

[MATHEMATICS] 09-01-2019_Morning

1. let $A = \left\{ \theta \in \left(-\frac{\pi}{2}, \pi \right) : \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{ is purely imaginary} \right\}$. Then the sum of the elements in A is ::

(A) $\frac{3\pi}{4}$ (B) π (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

Sol. C

$$\frac{3+2i\sin\theta}{1-2i\sin\theta} \in \text{Pure Imaginary}$$

$$\Rightarrow \operatorname{Re}\left(\frac{3+2i\sin\theta}{1-2i\sin\theta}\right) = 0$$

$$3 - 4\sin^2\theta = 0$$

$$\sin\theta = \pm \frac{\sqrt{3}}{2}$$

$$\theta \in \left\{ -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3} \right\}$$

2. for $x \in \mathbb{R} - \{0,1\}$. let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. if a function, $J(x)$ satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ then $J(x)$ is equal to :

(A) $f_1(x)$ (B) $f_3(x)$ (C) $f_2(x)$ (D) $\frac{1}{x} f_3(x)$

Sol. B

$$f_2(J(f_1(x))) = f_3(x)$$

$$1 - J\left(\frac{1}{x}\right) = \frac{1}{1-x}$$

$$J\left(\frac{1}{x}\right) = 1 - \frac{1}{1-x}$$

$$J\left(\frac{1}{x}\right) = \frac{1-x-1}{1-x}$$

$$J(x) = \frac{1/x}{1/x-1}$$

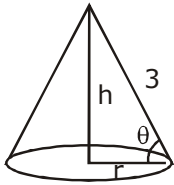
$$J(x) = \frac{1}{1-x}$$

$$\Rightarrow J(x) = f_3(x)$$

3. The maximum volume (in cu. m) of the right circular cone having slant height 3m is :

- (A) $\frac{4}{3}\pi$ (B) 6π (C) $2\sqrt{3}\pi$ (D) $3\sqrt{3}\pi$

Sol. C



$$h^2 + r^2 = 9$$

$$v = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow v = \frac{1}{3}\pi \cos^2 \theta \sin \theta$$

$$\frac{v}{9\pi} = \cos^2 \theta \sin \theta$$

$$\Rightarrow \frac{v^2}{81\pi^2} = \cos^4 \theta \sin^2 \theta$$

$$\frac{\frac{c^2}{2} + \frac{c^2}{2} + \sin^2 \theta}{3} \geq \sqrt[3]{\frac{v^2}{81\pi^2 4}}$$

$$\frac{v^2}{4.81\pi^2} \leq \sqrt[3]{\frac{v^2}{81\pi^2 4}}$$

$$\frac{v^3}{4.81\pi^2} \leq \frac{1}{27}$$

$$v^2 \leq \frac{4.81\pi^2}{27}$$

$$v^2 \leq 2\sqrt{3}\pi$$

4. If $y = y(x)$ is the solutions of the differential equation, $x \frac{dy}{dx} + 2y = x^2$ satisfying $y(1) = 1$, then

$y\left(\frac{1}{2}\right)$ is equal to :

- (A) $\frac{13}{16}$ (B) $\frac{1}{4}$ (C) $\frac{7}{64}$ (D) $\frac{49}{16}$

Sol. D

$$x \frac{dy}{dx} + 2y = x^2 \quad \frac{dy}{dx} + \left(\frac{2}{x}\right)y = x \quad |_{\text{LDE}}$$

$$\text{IF} = e^{\int \frac{2}{x} dx} = x^2$$

$$yx^2 = \int x^3 dx$$

$$yx^2 = \frac{x^4}{4} + C$$

$$C = \frac{3}{4}$$

$$yx^2 = \frac{x^4}{4} + \frac{3}{4}$$

$$y(1/2) \Rightarrow y \cdot \frac{1}{4} = \frac{1}{64} + \frac{3}{4}$$

$$y = \frac{1}{16} + 3$$

$$y = \frac{49}{16}$$

5. If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}$ ($x > \frac{3}{4}$) then x is equal to :

- (A) $\frac{\sqrt{145}}{12}$ (B) $\frac{\sqrt{145}}{10}$ (C) $\frac{\sqrt{146}}{12}$ (D) $\frac{\sqrt{145}}{11}$

