[MATHEMATICS] 10-01-2019_Morning

1. The mean of five observations is 5 and their variance is 9.20. if three of the given five observations are 1,3, and 8, then a ratio of other two observations is :

(A) 6 : 7 (B) 4 : 9 (C) 10 : 3 (D) 5 : 8

Sol. **B**

1, 3,x, x_2, 8 → 5 observer

Mean = $\frac{\sum x_i}{5}$ = 5 ⇒ $x_1 + x_2 = 13$

var. = σ^2 = $\frac{\sum x_i^2}{5}$ – 25 = 9.20

⇒ $\sum x_i^2$ = 171 ⇒ $x_1^2 + x_2^2$ = 171 – 1–9–64
= 97

$(x_1 + x_2)^2 - 2x_1x_2 = 97$
$x_1 x_2 = 36$
$x_1 : x_2$ = 4 : 9 as sum = 13 & pr = 36

2. The sum of all two digit positive numbers which when divided by 7 yeild 2 or 5 as remainder is :

(A) 1465 (B) 1256 (C) 1356 (D) 1365

Sol. **C**

$\sum_{r=2}^{13}(7r+2)$ & $\sum_{r=1}^{13}(7r+5)$ = 702
= 654
Total = 654 + 702 = 1356

3. If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a,b,c) ?

(A) (1,1,3) (B) $\left(\frac{1}{2}, 2, 3\right)$ (C) $\left(\frac{1}{2}, 2, 0\right)$ (D) (1,1,0)

Sol. **D**

Parabola $y^2 = 4b(x - c)$ & $y^2 = 3$ ax
have common normal other than x asix normals are :
$y = m(x - c) - 2bm - bm^3$
$y = mx - 4am - 2am^3$
$(C+2b)m + bm^3 = 4am + 2am^3$
$(4a - C - 2b)m = (b - 2a)m^3$

$\Rightarrow m^2 = \frac{c}{2a-b} - 2 > 0$

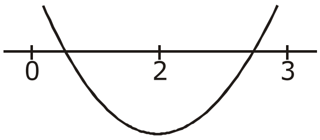
$\Rightarrow \frac{c}{2a-b} > 2$

only (4) option is true

4. Consider the quadratic equation $(c - 5)x^2 - 2cx + (c - 4) = 0$, $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0,2) and its other root lies in the interval (2,3). Then the number of element in S is :

(A) 18 (B) 11 (C) 10 (D) 12

Sol. **B**



f(0)(2) < 0
& f(2) f(3) < 0
$\Rightarrow$ (c – 4)(c – 24) < 0 & (c – 24)(4c – 49) < 0

$\frac{49}{4} < C < 24$

S = {13, 14, 15, 16....23} $\Rightarrow$ No. = 11

5. If the third term in the binomial expansion of $\left(1 + x^{\log_2 x}\right)^5$ equals 2560, the a possible value of x is :

(A) $\frac{1}{8}$ (B) $2\sqrt{2}$ (C) $\frac{1}{4}$ (D) $4\sqrt{2}$

Sol. **C**

$T_3\ {}^5C_2\left(x^{\log_2 x}\right)^2 = 2560$

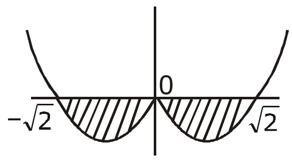
$2(\log_2 x)^2 = \log_2 256 = 8$

$\log_2 x = 2$ or $-2 \Rightarrow 4$ or $\frac{1}{4}$

6. Let I = $\int_a^b \left(x^4 - 2x^2\right)dx$. If I is minimum then the ordered pair (a,b) is :

(A) $\left(\sqrt{2}, -\sqrt{2}\right)$ (B) $\left(-\sqrt{2}, 0\right)$ (C) $\left(0, \sqrt{2}\right)$ (D) $\left(-\sqrt{2}, \sqrt{2}\right)$

Sol. **D**



as Area given is Negative so it will be Minimum when we take longest Integrative possible and in given option longest interval is (4)

