Claas slides

Discriminant :

$\rightarrow \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Outcomes of Discriminant

1 → $b^2 - 4ac > 0$, 2 distinct real roots

2. → $b^2 - 4ac = 0$, 1 real roots

3 → $b^2 - 4ac < 0$, no real roots

Ex. Solve if $kx^2 + 3kx - 2k^2 = 0$, has real, distinct roots.

$kx^2 + 3kx - 2k^2 > 0$

$ax^2 + bx + c$

$b^2 - 4ac$

$(3k)^2 - 4(k)(-2k^2) > 0$

$9k^2 - 4k(-2k^2) > 0$

$9k^2 + 8k^3 > 0$ Discriminant

$9k^2 > -8k^3$

$9 > -8k$

$-\frac{9}{8} > k$

$k < -\frac{9}{8}$

Q.

Find the values of k for which the quadratic function

$f(x) = 2kx^2 + kx - k + 2$ has real roots.

$ax^2 + bx + c$

$b^2 - 4ac$

$k^2 - 4(2k)(-k+2)$

$k^2 - 8k(-k+2)$

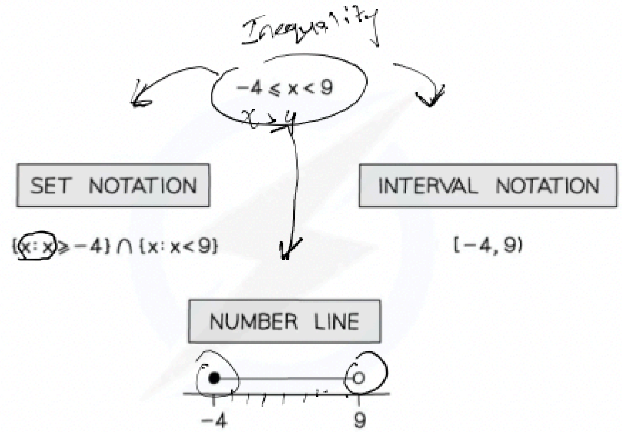
$k^2 + 8k^2 - 16k > 0$

$9k^2 - 16k > 0$

$9k^2 > 16k$

$9k > 16$

$k > \frac{16}{9}$



Ex

a) Find the discriminant for the quadratic function $x^2 + 8x + 15$.

b) Write down the number of real solutions to the equation $x^2 + 8x + 15 = 0$.

a) $b^2 - 4ac$

> 4

b) 2 distinct real roots.

Q1.

a) Write down, in terms of k, the discriminant of $x^2 + 8x + 4k$.

b) Hence find the values of for k which the equation $x^2 + 8x + 4k = 0$ has two real and distinct solutions.

a) Ans $\boxed{64 - 16k}$ (1)

b) $64 - 16k > 0$

$64 > 16k$

$\frac{64}{16} > k$

$k < \frac{64}{16}$ 4

Ans. $\boxed{k < 4}$ (2)

$3x + 2 \geq 11 - 2x$ 3

$3x + 2x + 2 \geq 11$

$5x \geq 11 - 2$

Q2.

Solve the inequality $3x + 4 \leq 5(x - 1)$.

1. Linear inequality.
Expand.

$3x + 4 \leq 5x - 5$

$4 \leq 5x - 3x - 5$

$4 \leq 2x - 5$

$9 \leq 2x$

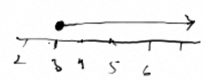
$\frac{9}{2} \leq x$

$x \geq \frac{9}{2}$

Ex: $3x + 2 \geq 11 - 2x$

$3x \geq 11 - 2$

$x \geq \frac{9}{3}$ 3 $\quad x \geq 3$



1) Linear inequality → 1 answer

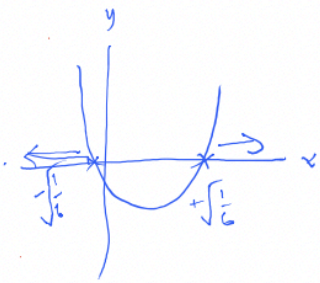
2) Quadratic inequality → 2 values

EX

Solve the inequality $\frac{5}{3x^2+2} \leq 2$.

