Polarization Entanglement Storage in Ensemble-Based Atomic Memories

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Quantum communication exploits quantum mechanical resources, such as entanglement, to achieve tasks unrealizable by classical means, such as accurate teleportation of quantum states and unconditionally secure public key distribution. For such applications, the fundamental problem of quantum communication is the distribution, over optical channels, of entanglement between distant nodes. However, the fidelity of entanglement between such nodes decreases exponentially with channel length. The possibility of creating *scalable* optical quantum networks requires that we overcome this difficulty by storing and processing quantum information locally in a quantum memory, first, as a repeater increasing network scalability, and second, as a light-matter interface to a quantum computer [Kim08].

The aim of this thesis is to address the following open problem: the theoretical limits of atomic-ensemble quantum memories that store polarization entanglement. It is part of a larger research program in RLE's Optical and Quantum Communications Group investigating the system performance of ensemble-based hybrid systems in quantum communication. The following briefly describes a preliminary model of a quantum memory used in communication with attention paid to the formalism of our treatment, specifically the marriage of collective interactions of atomic ensembles with quantum light fields, and the Gaussian state analysis of entanglement. We will also relate pre-existing memory and repeater architectures to this work, and discuss in greater detail our specific formulation of the problem.

1 Collective Interactions and Entanglement Fidelity

An illustration of a model quantum communication system is shown in Fig. 1, in the case of a single-trapped atom. Through type-II parametric downconversion, a maximally-entangled state can be produced of the form,

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|\sigma_+\rangle_1 |\sigma_-\rangle_2 + e^{i\phi} |\sigma_-\rangle_1 |\sigma_+\rangle_2\right) \tag{1}$$

where σ_+ (σ_-) indicates right (left) circular polarization, and ϕ is a phase offset. An arbitrary polarization of any photon entering the cavity can be stored in the basis of right and left circular polarizations, such that

$$|\psi_2\rangle = \alpha\,|\sigma_+\rangle + \beta\,|\sigma_-\rangle\,. \tag{2}$$

Through a Raman Λ-type interaction, a signal or idler photon effectively transfers its entanglement to the degenerate B magnetic hyperfine levels, and subsequently to D through a coherently-driven transition. The extra advantage we get from using an ensemble is that a collective atomic state is produced by a single excitation, i.e., we don't know which atom has been excited from its ground state $|g\rangle$ at A to the metastable state $|s\rangle$ at D.

To illustrate this for an atomic ensemble, consider N_a atoms prepared in their ground states, a collective state denoted by $|0\rangle_a = |g\rangle^{\otimes N_a}$. Coherently pumping the ensemble results in an inelastic Raman scattering event that is collectively enhanced by constructive interference within the ensemble [Dic54]. The resulting forward-scattered Stokes light results from coherent spontaneous emission in the ensemble, and the correlated ensemble excitation is a collective spin state,

$$|1\rangle_a = \hat{S}^\dagger\,|0\rangle_a = \frac{1}{\sqrt{N_a}}\sum_{i=1}^{N_a} |g\rangle_1 \cdots |s\rangle_i \cdots |g\rangle_{N_a}\,. \tag{3}$$

where $\hat{S} = (1/\sqrt{N_a})\sum_i |g\rangle_i \langle s|$. Because the excitation is composed of many atoms, the collective spin excitation is protected against the loss of individual atoms in the ensemble, increasings its robustness for storage. In the weak interaction limit, in which most of the atoms remain in their ground state, the spin excitation $\hat{S}$ is effectively a ladder operator, as $[\hat{S}, \hat{S}^\dagger] = \sum_i \left(|g\rangle_i \langle g| - |s\rangle_i \langle s|\right)/N_a \approx 1$, and the outgoing Stokes light and spin excitation are in a two-mode squeezed state [DLCZ01].