7. Let $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k}$ and $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of (λ_1, λ_2, λ_3) is :

(A) (1,3,1) (B) $\left(\frac{1}{2}, 4, -2\right)$ (C) (1,5,1) (D) $\left(-\frac{1}{2}, 4, 0\right)$

Sol. **B**

(1) $4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k}$

= $4\hat{i} + 2\lambda\hat{j} + 6\hat{k}$

= $3 - \lambda_2 = 2\lambda_1 \Rightarrow 2\lambda_1 + \lambda_2 = 3$
(2) $\vec{a}.\vec{c} = 0 \Rightarrow 6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$
$2\lambda_1 + \lambda_3 = -1$
$(\lambda_1, 3 - 2\lambda_1, -1 - 2\lambda_1)$ is $(\lambda_1, \lambda_2, \lambda_3)$
by options (B) is correct

8. If the system of equations
$x + y + z = 5$
$x + 2y + 3z = 9$
$x + 3y + \alpha z = \beta$
has infinitely many solutions, then $\beta - \alpha$ equals :
(A) 5 (B) 21 (C) 18 (D) 8

Sol. D

(1) $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = (\alpha - 1) - 4 = (\alpha - 5)$

for ∞ solutions $D = 0 \Rightarrow \alpha = 5$
(2) Now

$D_1 = \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0 \Rightarrow 2 + \beta - 15 = 0$

$\beta = 13$

(3) put $\beta = 13$ in $D_2 = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 9 & 3 \\ 1 & 13 & 5 \end{vmatrix} = 0$ & $D_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 9 \\ 1 & 3 & 13 \end{vmatrix} = 0$

$\Rightarrow \beta - \alpha = 13 - 5 = 8$

9. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is :
(A) 42 (B) 102 (C) 38 (D) 1

Sol. C
n(A) = No. of student taken maths = 70
n(B) = Physics = 46
n(c) = chemistry = 28

$n(A \cap B) = 23,$

$n(B \cap C) = 9,\ n(A \cap C) = 14$

$n(A \cap B \cap C) = 4,$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap C) - n(B \cap C) - n(A \cap B) + n(A \cap B \cap C)$
$= 70 + 46 + 28 - 23 - 9 - 14 + 4 = 102$

$\Rightarrow$ Total $n(A \cup B \cup C) = 140 - 102 = 38$ = Not opted any course

10. Consider a triangular plot ABC with sides AB = 7 m, BC = 5m and CA = 6m. A vertical lamp - post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp - post is :

(A) $2\sqrt{21}$ (B) $\frac{2}{3}\sqrt{21}$ (C) $7\sqrt{3}$ (D) $\frac{3}{2}\sqrt{21}$

Sol. **B**

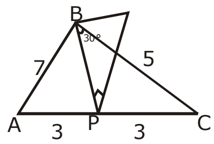
BD = hcot30° = $h\sqrt{3}$

So, $7^2 + 5^2 = 2\left(\left(h\sqrt{3}\right)^2 + 3^2\right)$

$\Rightarrow 37 = 3h^2 + 9.$

$\Rightarrow 3h^2 = 28$

h = $\sqrt{\frac{28}{3}} = \frac{2}{3}\sqrt{21}$



11. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is :

(A) $\frac{3\pi}{8}$ (B) π (C) $\frac{\pi}{2}$ (D) $\frac{5\pi}{4}$

Sol. **c**

$1 - \cos^2(2\theta) + \cos^4(2\theta) = \frac{3}{4}$

$4\cos^4(2\theta) - 4\cos^2(2\theta) + 1 = 0$

$(2\cos^2(2\theta) - 1)^2 = 0$

$\cos^2(2\theta) = \frac{1}{2} = \cos^2\frac{\pi}{4} \Rightarrow 2\theta = n\pi \pm \frac{\pi}{4}$

