Solve the Matricies - Sample

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Show all work for full credit. Use seperate sheet of paper if needed.

Problem 1:
Solve the following system of equations using matrices:

$$\begin{aligned} 3x - y &= 5, \\ 2x + 4y &= 12 \end{aligned}$$

Problem 2: Solve the following system of equations using matrices:

$$\begin{aligned} x + 2y - z &= 4, \\ 2x - y + 3z &= -1, \\ -3x + 4y + z &= 3 \end{aligned}$$

Problem 3: Solve the following system of equations using matrices:

$$\begin{aligned} 4x + 5y &= 20, \\ -2x + 3y &= 9 \end{aligned}$$

Problem 4: Solve the following system of equations using matrices:

$$\begin{aligned} x + 2y &= 3, \\ 2x + 3y &= 5 \end{aligned}$$

Solutions

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Problem 1:

1. Write the system as an augmented matrix:

$$\left[\begin{array}{cc|c} 3 & -1 & 5 \\ 2 & 4 & 12 \end{array}\right]$$

2. Perform row operations to make the leading entry of the first row a 1. Divide the first row by 3:

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{5}{3} \\ 2 & 4 & 12 \end{array}\right]$$

3. Make the first element of the second row 0 by replacing the second row with $R_2 - 2R_1$:

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{5}{3} \\ 0 & \frac{14}{3} & \frac{24}{3} - \frac{10}{3} \end{array}\right]$$

Simplifying, we get:

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{5}{3} \\ 0 & \frac{14}{3} & \frac{14}{3} \end{array}\right]$$

4. Make the second entry of the second row a 1 by dividing the second row by $\frac{14}{3}$:

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{5}{3} \\ 0 & 1 & 1 \end{array}\right]$$

5. Eliminate the -1/3 coefficient in the first row by replacing R_1 with $R_1 + \frac{1}{3}R_2$:

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

The resulting matrix is in RREF, indicating that the solution to the system of equations is $x = 2$ and $y = 1$.

Problem 2: 1. Write the system as an augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 2 & -1 & 3 & -1 \\ -3 & 4 & 1 & 3 \end{array}\right]$$

2. Use row operations to form a leading 1 in the first row, first column. The matrix is already in the desired form for the first row.

3. Make the first element of the second and third rows 0 by replacing R_2 with $R_2 - 2R_1$ and R_3 with $R_3 + 3R_1$:

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & -5 & 5 & -9 \\ 0 & 10 & 2 & 15 \end{array}\right]$$

4. Make the second element of the second row a 1 by dividing the second row by -5:

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & 1 & -1 & \frac{9}{5} \\ 0 & 10 & 2 & 15 \end{array}\right]$$

5. Eliminate the second column elements in rows 1 and 3 by row operations: replace R_1 with $R_1 - 2R_2$ and R_3 with $R_3 - 10R_2$:

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & \frac{8}{5} \\ 0 & 1 & -1 & \frac{9}{5} \\ 0 & 0 & 12 & 6 \end{array}\right]$$

6. Make the third element of the third row a 1 by dividing the third row by 12:

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & \frac{8}{5} \\ 0 & 1 & -1 & \frac{9}{5} \\ 0 & 0 & 1 & \frac{1}{2} \end{array}\right]$$

7. Eliminate the third column elements in rows 1 and 2 to get the final RREF form by replacing R_1 with $R_1 - R_3$ and R_2 with $R_2 + R_3$:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{7}{5} \\ 0 & 1 & 0 & \frac{11}{5} \\ 0 & 0 & 1 & \frac{1}{2} \end{array}\right]$$

The resulting matrix is in RREF, indicating that the solution to the system of equations is $x = \frac{7}{5}$, $y = \frac{11}{5}$, and $z = \frac{1}{2}$.

Problem 3: 1. Write the system as an augmented matrix:

$$\left[\begin{array}{cc|c} 4 & 5 & 20 \\ -2 & 3 & 9 \end{array}\right]$$

2. To simplify the matrix, we start by making the leading coefficient of the first row a 1. Divide the first row by 4:

$$\left[\begin{array}{cc|c} 1 & \frac{5}{4} & 5 \\ -2 & 3 & 9 \end{array}\right]$$

3. Eliminate the first element of the second row by adding 2 times the first row to the second row:

$$\left[\begin{array}{cc|c} 1 & \frac{5}{4} & 5 \\ 0 & \frac{11}{2} & 19 \end{array}\right]$$

4. Make the second element of the second row a 1 by dividing the second row by $\frac{11}{2}$:

$$\left[\begin{array}{cc|c} 1 & \frac{5}{4} & 5 \\ 0 & 1 & \frac{38}{11} \end{array}\right]$$

5. Eliminate the second element in the first row by subtracting $\frac{5}{4}$ times the second row from the first row:

$$\left[\begin{array}{cc|c} 1 & 0 & \frac{20}{11} \\ 0 & 1 & \frac{38}{11} \end{array}\right]$$

The resulting matrix is in RREF, indicating that the solution to the system of equations is $x = \frac{20}{11}$ and $y = \frac{38}{11}$.

Problem4: 1. Convert to augmented matrix:

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 2 & 3 & 5 \end{array}\right]$$

2. Perform row operations to get the leading 1 in the first column of the first row:

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 2 & 3 & 5 \end{array}\right]$$

Since the first row already has a leading 1, we move to the next step.

3. Make the first element of the second row 0 by using $R_2 - 2R_1 \rightarrow R_2$:

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & -1 & -1 \end{array}\right]$$

4. Convert the second row to have a leading 1 by multiplying R_2 by -1:

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & 1 \end{array}\right]$$

5. Eliminate the y term from the first row by subtracting 2 times the second row from the first row, $R_1 - 2R_2 \rightarrow R_1$:

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right]$$

The resulting augmented matrix is in RREF, indicating the solution to the system is $x = 1$ and $y = 1$.

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Find the Determinants

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