Sol. A

$$\cos^{-1}\left(\frac{2}{3x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{3}{4x}\right)$$

$$\cos\left[\cos^{-1}\left(\frac{2}{3x}\right)\right] = \cos\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{3}{4x}\right)\right]$$

$$\frac{2}{3x} = \sin\left(\cos^{-1}\frac{3}{4x}\right)$$

$$\frac{2}{3x} = \cos\left(\sin^{-1}\sqrt{1 - \left(\frac{3}{4x}\right)^2}\right)$$

$$\left(\frac{2}{3x}\right)^2 = 1 - \left(\frac{3}{4x}\right)^2$$

$$\begin{aligned}\frac{4}{9x^2} + \frac{9}{16x^2} &= 1 &\Rightarrow x^2 &= \frac{4}{9} + \frac{9}{16} \\ & &\Rightarrow x^2 &= \frac{64 + 81}{16 \cdot 9} \\ & &\Rightarrow x^2 &= \sqrt{\frac{145}{12}}\end{aligned}$$

6. Three circles of radii a, b, c ($a < b < c$) touch each other externally. If they have x - axis as a common tangent, then :

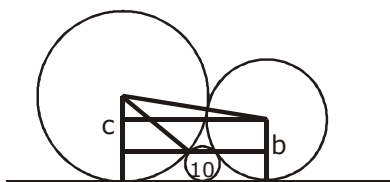
(A) a, b, c are in A.P

(B) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are In A.P.

(C) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

(D) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

Sol. D



$$\begin{aligned}\sqrt{(b+c)^2 - (c-b)^2} &= \sqrt{(c+a)^2 - (c-a)^2} + \sqrt{(b+a)^2 - (b-a)^2} \\ \sqrt{2bc} &= \sqrt{2ac} + \sqrt{2ab} \\ \frac{1}{\sqrt{a}} &= \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}\end{aligned}$$

7. Two cards are drawn successively with replacement from a well - shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then $P(X = 1) + P(X = 2)$ equals :

(A) $52/169$

(B) $24/169$

(C) $25/169$

(D) $49/169$

Sol. C

$$\begin{aligned}P(x = 1) + P(x = 2) \\ &= 2 \cdot \frac{4}{52} \times \frac{48}{52} + \frac{4}{52} \cdot \frac{4}{52} \\ &= \frac{400}{52 \cdot 52} = \frac{25}{169}\end{aligned}$$

8. Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true ?

(A) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

(B) The lines are all parallel.

(C) Each line passes through the origin

(D) the lines are not concurrent.

Sol. A

$$px + qy + r = 0$$

$$\frac{3}{4}p + \frac{2}{4}q + r = 0$$

$$\Rightarrow \text{Line Pass } \left(\frac{3}{4}, \frac{1}{2} \right)$$

9. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to :

(A) 6

(B) 4

(C) 8

(D) 14

Sol. C

$$\left\{ \frac{2^{403}}{15} \right\}$$

$$= \left\{ \frac{8(2^4)^{100}}{15} \right\}$$

$$\left\{ \frac{8(15+1)^{100}}{15} \right\}$$

$$\Rightarrow k = 8$$

10. consider a class of 5 girls and 7 boys. the number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team is :

(A) 200

(B) 300

(C) 500

(D) 350

Sol. B

5 G & 7 B



2 G + 3 B

$${}^5C_2 \cdot {}^7C_3 - {}^5C_2 \cdot {}^2C_2 \cdot {}^5C_1$$

$$= {}^5C_2 \{ {}^7C_3 - {}^5C_1 \}$$

$$= 10 \{ 35 - 5 \}$$

$$= 300$$

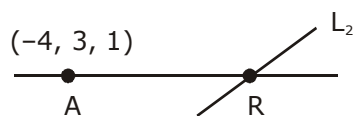
11. The equation fo the line passing through $(-4, 3, 1)$, parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is :