The input field in this interaction is one of a pair of polarization-entangled photons generated by the interference of a pair of optical parametric amplifiers (OPAs) as shown in Fig. 1. The output state of this process is given by expanding out the number ket representations of the OPAs to first order,

$$\begin{aligned}
|\psi\rangle_{\mathrm{SI}} &= \sum_n \sqrt{\frac{\bar{N}}{(\bar{N}+1)^{n+1}}}\,|n\rangle_{S_x}|n\rangle_{I_y} \otimes \sum_n (-1)^n \sqrt{\frac{\bar{N}}{(\bar{N}+1)^{n+1}}}\,|n\rangle_{S_y}|n\rangle_{I_x} \\
&\approx \frac{1}{\bar{N}+1}\,|0\rangle_{S_x}|0\rangle_{I_y}|0\rangle_{S_y}|0\rangle_{I_x} + \sqrt{\frac{\bar{N}}{(\bar{N}+1)^3}}\Big(|1\rangle_{S_x}|1\rangle_{I_y}|0\rangle_{S_y}|0\rangle_{I_x} - |0\rangle_{S_x}|0\rangle_{I_y}|1\rangle_{S_y}|1\rangle_{I_x}\Big) \\
&= |\mathrm{vac}\rangle + \sqrt{\frac{\bar{N}}{(\bar{N}+1)^3}}\left(|0\rangle_T|1\rangle_R - |1\rangle_T|0\rangle_R\right),
\end{aligned} \tag{4}$$

where $\bar{N}$ is the average photon number per mode; T and R denote the transmitter and receiver, respectively; and $|0\rangle_R = |1\rangle_{S_x}|0\rangle_{S_y}$, $|1\rangle_T = |0\rangle_{S_x}|1\rangle_{S_y}$, $|0\rangle_R = |1\rangle_{I_x}|0\rangle_{I_y}$, and $|1\rangle_R = |0\rangle_{I_x}|1\rangle_{I_y}$. Following measurement post-selection, this state is a maximally entangled singlet state of the form in Eqn. 1 [Sha02], and expansions to higher orders account for multiple-pair effects. A useful property of $|\psi\rangle_{\mathrm{SI}}$ is that its anti-normally ordered characteristic function is a zero-mean, jointly Gaussian distribution that remains invariant under linear transformations. Its joint density operator is $\hat{\rho}_{SI} = \hat{\rho}_{S_x I_y} \otimes \hat{\rho}_{S_y I_x}$, whose anti-normally ordered characteristic functions are given by

$$\begin{aligned}\chi_A^{\rho_{S_x I_y}}(\zeta_S, \zeta_I) &= \langle e^{\zeta_S^* \hat{a}_{S_x} + \zeta_I^* \hat{a}_{I_y}} e^{\zeta_S \hat{a}_{I_y}^\dagger + \zeta_S \hat{a}_{I_y}^\dagger} \rangle \\ &= e^{-\left(1+\bar{N}\right)\left(|\zeta_S|^2 + |\zeta_I|^2\right) + 2\bar{N}\mathrm{Re}(\zeta_S \zeta_I)}\end{aligned} \tag{5}$$

and

$$\begin{aligned}\chi_A^{\rho_{S_y I_x}}(\zeta_S, \zeta_I) &= \langle e^{\zeta_S^* \hat{a}_{S_y} + \zeta_I^* \hat{a}_{I_x}} e^{\zeta_S \hat{a}_{S_y}^\dagger + \zeta_I \hat{a}_{S_x}^\dagger} \rangle \\ &= e^{-\left(1+\bar{N}\right)\left(|\zeta_S|^2 + |\zeta_I|^2\right) - 2\bar{N}\mathrm{Re}(\zeta_S \zeta_I)},\end{aligned} \tag{6}$$

which contain all orders of $|\psi\rangle_{\mathrm{SI}}$. Following a linear transformation, the output state $\hat{\rho}_{\mathrm{out}}$ can be determined by taking inverse transform of the output characteristic function. In a memory or teleportation architecture, we want the output, represented by a pure or mixed state $\hat{\rho}_{\mathrm{out}}$, to have the highest possible fidelity with respect to its input state $\hat{\rho}_{\mathrm{in}}$. The trace separation quantifies this fidelity as $F(\hat{\rho}) = \mathrm{Tr}\sqrt{\sqrt{\hat{\rho}_{\mathrm{out}}}\hat{\rho}_{\mathrm{in}}\sqrt{\hat{\rho}_{\mathrm{out}}}}$, which reduces to a projection overlap $\sqrt{\langle\psi_{\mathrm{in}}|\, \hat{\rho}_{\mathrm{out}}\, |\psi_{\mathrm{in}}\rangle}$ when the input is the pure state $\hat{\rho}_{\mathrm{in}} = |\psi_{\mathrm{in}}\rangle \langle\psi_{\mathrm{in}}|$.