$\theta = \frac{n\pi}{2} \pm \frac{\pi}{8}$

n = 0

$\theta = \frac{\pi}{8}, \frac{-\pi}{8}$ (Reject)

$\theta = \frac{\pi}{2} - \frac{\pi}{8}, \frac{\pi}{2} + \frac{\pi}{8}$ (Reject)

sum = $\frac{\pi}{2} - \frac{\pi}{8} + \frac{\pi}{8} = \frac{\pi}{2}$

12. Consider the statement : "P(n) : $n^2 - n + 41$ is prime,. " then which one fo the following is true ?

(A) Both P(3) and P(5) are true (B) P(3) is false but P(5) is true

(C) Both P(3) and P(5) are false (D) P(5) is false but P(3) is true.

Sol. **A**

$p(n) = n^2 - n + 41$

n(5) = 61

n(3) = 47

13. If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, (k > 0) is 1 square unit. Then k is :

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{3}$

Sol. **A**

$$\frac{\frac{1}{k}\times\frac{1}{k}}{3} = 1$$

$$\frac{1}{k^2} = 3 \Rightarrow k^2 = \frac{1}{3} \Rightarrow k = \frac{1}{\sqrt{3}}$$

14. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then

$$\lim_{x\to1+}\frac{\left(1-|x|+\sin|1-x|\right)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

(A) does not exist (B) equals 1 (C) equals – 1 (D) equals 0

Sol. **D**

$$\lim_{x\to1+}\frac{\left(1-|x|+\sin|1-x|\right)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

$$= \lim_{x\to1^+}\frac{(1-x)+\sin(x-1)}{(x-1)(-1)}\sin\left(\frac{\pi}{2}(-1)\right)$$

$$= \lim_{x\to1^+}\left(1-\frac{\sin(x-1)}{(x-1)}\right)(-1) = (1-1)(-1) = 0$$

15. The plane passing through the point (4, – 1,2) and parallel to the lines $\frac{x+2}{3}=\frac{y-2}{-1}=\frac{z+1}{2}$ and $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{3}$ also passes through the point :

(A) (1,1,–1) (B) (–1,–1,1) (C) (–1,–1,–1) (D) (1,1,1)

Sol. **D**

let $\vec{n}$ be the normal vector to the plane passing through (4, – 1, 2) and parallel to the lines L_1 & L_2

then $\vec{n} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 2\\ 1 & 2 & 3\end{vmatrix}$

$\therefore \vec{n} = -7\hat{i}-7\hat{j}+7\hat{k}$

$\therefore$ Equation of plane is

$-7(x-4) - 7(y+1) + 7(z-2) = 0$

$\therefore x + y - z - 1 = 0$

Now check options

16. If the line 3x + 4y – 24 = 0 intersects the x - axis at the point A and the y - axis at the point B, then the incentre of the triangle OAB, where O is the origin, is :

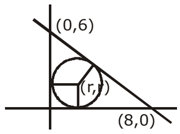
(A) (2,2) (B) (4,3) (C) (3,4) (D) (4,4)

Sol. **A**

$\frac{|3r+4r-24|}{5} = r \quad \Rightarrow \quad |7r - 24| = 5r$

$7r - 24 = \pm 5r$

$\Rightarrow r = 2 \;\&\; 14 \Rightarrow (2,2)$



17. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained. On them is noted. If the toss of the coin results in tail then a card from well - shuffled pack of nine cards numbered 1,2,3,4....,9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is :

(A) $\frac{15}{72}$ (B) $\frac{13}{36}$ (C) $\frac{19}{36}$ (D) $\frac{19}{72}$

Sol. **D**

Start —(1/2)→ dice has sum 7 or 8 = $\frac{11}{36}$

Start —(1/2)→ card has number 7 or 8 = $\frac{2}{9}$

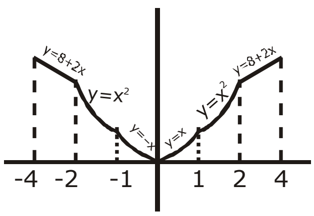
$$P(A) = \underset{\text{Head comes}}{\frac{1}{2}} \times \underset{\text{dice has 7 or 8}}{\frac{11}{36}} + \underset{\text{tail comes}}{\frac{1}{2}} \times \underset{\text{card has 7 or 8}}{\frac{2}{9}} = \frac{19}{72}$$