$$(A) \frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$$

$$(B) \frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$

$$(C) \frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$$

$$(D) \frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$$

Sol. B

Let R on L_2 ; $(-1 - 3t, 3 + 2t, 2 - t)$
for t $V_L \cdot n_p = 0$

$$\overrightarrow{AR} \cdot \overrightarrow{n_p} = 0$$

$$(3 - 3t) \cdot 1 + 2t \cdot 2 + (1 - t)(-1) = 0$$

$$3 - 3t + 4t - 1 + t = 0$$

$$t = -1$$

$$V_L = \langle 6, -2, 2 \rangle$$

- 12.** 5 students of a class have an average height 150 cm and variance 18 cm². A new student, whose height is 156 cm, joined them. The variance (in cm²) of the height of these six students is :
(A) 16 (B) 20 (C) 22 (D) 18

Sol. B

Let students are s_1, s_2, s_3, s_4, s_5
Given avg. high

$$\bar{x} = \frac{s_1 + s_2 + s_3 + s_4 + s_5}{5} = 150$$

$$\sum s_i = 750$$

& Variance

$$\frac{\sum (s_i)^2}{5} - (\bar{x})^2 = 18$$

$$\sum (s_i)^2 = 11250$$

height of new student is 156

Now new variance

$$= \frac{11250 + (156)^2}{6} - \left(\frac{750 + 156}{6} \right)^2 = 20$$

- 13.** For $x \neq n\pi + 1$, $n \in \mathbb{N}$ (the set of the natural numbers), the integral

$$\int x \sqrt{\frac{2 \sin(x^2 - 1) - \sin 2(x^2 - 1)}{2 \sin(x^2 - 1) + \sin 2(x^2 - 1)}} dx \text{ is equal to :}$$

(where c is a constant of integration.

$$(A) \log_e \left| \sec \left(\frac{x^2 - 1}{2} \right) \right| + C$$

$$(B) \log_e \left| \frac{1}{2} \sec^2 (x^2 - 1) \right| + C$$

$$(C) \frac{1}{2} \log_e \left| \sec^2 \left(\frac{x^2 - 1}{2} \right) \right| + C$$

$$(D) \frac{1}{2} \log_e \left| \sec (x^2 - 1) \right| + C$$

Sol. A

$$\text{Let } x^2 - 1 = \theta \Rightarrow 2x dx = d\theta$$

$$\frac{1}{2} \int \sqrt{\frac{2 \sin \theta - \sin 2\theta}{2 \sin \theta + \sin 2\theta}} d\theta$$

$$\frac{1}{2} \int \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

$$= \frac{1}{2} \int |\tan \theta / 2| d\theta$$

$$= \ln |\sec \theta / 2| + C$$

$$= \ln \left| \sec \left(\frac{x^2 - 1}{2} \right) + C \right|$$

14. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as

$$f(x) = \begin{cases} 5, & \text{if } x \leq 1 \\ a + bx, & \text{if } 1 < x < 3 \\ b + 5x, & \text{if } 3 \leq x < 5 \\ 30, & \text{if } x \geq 5 \end{cases} \quad \text{Then, } f \text{ is :}$$

- (A) Continuous if $a = 5$ and $b = 5$
 (B) not continuous for any values of a and b
 (C) continuous if $a = 0$ and $b = 5$
 (D) continuous if $a = -5$ and $b = 10$

Sol. B

$$\begin{aligned} x = 1 & \quad 5 = 5 = a + b \Rightarrow a + b = 5 \\ x = 3 & \quad a + 3b = b + 15 = b + 15 \Rightarrow a + 2b = 15 \\ x = 5 & \quad b + 25 = 30 = 30 \\ \Rightarrow & \quad b = 5 \end{aligned}$$

15. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2 \theta} - \frac{y^2}{\sin^2 \theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval :

- (A) $(3/2, 2]$ (B) $(3, \infty)$ (C) $(1, 3/2]$ (D) $(2, 3]$

Sol. B

$$e_H > 2$$

$$\Rightarrow \sqrt{1 + \frac{\sin^2 \theta}{\cos^2 \theta}} > 2$$

$$\sec \theta > 2$$

$$0 < \cos \theta < \frac{1}{2}$$

$$\frac{\pi}{3} < \theta < \frac{\pi}{2}$$

Now

$$LR = \frac{2b^2}{a} = \frac{(2b)^2}{2a} = \frac{(\sin \theta)^2}{2 \cos \theta}$$

$$LR = 2 \frac{\sin^2 \theta}{\cos \theta}$$

$$LR = 2 \tan \theta \sin \theta$$

$$\Rightarrow LR \in (3, \infty)$$

16. if θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan \theta|$ is equal to :