1.1 Pre-Existing Architectures

1.1.1 Cavity Quantum Memories (MIT-NU) and Coherently-Driven Atomic Ensembles (DLCZ)

Our analysis merges the approaches of trapped single atoms in cavity quantum electrodynamics (QED) proposed by MIT and Northwestern University (MIT-NU) and the ensemble-based repeater architecture proposed by Duan, Lukin, Cirac, and Zoller (DLCZ) [LSSH01] [DLCZ01]. The MIT-NU and DLCZ protocols both utilize spontaneous Raman transitions to mediate atomic storage. Whereas the former has the advantage of storing externally generated entanglement and verifying its success through a cycling resonant fluorescence transition, it is prohibitively difficult to implement because of the strong coupling requirements in cavity QED.

In contrast, the DLCZ protocol creates a coherent atomic excitation, as in Eqn. 3, not by an external input photon, but by the ensemble itself interacting with a classical (write) field. The entangled state is generated probabilistically (but heralded) through postselection and measurement quantum interference, as shown in Fig. 2. In the ideal case of low excitation probability, a photodetection at either of the two detectors projects the two ensembles into a maximally-entangled singlet state of excitations of the form in Eqn. 1. Although scalably resilient to issues that might plague such protocols, such as propagation loss and photodetector dark counts, DLCZ requires stable phase coherence and number-resolving photodetectors, neither of which are easy to implement in practice. By enabling the storage of externally-generated entanglement in a DLCZ-type protocol, we will address new error models for entanglement fidelity in quantum memories.

1.1.2 Stimulated Raman Transitions and Electomagnetically Induced Transparency (EIT)

As an aside, it is worth noting a competing approach to photon storage that may serve as a useful comparison in the future, namely, the usage of stimulated Raman transitions and electromagnetically induced transparency (EIT) to increase the coupling of an input quantum field with an atomic ensemble [FL02] [Luk03] [GAF+07] [GALS07]. In this approach, an external coherent control field couples the $|e\rangle - |s\rangle$ transition in a Λ-type atom, adiabatically reducing the group velocity of a single photon wavepacket and trapping it within the ensemble. Such an approach is deterministic, with high throughput, but admits neither easy verification (as in MIT-NU) nor heralding (as in DLCZ). Like the DLCZ protocol above, its sensitivity to externally generated entanglement is an open question.

2 Plan of Attack

Using Gaussian state outputs from the OPA's, we can quantify fidelity loss in entanglement distribution by accounting for multiple-pair effects, fiber propagation loss, photodetection limitations, and phase mismatch between atomic ensembles. We have already demonstrated the validity of a Gaussian state analysis for finite atomic ensembles, and are currently working to describe the input-output behavior of an cavity-confined atomic ensemble in terms of an linear time-invariant (LTI) state-space transformation.

2.1 Proposed Solution

2.1.1 DLCZ with Quantum Field Inputs

Our first task is to abstract a model for the interaction of input quantum field into our sample quantum memory. A basis for this model is inspired by recent experimental work on heralded single-photon atomic memories and interfaces from [STTVac07] [TGS+09], which utilized two spatially-overlapping atomic ensembles to absorb arbitrarily polarized single photons. Heralding was observed (although rare, at rate of 10^{-6}, using pulsed coherent states ($\bar{N} \approx 500$) with an absorption probability $\alpha = 0.01$. Despite operating in an effective single-photon regime, multiple photon inputs were still present, a problem we wish to analyze in the case of a parametric downconverter input. We consider an ensemble of Λ-type atoms confined in a single-sided, low-finesse ring cavity, as shown in Fig. 3. The $|g\rangle - |e\rangle$ and $|e\rangle - |s\rangle$ transitions are coupled to the cavity modes $\hat{a}$ and $\hat{b}$, respectively, each with coupling coefficient g_c. Under the rotating wave approximation, the interaction Hamiltonian for the collective interaction process is given by

$$\hat{H} = \hbar\Gamma \left(\hat{a}\hat{S}^\dagger\hat{b}^\dagger + \hat{b}\hat{S}\hat{a}^\dagger \right) \tag{7}$$

