Quadratic Equations

Solve by Completing the Square

Name:______________________________ Date:____________

How to Complete the Square

Step 1: If the equation is in the form $ax^2 + bx + c = 0$, divide each term by 'a'. The leading coefficient must be 1.

Step 2: Rewrite the equation in the form $x^2 + bx = c$ by adding or subtracting 'c' to both sides of the equation.

Step 3: Take the b term, divide it by 2, $\frac{b}{2}$, then square it, $\left(\frac{b}{2}\right)^2$.

Step 4: Add this result to both sides of the equation

$$x^2+bx+\left(\frac{b}{2}\right)^2=c+\left(\frac{b}{2}\right)^2$$

Step 5: Rewrite the perfect square on the left to the form $\left(x+\frac{b}{2}\right)^2$.

$$\left(x+\frac{b}{2}\right)^2=c+\left(\frac{b}{2}\right)^2$$

Step 6: Take the square root of both sides (Square Root Property)

$$\sqrt{\left(x+\frac{b}{2}\right)^2}=\sqrt{c+\left(\frac{b}{2}\right)^2}$$

Step 7: Isolate x on the left by subtracting or adding on both sides.
Step 8: Solve for x.

Practice

Solve the quadratic equations by completing the square

1) $2x^2 + 8x - 10 = 0$
2) $5x^2 - 10x - 5 = 0$
3) $4x^2 - x + 1 = 0$
4) $3x^2 - 3x + 1 = 0$
5) $6x^2 + 2x - 1 = 0$
6) $3x^2 - 7x - 3 = 0$

Solve Quadratic Equations by Completing the Square Solutions

Problem 1: Solve $2x^2 + 8x - 10 = 0$

Since $a \neq 1$, divide through by 2:

$$\frac{2x^2}{2} + \frac{8}{2}x - \frac{10}{2} = \frac{0}{2}$$

$$x^2 + 4x - 5 = 0$$

Isolate the variable terms by adding 5 to both sides:

$x^2 + 4x - 5 + 5 = 0 + 5$

$x^2 + 4x = 5$

Complete the square by adding the square of half the coefficient of x to both sides:

$x^2 + 4x + 4 = 9$

This forms a perfect square on the left side:

$(x + 2)^2 = 9$

Taking the square root of both sides:

$\sqrt{(x + 2)^2} = \sqrt{9}$

$x + 2 = \pm 3$

Subtract 2 from both sides to solve for x:

$x + 2 - 2 = \pm 3 - 2$

$x = -2 \pm 3$

Solving for x provides the solutions:

$x = 1 \quad \text{or} \quad x = -5$

Problem 2: Solve $5x^2 - 10x - 5 = 0$

First, divide the entire equation by 5 to simplify:

$$\frac{5x^2}{5} - \frac{10x}{5} - \frac{5}{5} = \frac{0}{5}$$

$$x^2 - 2x - 1 = 0$$

Isolate the variable terms by adding 1 to both sides:

$$x^2 - 2x - 1 + 1 = 0 + 1$$

$$x^2 - 2x = 1$$

Complete the square by adding the square of half the coefficient of x (which is $1^2 = 1$) to both sides:

$$\left(\frac{2}{1}\right)^2 = 1^2 = 1$$

$$x^2 - 2x + 1 = 2$$

This forms a perfect square on the left side:

$$(x - 1)^2 = 2$$

Taking the square root of both sides gives:

$$\sqrt{(x - 1)^2} = \sqrt{2}$$

$$x - 1 = \pm\sqrt{2}$$

$$x - 1 + 1 = \pm\sqrt{2} + 1$$

Solve for x by adding 1 to both sides:

$$x = 1 \pm \sqrt{2}$$

Thus, the solutions are:

$$x = 1 + \sqrt{2} \quad \text{and} \quad x = 1 - \sqrt{2}$$

Problem 3: Solve $4x^2 - x + 1 = 0$

First, divide the entire equation by 4 to simplify:

$$\frac{4x^2}{4} - \frac{x}{4} + \frac{1}{4} = 0$$

$$x^2 - \frac{1}{4}x + \frac{1}{4} = 0$$

Isolate the variable terms by subtracting $\frac{1}{4}$ from both sides:

$$x^2 - \frac{1}{4}x + \frac{1}{4} - \frac{1}{4} = 0 - \frac{1}{4}$$

$$x^2 - \frac{1}{4}x = -\frac{1}{4}$$

Complete the square by adding the square of half the coefficient of x (which is $\left(\frac{-1}{8}\right)^2 = \frac{1}{64}$) to both sides:

$$\left(\frac{-\frac{1}{4}}{2}\right) = \left(\frac{-1}{8}\right)^2 = \frac{1}{64}$$

$$x^2 - \frac{1}{4}x + \frac{1}{64} = -\frac{1}{4} + \frac{1}{64}$$

This forms a perfect square on the left side:

$$\left(x - \frac{1}{8}\right)^2 = -\frac{1}{4} + \frac{1}{64}$$

Simplifying the right side:

$$\left(x - \frac{1}{8}\right)^2 = \frac{-16 + 1}{64}$$

$$\left(x - \frac{1}{8}\right)^2 = \frac{-15}{64}$$

Since the right side of the equation is negative, this equation does not have real solutions. For real solutions, the expression inside the square root must be non-negative. Therefore, this quadratic equation has complex solutions, which we find by taking the square root of both sides:

$$x - \frac{1}{8} = \pm\sqrt{\frac{-15}{64}}$$

$$x = \frac{1}{8} \pm i\frac{\sqrt{15}}{8}$$

Thus, the solutions are:

$$x = \frac{1}{8} + i\frac{\sqrt{15}}{8} \quad \text{and} \quad x = \frac{1}{8} - i\frac{\sqrt{15}}{8}$$