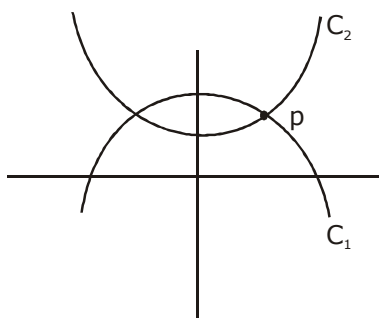
(A) $\frac{4}{9}$

(B) $\frac{8}{17}$

(C) $\frac{8}{15}$

(D) $\frac{7}{17}$

Sol. C



$$C_1 : y = 10 - x^2$$

$$C_2 : y = 2 + x^2$$

for P.O.I

$$10 - x^2 = 2 + x^2$$

$$x^2 = 4$$

$$x = \pm 2$$

$$P : (2, 6)$$

$$\tan \theta = \left| \frac{-2.2 - 2.2}{1 - (2.2).(2.2)} \right|$$

$$\tan \theta = \frac{8}{15}$$

17. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} \times \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{c} = 4$, then $|\vec{c}|^2$ is equal to :

(A) $\frac{19}{2}$

(B) 8

(C) $\frac{17}{2}$

(D) 9

Sol. A

$$\vec{a} \times \vec{c} + \vec{b} = \vec{0}$$

$$\vec{a} \times (\vec{a} \times \vec{c} + \vec{b}) = \vec{0}$$

$$(\vec{a} \cdot \vec{c})\vec{a} - a^2\vec{c} + \vec{a} \times \vec{b} = \vec{0}$$

$$\vec{c} = \frac{4\vec{a} + \vec{a} \times \vec{b}}{|\vec{a}|^2}$$

$$\vec{c} = \frac{\langle 4, -4, 0 \rangle + \langle -1, -1, 2 \rangle}{2}$$

$$\bar{c} = \frac{<+3, -5, 2>}{2}$$

$$|\bar{c}|^2 = \frac{9+25+4}{4}$$

$$|\bar{c}|^2 = \frac{19}{2}$$

18. If a, b and c be three distinct real numbers in G.P. and $a + b + c = xb$, then x cannot be :
 (A) -2 (B) -3 (C) 2 (D) 4

Sol. C

a, b, c are in G.P.

$\frac{b}{r}, b, br$ are in G.P.

$$\text{Now } \frac{b}{r} + b + br = xb$$

$$x - 1 = r + \frac{1}{r}$$

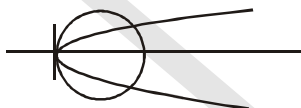
$$x \geq 3 \text{ or } x \leq -1$$

19. Equation of a common tangent to the circle $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is :
 (A) $2\sqrt{3}y = -x - 12$ (B) $2\sqrt{3}y = 12x + 1$ (C) $\sqrt{3}y = x + 3$ (D) $\sqrt{3}y = 3x + 1$

Sol. C

$$C_1 : x^2 + y^2 - 6x = 0$$

$$P : y^2 = 4x$$



$$C_1 : (x-3)^2 + y^2 = 3^2$$

$$P : y^2 = 4x$$

$$T | C_1 \Rightarrow y = m(x-3) \pm 3\sqrt{1+m^2}$$

$$T | P \Rightarrow y = m \times + \frac{1}{m}$$

$$\Rightarrow \frac{1}{m} = -3m \pm 3\sqrt{1+m^2}$$

$$\left(\frac{1}{m} + 3m\right)^2 = 9 + 9m^2$$

$$\frac{1}{m^2} + 9m^2 + 6 = 9 + 9m^2$$

$$m^2 = \frac{1}{3}$$

$$m = \pm \frac{1}{\sqrt{3}}$$

$$T : y = \pm \frac{x}{\sqrt{3}} + \sqrt{3}$$

- 20.** The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y -axis also passes through the point :

