System of Equations Word Problems - Sample Worksheet

MathHub.Club

Name: _____

Show all work for full credit. Use seperate sheet of paper if needed.

Juice Blend Problem

A local juice bar is planning to introduce a special blend of juice that combines their premium and regular juices. The premium juice is priced at $0.72 per quart, while the regular juice is available at $0.40 per quart. The goal is to produce 50 quarts of the blend, which should have a selling price of $0.60 per quart to attract customers. How much of each type of juice should the juice bar mix to achieve the desired blend?

Kayaking Adventure Problem

During a kayaking trip, a group of friends observes that they can move downstream at a speed of 12 miles per hour with the river's current. However, when they turn around to row upstream, it takes them twice as long to cover the same distance. The kayakers is curious about the speed of the river's current.

Friends Meet In The Middle Problem

Two friends, Alex and Blake, live in towns that are 240 miles apart. They decide to meet somewhere in between. At the same time, Alex starts driving an electric car towards Blake's town at a speed of 45 miles per hour, while Blake starts driving towards Alex's town at a speed that is half as fast as Alex's. Where will they meet, and how far will it be from Alex's starting point? (Round answer to the nearest hundreth if needed.)

Juice Blend Solution

Let x be the amount of premium juice (in quarts) and y be the amount of regular juice (in quarts). We can set up the following system of equations based on the problem statement:

1. The total amount of the blend is 50 quarts:

$$x + y = 50$$

2. The cost per quart of the blend is \$0.60:

$$0.72x + 0.40y = 50 \times 0.60$$

Solving the system of equations:

$$\begin{aligned} x + y &= 50 \\ 0.72x + 0.40y &= 30 \end{aligned}$$

Multiplying the second equation by 100 for simplicity:

$$\begin{aligned} x + y &= 50 \\ 72x + 40y &= 3000 \end{aligned}$$

Using the method of substitution or elimination, we find:

$$\begin{aligned} x &= \frac{125}{4} \\ y &= \frac{75}{4} \end{aligned}$$

Therefore, the juice bar needs to mix 31.25 quarts of premium juice and 18.75 quarts of regular juice to make the 50-quart blend that can be sold for \$0.60 per quart.

Kayaking Adventure Solution

- Let $x + y$ represent the kayaking speed downstream in miles per hour.
- Let $x - y$ represent the kayaking speed upstream in miles per hour.
- The speed downstream is 12 miles per hour, so $x + y = 12$.
- The time to kayak upstream is twice the time to kayak downstream for the same distance.

Objective:

Determine the speed of the river's current, y.

Solution:

Given that the time to kayak a certain distance upstream is twice the time to kayak the same distance downstream, and using the relationship between speed, distance, and time, we have:

$$\frac{1}{x-y} = 2 \cdot \frac{1}{x+y}$$

Given $x + y = 12$, we can substitute into our equation:

$$\frac{1}{x-y} = 2 \cdot \frac{1}{12}$$

Solving the equation

$$\frac{1}{x-y} = \frac{1}{6}$$

for x and y, under the constraint $x + y = 12$.
Cross multiply

$$\frac{1}{x-y} = \frac{1}{6}$$

simplifies to $x - y = 6$.
Therefore:

$$x - y = 6$$
$$x + y = 12$$

Solving this system of equations:

$$2x = 18$$
$$x = 9$$
$$y = 3$$

Conclusion: The speed of the river's current, y, is 3 miles per hour.

Friends Meet In The Middle Solution

- The distance between their towns is 240 miles.
- Alex's speed is $x = 45$ mph.
- Blake's speed is $y = \frac{1}{2}x = 22.5$ mph.

Solution

To describe their journey, we set up the following system of equations:

$$\begin{aligned} x &= 45, \\ y &= \frac{x}{2}. \end{aligned}$$

Remember: distance = rate x time. Given that they meet after the same time t, the distances covered by Alex and Blake can be expressed as $d_{\text{Alex}} = xt$ and $d_{\text{Blake}} = yt$, respectively. Since they start 240 miles apart, we have:

$$xt + yt = 240.$$

Substituting the values of $x = 45$ and $y = 22.5$:

$$45t + 22.5t = 240.$$

Solving for t:

$$67.5t = 240, \quad t = \frac{240}{67.5}, \quad t = 3.56 \text{ hours}.$$

To find the distance from Alex's starting point:

$$d = xt = 45 \times 3.56 = 160.2 \text{ miles} = 160\frac{1}{5} \text{ miles}.$$

Conclusion: Alex and Blake will meet $160\frac{1}{5}$ miles from Alex's starting point.