$5 \leq 2(3x^2+2)$

$0 \leq 2(3x^2+2) - 5$



x - values

$5 = 2(3x^2+2)$

$5 = 6x^2+4$

$6x^2 - 1$

$1 = 6x^2$

$\pm\sqrt{\frac{1}{6}} = x$

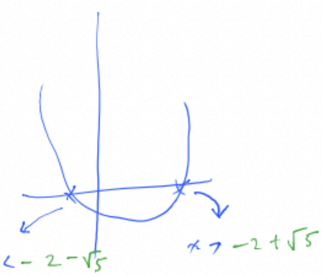
$x \leq -\sqrt{\frac{1}{6}}$

$x \geq +\sqrt{\frac{1}{6}}$

Q3

Solve the inequality $(x+2)^2 > 5$.

1. Solve x-values → Use Quadratic formula
2. put inequality sign



$(x+2)^2 > 5$

$(x+2)(x+2) > 5$

$x^2 + 2x + 2x + 4 > 5$

$x^2 + 4x - 1$

$\frac{-4 \pm \sqrt{4^2 - 4(1)(-1)}}{2(1)} = -2 + \sqrt{5}, \ -2 - \sqrt{5}$

$x < -2$

EX

$19 - 2x \geq 5$ ①

$9 < x + 5$ ②

$-2x \geq -14$

$x \geq 7$

$19 - 5 \leq 2x$

$14 \leq 2x$

$\frac{14}{2} \leq x$

$x \geq 7$

$9 - 5 < x$

$4 < x$

$\boxed{4 < x \geq 7}$

Notes

Discriminants

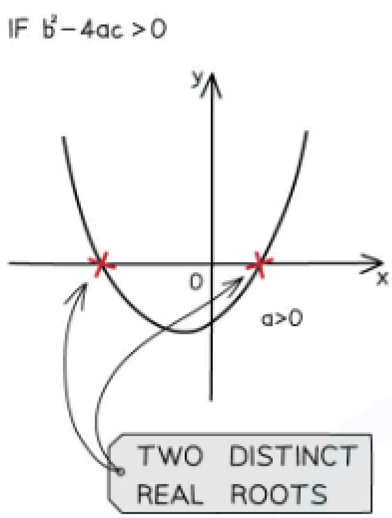
What is a discriminant?

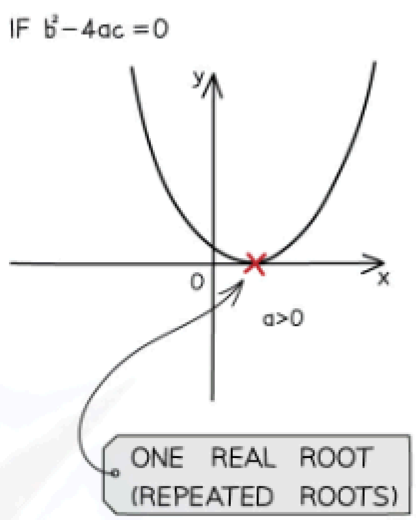
- The discriminant is the part of the quadratic formula that is under the square root sign $b^2 - 4ac$
- It is sometimes denoted by the Greek letter Δ (capital delta)

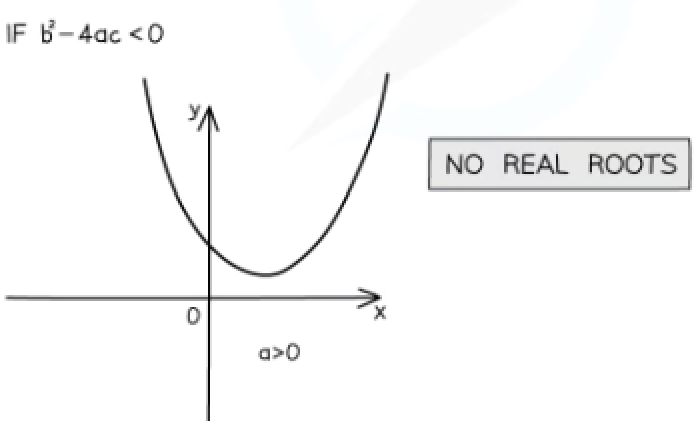
How does the discriminant affect graphs and roots?