(A) $(-3, 1, 1)$ (B) $(3, 2, 1)$ (C) $(3, 3, -1)$ (D) $(-3, 0, -1)$

Sol. B

$$P : P_1 + \lambda P_2 = 0$$

$$P : (x + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z + (-1 + 4\lambda) = 0$$

for λ :

$$\vec{n}_p \cdot \hat{j} = 0$$

$$1 + 3\lambda = 0 \Rightarrow \lambda = -1/3$$

$$P : \pm \frac{1}{3}x + \frac{4}{3}z - \frac{7}{3} = 0$$

$$P : x + 4z - 7 = 0$$

Now check options

21. $\lim_{y \rightarrow 0} \frac{\sqrt{1 + \sqrt{1 + y^4}} - \sqrt{2}}{y^4}$

(A) exists and equals $\frac{1}{4\sqrt{2}}$

(B) does not exist

(C) exists and equals $\frac{1}{2\sqrt{2}}$

(D) exists and equals $\frac{1}{2\sqrt{2}(\sqrt{2} + 1)}$

Sol. A

$$\lim_{y \rightarrow 0} \frac{\sqrt{1 + \sqrt{1 + y^4}} - \sqrt{2}}{y^4}$$

$$\lim_{y \rightarrow 0} \left(\frac{1 + \sqrt{1 + y^4} - 2}{y^4} \right) \left(\frac{1}{\sqrt{1 + \sqrt{1 + y^4}} + \sqrt{2}} \right)$$

$$\lim_{y \rightarrow 0} \frac{1 + y^4 - 1}{y^4} \left(\frac{1}{\sqrt{1 + (y^4)} + 1} \right) \left(\frac{1}{\sqrt{1 + \sqrt{1 + y^4}} + \sqrt{2}} \right)$$

$$= \frac{1}{2} \cdot \frac{1}{2\sqrt{2}} = \frac{1}{4\sqrt{2}}$$

22. If $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to :

(A) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (C) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

Sol. A

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$A^n = \begin{bmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{bmatrix}$$

$$A^{-50} = \begin{bmatrix} \cos(-50)\theta & -\sin(-50)\theta \\ \sin(-50)\theta & \cos(-50)\theta \end{bmatrix}$$

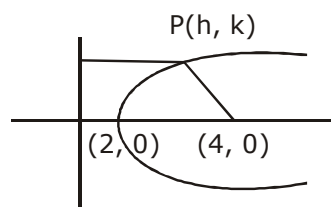
$$A^{-50} = \begin{bmatrix} \cos \frac{50\pi}{12} & \sin \frac{50\pi}{12} \\ -\sin \frac{50\pi}{12} & \cos \frac{50\pi}{12} \end{bmatrix}$$

$$A^{-50} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

23. Axis of a parabola lies along x - axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x - axis then which of the following points does not lie on it ?

(A) (8,6) (B) $(5, 2\sqrt{6})$ (C) (4,-4) (D) $(6, 4\sqrt{2})$

Sol. A



$$y^2 = 8(x - 2)$$

24. Let $a_1, a_2, \dots, a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to :
 (A) 42 (B) 47 (C) 57 (D) 52

Sol. **D**

$$S - 27 = 75$$

$$\frac{30}{2} [2a_1 + 29d] - 2 \left\{ \frac{15}{2} \right\} [2a_1 + (14)2d] = 75$$

$$15d = 75$$

$$d = 5 \quad \dots (i)$$

$$\& a_5 = 27$$

$$a_1 + 4d = 27 \quad \dots (2)$$

$$a_1 = 7$$

$$\text{Now } a_{10} = a_1 + 9d \\ = 7 + 45 = 52$$

25. the system of linear equations

$$x + y + z = 2$$

$$2x + 3y + 2z = 5$$

$$2x + 3y + (a^2 - 1)z = a + 1$$

$$(A) \text{ has a unique solutions for } |a| = \sqrt{3} \quad (B) \text{ is inconsistent when } |a| = \sqrt{3}$$

