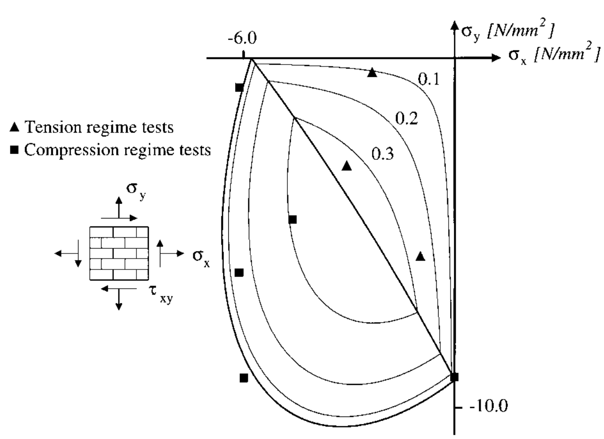

Figure 11—Calculated composite failure criterion for hollow concrete block masonry of Lurati *et al.* (1990), with iso-shear stress lines. Contour spacing: 0.1 f_{my} (1 N/mm² = 145 lbf/in.²)

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NOTATIONS

f_1 = yield function in tension.
f_2 = yield function in compression.
f_{mx} = compressive strength in the *x* direction.
f_{my} = compressive strength in the *y* direction.

f_t = isotropic tensile strength.

f_{tx} = tensile strength in the *x* direction.

f_{ty} = tensile strength in the *y* direction.

s_x = normal stress along the *x* axis.

s_y = normal stress along the *y* axis.

t_{xy} = shear stress (acting along the x axis in a plane normal to the *y* axis).

q = rotation angle between the principal stress axes and the material axes.

s_1 = maximum principal stress.

s_2 = minimum principal stress.

a = material parameter that controls the shear contribution to tensile failure.

b = material parameter that controls the rotation of the compressive part of the yield surface around the shear axis.

g = material parameter that controls the shear contribution to tensile failure.