18. Let $f(x) = \begin{cases} \max\{|x|, x^2\}, & |x| \le 2 \\ 8 - 2|x|, & 2 < |x| \le 4 \end{cases}$. Let S be the set of points in the interval (–4,4) at which f is not differentiable. Then S.

(A) equals {–2, –1, 0,1,2}
(B) is an empty set
(C) equals {–2,2}
(D) equals {–2,–1,1,2}

Sol. **A**

$$\begin{cases} 8+2x, & -4 \le x < -2 \\ x^2, & -2 \le x \le -1 \\ |x|, & -1 < x < 1 \\ 8-2x, & 2 < x \le 4 \end{cases}$$



f(x) is not differentiable at x = = {–2,–1, 0,1,2}

S = {–2,–1,0,1,2}

19. Let f : R → R be a function such that $f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3), x \in R$. Then f(2) equals :
(A) 30 (B) 8 (C) –2 (D) –4

Sol. **C**

$f(x) = x^3 + x^2f'(x) + x.f''(2) + f'''(3), x \in R$
$f'(x) = 3x^2 + 2xf'(1) + f''(2)$
$f''(x) = 6x + 2f'(1)$ & $f'''(x) = 6$
Put x = 1 in f'(x) & x = 2 in f''(x) & find f'(1), f''(2)
$\Rightarrow f'(1) = 3 + 2f'(1) + f''(2)$

$$\begin{cases} f'(1) = 5 \\ f''(2) = 2 \\ f''(3) = 6 \end{cases}$$

$f''(2) = 12 + 2f'(1)$
$f'(x) = x^3 - 5x^2 + 2x + 6$
$\Rightarrow f(2) = -2$

20. If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to :
(A) $\frac{3}{4}$ (B) $\frac{7}{4}$ (C) $\frac{5}{4}$ (D) $\frac{3}{2}$

Sol. **B**

(1) $0 < r < 1$
$r + r^2 > 1$

$$\left(r-\left(\frac{-1-\sqrt{5}}{2}\right)\right)\left(r-\left(\frac{-1+\sqrt{5}}{2}\right)\right) > 0$$

$$\frac{\sqrt{5}-1}{2} < r < 1 \quad \text{...(1)}$$

(2) $r > 1$
$r^2 - r - 1 < 0$

$$\left(r-\left(\frac{1+\sqrt{5}}{2}\right)\right)\left(r-\left(\frac{1-\sqrt{5}}{2}\right)\right) < 0$$

$$\frac{1-\sqrt{5}}{2} < r < \frac{1+\sqrt{5}}{2}$$

$$\left(1 < r < \frac{1+\sqrt{5}}{2}\right) \quad \text{....(B)}$$

By (A) & (B)

$$r \in \left(\frac{-1+\sqrt{5}}{2}, 1\right) \cup \left(1, \frac{1+\sqrt{5}}{2}\right)$$

21. The equation of a tangent to the hyperbola $4x^2 - 5y^2 = 20$ parallel to the line x – y = 2 is :
(A) x – y + 9 = 0 (B) x – y – 3 = 0 (C) x – y + 7 = 0 (D) x – y + 1 = 0

Sol. **D**

H : $\frac{x^2}{5} - \frac{y^2}{4} = 1$

equation of tangent $\Rightarrow y = mx \pm \sqrt{a^2m^2 - b^2}$ & m = 1

$y = x \pm \sqrt{5-4} \Rightarrow y = x \pm 1$
$x - y \pm 1 = 0$

22. Let z_1 and z_2 be any two non- zero complex numbers such that $3|z_1| = 4|z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then :