$$(C) \text{ has infinitely many solutions for } a = 4$$

$$(D) \text{ is inconsistent when } a = 4$$

Sol. **B**

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2 - 1 \end{vmatrix}$$

$$\Rightarrow 3a^2 - 3 - 6 - 2a^2 + 2 + 4 + 2a^2 - 2 - 4 = 0$$

$$\Rightarrow 3(a^2 - 3) = 0$$

$$\Rightarrow a^2 = 3 \Rightarrow |a| = \sqrt{3}$$

$$\Rightarrow \text{in constant}$$

$$\Delta = 0$$

26. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to :
 (A) 512 (B) -512 (C) 256 (D) -256

Sol. **D**

$$x^2 + 2x + 1 = -1$$

$$(x + 1)^2 = i^2$$

$$x = -(1 + i) - (1 - i)$$

$$\alpha^{15} + \beta^{15} = 2^{15/2} \left\{ \frac{(-1+i)^{15}}{\sqrt{2}} + \frac{(-1-i)^{15}}{\sqrt{2}} \right\}$$

$$= 2^{15/2} \left\{ 2 \cos \left(\frac{3\pi}{4} \cdot 15 \right) \right\}$$

$$2^{15/2} \left\{ -2 \cos \left(\frac{45\pi}{4} \right) \right\}$$

$$= -2^{15/2} \cdot 2 \cdot 2^{1/2} = -256$$

27. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is :

(A) $(\wedge, \wedge)$ (B) $(\wedge, \vee)$ (C) $(\vee, \vee)$ (D) $(\vee, \wedge)$

Sol. B

p	q	$\sim p$	$p \wedge q$	$\sim p \vee q$	$(p \wedge q) \wedge (\sim p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	F
F	T	T	F	T	F
F	F	T	F	T	F

28. The value of $\int_0^{\pi} |\cos x|^3 dx$ is :

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{4}{3}$ (D) $\frac{2}{3}$

Sol. A

$$\int_0^{\pi} |\cos x|^3 dx$$

$$\int_0^{\pi/2} (\cos x)^3 dx - \int_{\pi/2}^{\pi} (\cos x)^3 dx$$

$$= \frac{2}{3} - \left(-\frac{2}{3}\right)$$

$$= \frac{4}{3}$$

29. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals :

(A) $13 - 4\cos^2\theta + 6\cos^4\theta$ (B) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$
 (C) $13 - 4\cos^6\theta$ (D) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

Sol. C

$$= 3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$$

$$= 3\{1 - \sin 2\theta\}^2 + 6\{1 + \sin 2\theta\} + 4\sin^6\theta$$

$$= 9 + 3\sin^2 2\theta + 4\sin^6\theta$$

$$= 9 + 4\sin^2\theta\{3\cos^2\theta + \sin^4\theta\}$$

$$= 9 + 4(1 - c^2)\{3c^2 + (1 - c^2)^2\}$$

$$= 9 + 4(1 - c^2)(3c^2 + c^2 - 2c^2 + 1)$$

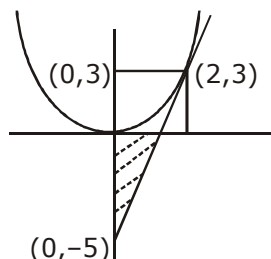
$$= 13 - 4\cos^6\theta$$

30. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point $(2, 3)$ to it and the y -axis is :

(A) $\frac{14}{3}$ (B) $\frac{32}{3}$ (C) $\frac{56}{3}$ (D) $\frac{8}{3}$

Sol. D

C : $y = x^2 - 1$



$$T_{(2,3)} : y - 3 = 4(x - 2)$$

$$y - 4x + 5 = 0$$

$$A = \frac{1}{2} \cdot 2 \cdot 8 - \int_{-1}^3 x \, dy$$

$$A = 8 - \int_{-1}^3 \sqrt{1+y} \, dy$$

$$A = 8 - \frac{23}{3} (1+y)^{3/2} \Big|_{-1}^3$$

$$= 8 - \frac{2}{3} \{2^3\}$$

$$= \left| 8 - \frac{16}{3} \right| = \frac{8}{3}$$