There are three options for the outcome of the discriminant:

- If $b^2 - 4ac > 0$ the quadratic crosses the x-axis **twice** meaning there are **two distinct real roots**
- If $b^2 - 4ac = 0$ the quadratic touches the x-axis **once** meaning there is **one real root** (also called repeated roots)
- If $b^2 - 4ac < 0$ the quadratic **does not cross** the x-axis meaning there are no real roots

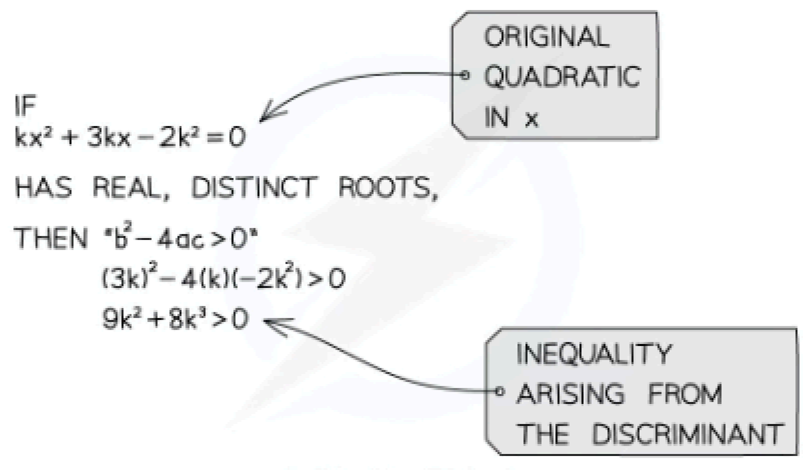






Discriminant and inequalities

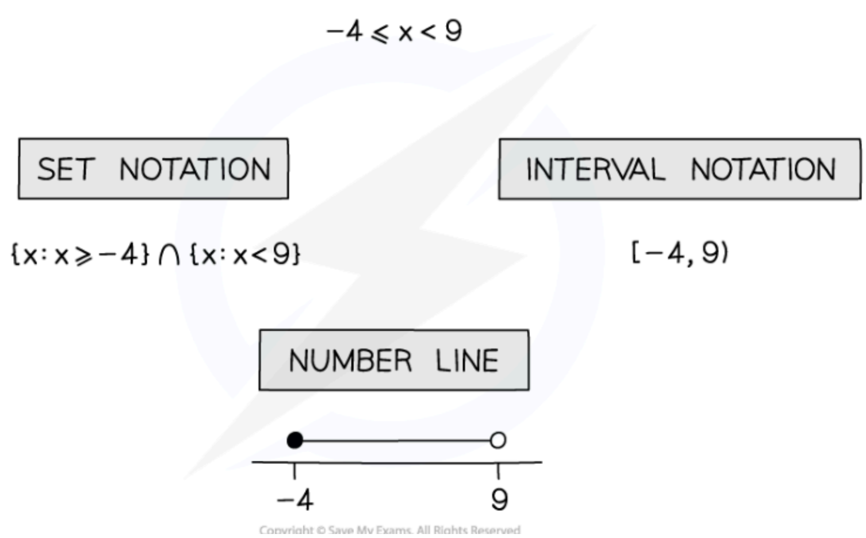
- You need to be able to **set up** and **solve** equations and inequalities (often quadratic) arising from the discriminant
- Sketch the quadratic and decide whether you're looking above or below zero to write your solutions correctly



Linear Inequalities

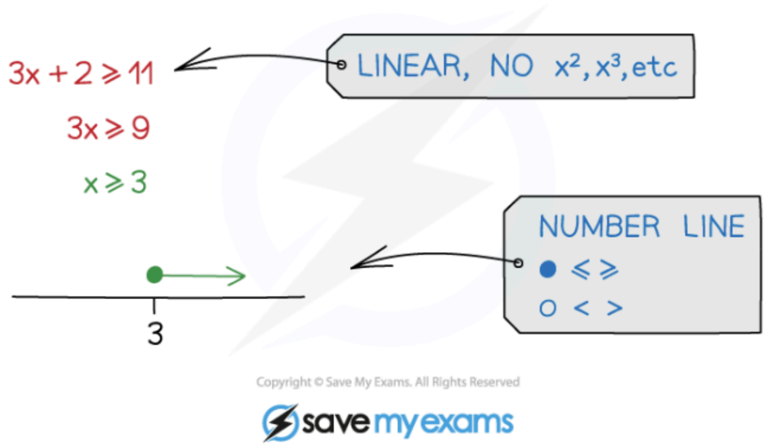
What are linear inequalities?