(A) Im(z) = 0 (B) $|z| = \sqrt{\frac{5}{2}}$ (C) Re(z) = 0 (D) $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$

Sol. **Bonus(All options are wrong)**

$\left|\frac{z_1}{z_2}\right| = \frac{4}{3} \Rightarrow \left|\frac{3z_1}{2z_2}\right| = \frac{3}{2} \times \frac{4}{3}$

using polar form :

$\frac{3z_1}{2z_1} = 2\text{cis}\theta = 2\cos\theta + 3i\sin\theta$

$\frac{2z_2}{3z_1} = \frac{1}{2}\left(\frac{1}{\cos\theta + i\sin\theta}\right) = \frac{1}{2}(\cos\theta - i\sin\theta)$

$z = \frac{5}{2}\cos\theta + \frac{3}{2}i\sin\theta$

all options are wrong

23. A point P moves on the line 2x – 3y + 4 = 0. if Q (1,4) and R(3, – 2) are fixed points, then the locus of the centroid of $\triangle PQR$ is a line :

(A) parallel to x - axis (B) with slope $\frac{3}{2}$

(C) parallel to y - axis (D) with slope $\frac{2}{3}$

Sol. **D**

$P = (\alpha, \beta)$

$\frac{\alpha + 1 + 3}{3} = h / \frac{\beta + 4 - 2}{3} = k$

$\alpha = (3h - 4)$, $\beta = 3k - 2$ & (α, β)

$2(3h - 4) - 3(3k - 2) + 4 = 0$

$6x - 9y + 2 = 0$

24. If $\frac{dy}{dx} + \frac{3}{\cos^2 x}y = \frac{1}{\cos^2 x}$, $x \in \left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$ and $y\left(\frac{\pi}{4}\right) = \frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals :

(A) $\frac{1}{3}$ (B) $\frac{1}{3} + e^6$ (C) $\frac{1}{3} + e^3$ (D) $-\frac{4}{3}$

Sol. **B**

(1) IF = $e^{3\int \sec^2 x dx} = e^{3\tan x}$

(2) $y.e^{3\tan x} = \int \sec^2 x.e^{3\tan x}dx$

$y.e^{3\tan x} = \frac{1}{3}e^{3\tan x} + C$. $\left(y\left(\frac{\pi}{4}\right) = \frac{4}{3}\right)$

$\Rightarrow \frac{4}{3}.e^{3\tan\frac{\pi}{4}} = \frac{1}{3}e^{3\tan\frac{\pi}{4}} + C \Rightarrow C = e^3$

then $y\left(-\frac{\pi}{4}\right)$, $y.e^{-3} = \frac{1}{3}e^{-3} + e^{3} = \frac{1+3e^{6}}{3e^{3}} \Rightarrow y = \frac{1}{3} + e^{6}$

25. If $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$, then k equals :

(A) 100 (B) 50 (C) 200 (D) 400

Sol. **A**

$= \frac{1}{(21)^3}\left(\frac{(20)(21)}{2}\right)^2 = \frac{k}{21}$

$\Rightarrow k = 100$

26. Let A be a point on the line $\vec{r} = (1-3\mu)\hat{i} + (\mu-1)\hat{j} + (2+5\mu)\hat{k}$ and B(3,2,6) be a point in the space. Then the value of μ for which the vector $\overrightarrow{AB}$ is parallel to the plane x – 4y + 3z = 1 is :

(A) $\frac{1}{8}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{-1}{4}$

Sol. $\vec{r} = <1,-1,2> + \mu<-3, +1, 5>$

$\frac{x-1}{-3} = \frac{y+1}{+1} = \frac{z-2}{5} = \mu = k$

A <–3k + 1, k – 1, 5k + 2>

& B = <3,2,6>

$\overrightarrow{AB}$ = < –3 k – 2, +k – 3, 5k – 4>

then 1(–3k – 2) – 4(k – 3) + 3(5k – 4) = 0

$k = \frac{1}{4} = \mu$

27. Let d $\in$ R, and

$A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix}$ $\theta \in [0, 2\pi]$. If the minimum value of det(A) is 8, then a value of d is :