where $\Gamma = g_c^2 N_a/\Delta$ (Δ is the detuning from the two-photon resonance). The input output expressions for single-sided optical cavities with decay rate κ and input states $\hat{a}_{\text{in}}$ (downconverter) and $\hat{b}_{\text{in}}$ (in vacuum) are

$$\begin{aligned} \hat{a}_{\text{out}}(t) &= \sqrt{\kappa}\hat{a}(t) - \hat{a}_{\text{in}}(t) \\ \hat{b}_{\text{out}}(t) &= \sqrt{\kappa}\hat{b}(t) - \hat{b}_{\text{in}}(t), \end{aligned} \tag{8}$$

and the equations of motion for the internal state operators are,

$$\begin{aligned} \frac{d\hat{a}}{dt} &= -i\Gamma\hat{S}\hat{b} - \frac{\kappa}{2}\hat{a} + \sqrt{\kappa}\hat{a}_{\text{in}}(t) \\ \frac{d\hat{b}}{dt} &= -i\Gamma\hat{S}^{\dagger}\hat{a} - \frac{\kappa}{2}\hat{b} + \sqrt{\kappa}\hat{b}_{\text{in}}(t) \\ \frac{d\hat{S}^{\dagger}}{dt} &= i\Gamma\hat{b}\hat{a}^{\dagger}. \end{aligned} \tag{9}$$

In principle, Eqns. 8 and 9 are all that are needed to determine $\hat{a}_{\text{out}}(t)$ and $\hat{b}_{\text{out}}(t)$. In practice, Eqn. 9 is an operator-valued system of nonlinear differential equations.

Our ultimate goal is to find a linearized, approximate solution consistent with the operation of a quantum memory. Furthermore, such a solution must be consistent with the semiclassical limit of coherent pumping at the $|g\rangle - |e\rangle$ transition, which reduces Eqn. 7 to two-mode parametric amplification between the collective mode $\hat{S}$ and the outgoing Stokes light $\hat{b}_{\text{out}}$. There are a variety of approaches for linearizing these expressions, including, but not limited to, short-time Taylor series approximation, weak field approximation, and symmetrization using Lie group representations [DSI06]. Absent a solution (for now), we will likely try a multi-port beamsplitter *anzatz* expressing $\hat{b}_{\text{out}}$ as a function of the state operators. This model will be combined with a teleportation architecture based on the DLCZ and MIT-NU approaches for our final fidelity analysis.

3 Summary

In summary, we have proposed an approach for addressing entanglement storage in ensemble-based atomic memories, particularly errors arising from multiple pair effects in entanglement generation.

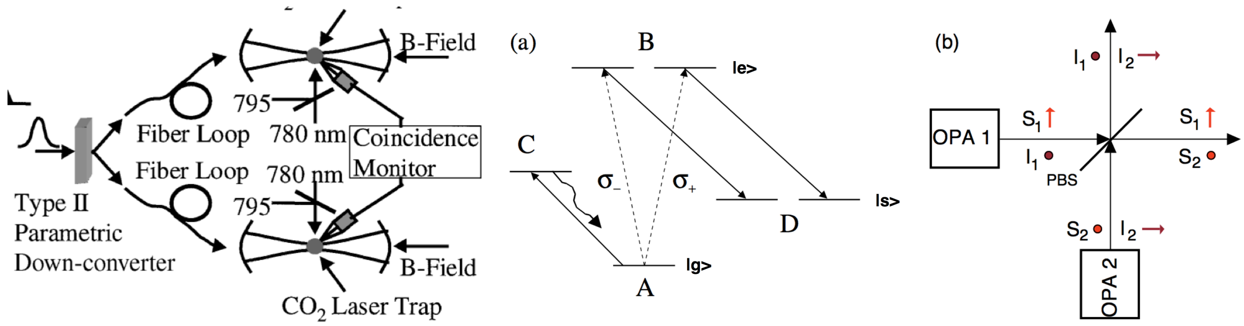

Figure 1: Components of a quantum repeater node in the MIT-NU architecture. (a) Parametric downconversion creates pairs of polarization-entangled photons, sending the idler photon to ensemble 1 and the signal photon to ensemble 2. Each trap contains a single ultra-cold rubidium atom cooled to its hyperfine ground state. In the energy level diagram, the AB-transition absorbs 795nm photons, and the BD-transition is coherently driven, thereby enabling storage at D. (b) Polarization-entangled photon pairs generated by a pair of two-mode optical parametric amplifiers (OPAs) and a beamsplitter (PBS). The polarizations $\hat{x}$ and $\hat{y}$ are denoted by arrows and bullets, respectively. Figures taken from [LSSH01] and [SW00].