- Linear inequalities are similar to equations but answers take a range of values
- Linear means there will be no terms other than degree 1
  - no squared terms or higher powers, no fractional or negative powers
- Inequalities use the symbols following symbols
  - $>$ Greater than e.g. $5 > 3$
  - $<$ Less than e.g. $-8 < 7$
  - $\geq$ Greater than or equal to
  - $\leq$ Less than or equal to
- Inequalities can be represented in many ways using number lines, set notation and interval notation



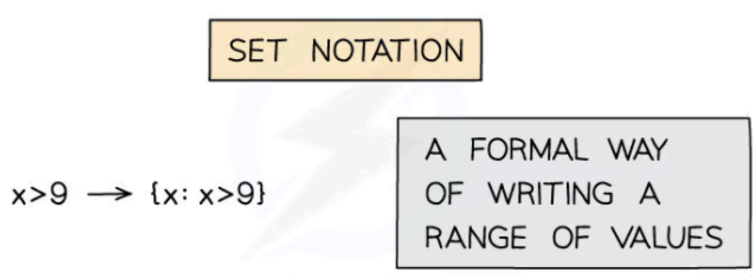
Number line diagrams

- Number line diagrams are made up from circles and lines set above a number line
  - A filled-in circle or empty circle above a number denotes whether the number is included or not
    - filled in for the greater/less than or equal to symbols $\le$ $\ge$
    - empty for the greater/less than symbols $<$ $>$
  - Arrows show the range of values that are allowed



Set notation

- Set notation is a formal way of writing a range of values
- Use of curly brackets { }
- Intersection ∩ and union ∪ may be used
- Not to be confused with interval notation

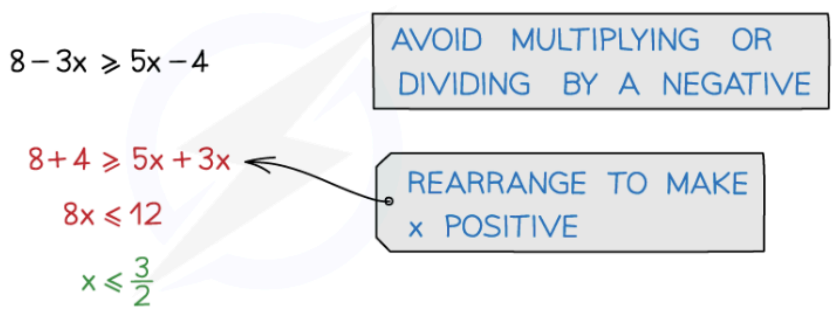


Skills for solving linear inequalities

- representing and interpreting inequalities displayed on a number line
- **writing** and **interpreting** set notation
  - eg $\{x : x > 1\} \cap \{x : x \leq 7\}$ is the same as $1 < x \leq 7$
- **writing** and **interpreting** interval notation
  - eg $[-4, 6)$ is the same as $-4 \leq x < 6$

How do I solve linear inequalities?

- Treat the inequality as an equation and solve
  - avoid multiplying or dividing by a negative
  - if unavoidable, “flip” the inequality sign so $< \rightarrow >$, $\geq \rightarrow \leq$, etc
  - try to rearrange to make the x term positive



Worked example

Given that $\{x : x \geq 0\}$, solve the simultaneous inequalities $19 - 2x \geq 5$ and $9 < x + 5$, giving your answer in set notation.