(A) –7 (B) $2\left(\sqrt{2}+2\right)$ (C) $2\left(\sqrt{2}+1\right)$ (D) –5

Sol. **D**

$\det A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix}$

$R_1 \rightarrow R_1 + R_3 - 2R_2$

$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin\theta+2 & d \\ 5 & 2\sin\theta-d & 2+2d-\sin\theta \end{vmatrix}$

$= d^2 + 4d + 4 - \sin^2\theta = (d+2)^2 - \sin^2\theta)$ min. at $\sin\theta = 1$

$=(d+2)^2 - 1 = 8$ (given)

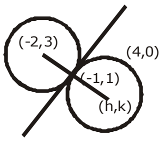
d = 1 or – 5

28. If a circle C passing through the point (4,0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, –1), then the radius of C is :

(A) $\sqrt{57}$ (B) 5 (C) $2\sqrt{5}$ (D) 4

Sol. **B**

Let the centre of circle (–2,3)



F.O.T $\rightarrow$ x.1 + y(–1) + 2(x + 1) –3(y – 1) – 12 = 0

3x – 4y – 7 = 0

$m_T = \dfrac{3}{4}$

$\dfrac{k-3}{h+2} \times \dfrac{3}{4} = -1 (\therefore m_T . m_N = -1)$

k + 3h – 7 = 0 ....(1)

distance of (h,k) from (–1,1) is equal to the distance from (4,0)

$(h - 1)^2 + (k + 1)^2 = (h - 4)^2 + (k - 0)^2$

–2h + 2k + 2 = – 8h + 16

–2h + 2k + 2 = – 8h + 16

6h + 2k –14 = 0 ...(2)

from equation (1) & (2)

h =4

k = – 5

then radius

$r = \sqrt{(4-4)^2 + (-5)^2} \Rightarrow r = 5$

29. The shortest distance between the point $\left(\dfrac{3}{2}, 0\right)$ and the curve $y = \sqrt{x}$, (x > 0) is :

(A) $\dfrac{3}{2}$ (B) $\dfrac{5}{4}$ (C) $\dfrac{\sqrt{5}}{2}$ (D) $\dfrac{\sqrt{3}}{2}$

Sol. **C**

Let $Pt\left(t, \sqrt{t}\right)$

distance formula using

$\left(t, \sqrt{t}\right) - \left(\dfrac{3}{2}, 0\right)$

$Z = \left(t - \dfrac{3}{2}\right)^2 + \left(\sqrt{t} - 0\right)^2$

$\dfrac{dz}{dt} = 2\left(t - \dfrac{3}{2}\right) + 1 = 0 \Rightarrow t = 1$

$pt = \left(1, \sqrt{1}\right) = (1,1)$

Sh. distance $= \sqrt{\dfrac{5}{4}} = \dfrac{\sqrt{5}}{2}$

30. Let $n \geq 2$ be a natural number and $0 < \theta < \pi/2$. Then $\int \frac{\left(\sin^n\theta - \sin\theta\right)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}d\theta$ is equal to :

(where C is a constant of integration)

(A) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(B) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(C) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(D) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

Sol. D

$\sin^n\theta$ common :

$$\int \frac{\sin\theta\left(1-\sin^{1-n}\theta\right)^{1/n}\cos\theta}{\sin^{n+1}\theta}d\theta$$

$1 - \sin^{1-n}\theta = t$

$-(1-n)\sin^{-n}\theta\cos\theta\, d\theta = dt$

$$\frac{\cos\theta d\theta}{\sin^n\theta} = \frac{dt}{n-1}$$

$$\frac{1}{n-1}\int (t)^{1/n}dt$$

$$\frac{1}{(n-1)}\left(\frac{t^{\frac{1}{n}}+1}{\frac{1}{n}+1}\right)+C$$