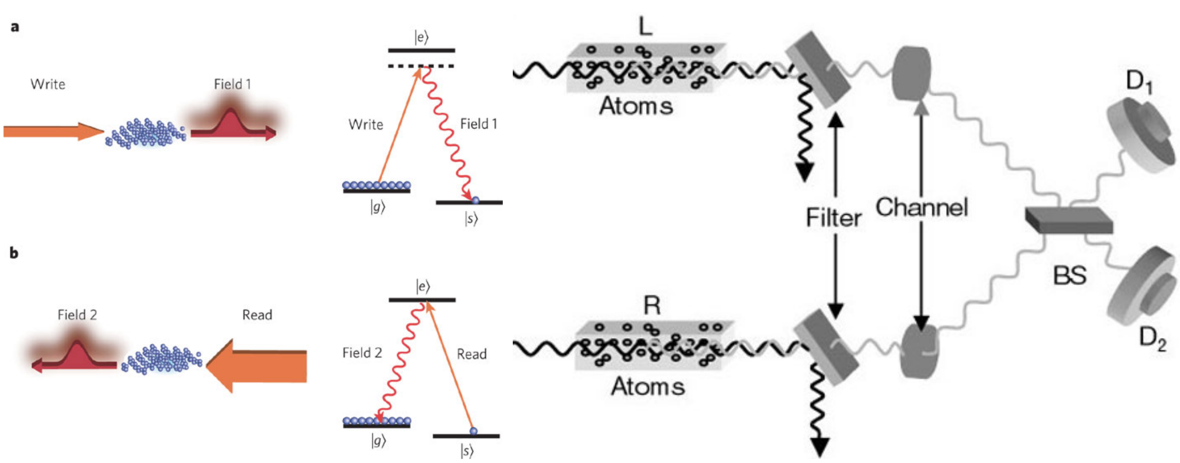

Figure 2: Entanglement with the DLCZ protocol. (Left) Weak laser pulses induce reading and writing through spontaneous Raman transitions. (Right) Measurement-induced interference results in a single-excitation entangled state. Figures taken from [Kim08] and [DLCZ01].

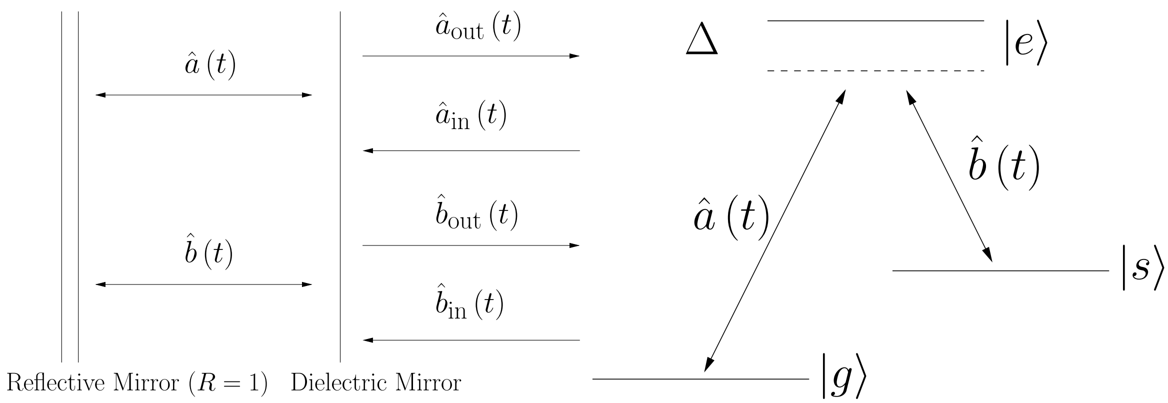

Figure 3: DLCZ with quantum field inputs. (Left) Input-output formalism for a single-sided, two-mode ring cavity. (Right) Interaction in a three-mode parametric amplifier

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