$19 - 2x \geqslant 5$

$19 - 5 \geqslant 2x$

$2x \leqslant 14$

$x \leqslant 7$

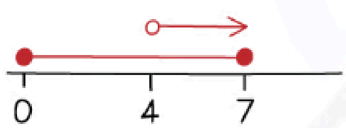
$0 \leqslant x \leqslant 7$

SOLVE EACH INEQUALITY SEPARATELY, DEALING WITH NEGATIVES CAREFULLY

THE QUESTION SAID "$x \geqslant 0$"

$9 < x + 5$

$x > 4$



DRAW A QUICK DIAGRAM TO SEE WHERE ANSWERS COINCIDE (OVERLAP)

$4 < x \leqslant 7$

$\{x : x > 4\} \cap \{x : x \leqslant 7\}$

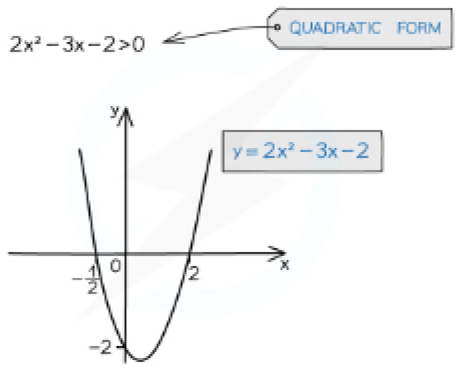
FINAL ANSWER IN SET NOTATION

Quadratic Inequalities

Quadratic inequalities

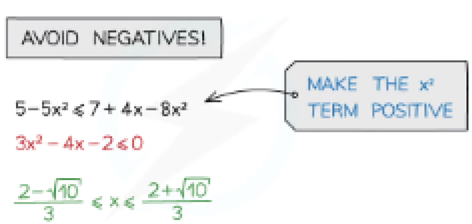
- Similar to quadratic equations quadratic inequalities just mean there is a range of values that satisfy the solution
- Sketching a quadratic graph is essential

Can involve the **discriminant** or **applications** in **mechanics** and **statistic**



How do I solve quadratic inequalities?

- **STEP 1**: **Rearrange** the inequality into quadratic form with a **positive squared term**
  - $ax^2 + bx + c > 0$ (>, <, ≤ or ≥)
- **STEP 2**: Find the **roots** of the quadratic equation
  - **Solve** $ax^2 + bx + c = 0$ to get x_1 and x_2 where $x_1 < x_2$
- **STEP 3**: **Sketch** a graph of the quadratic and **label the roots**
  - As the squared term is positive it will be **"U" shaped**
- **STEP 4**: **Identify** the **region** that satisfies the inequality
  - For $ax^2 + bx + c > 0$ you want the region **above** the *x*-axis
    - The solution is $x < x_1$ **or** $x > x_2$
  - For $ax^2 + bx + c < 0$ you want the region below the *x*-axis
    - The solution is $x > x_1$ **and** $x < x_2$
    - This is more commonly written as $x_1 < x < x_2$
-
  - **avoid** multiplying or dividing by a negative number
    If unavoidable, "**flip**" the inequality sign so $< \to >$, $\geq \to \leq$, etc
  - **avoid** multiplying or dividing by a **variable** (**x**) that **could** be negative
    (multiplying or dividing by x^2 guarantees positivity (unless **x** could be 0) but this can create extra, invalid solutions)
  - **do** rearrange to make the x^2 term positiveBe careful:



Worked example

Find the set of values for which $3x^2 + 2x - 6 > x^2 + 4x - 2$ giving your answer in set notation.

$3x^2 + 2x - 6 - x^2 - 4x + 2 > 0$

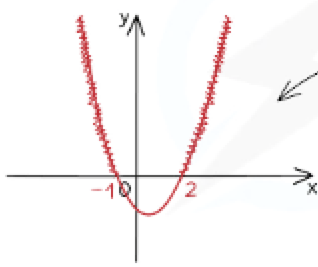
$2x^2 - 2x - 4 > 0$

REARRANGE TO QUADRATIC FORM

$2(x^2 - x - 2) > 0$

$x^2 - x - 2 > 0$

$(x - 2)(x + 1) > 0$



CRUCIAL: SKETCH THE GRAPH TO SEE WHERE THE SOLUTIONS ARE

$x < -1$ OR $x > 2$

$\{x : x < -1\} \cup \{x : x > 2\}$

FINAL ANSWER IN SET NOTATION