Ozobot Bit Classroom Application:

Inverse Square Law of Light Simulation

Created by

Richard Born

Associate Professor Emeritus

Northern Illinois University

richb@rborn.org

Topics

Physical Science, Physics, Inverse Square Law of Light, Illumination, Interpolation, Extrapolation, Data Collection, Graphing, Curve Fitting, Excel, Logger Pro

Ages

Grades 7-10 (Exercises 1-6)

Grades 11-12 (Exercises 1-8)

Duration

Grades 7-10 (30 minutes)

Grades 11-12 (45 minutes)

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rborn@niu.edu

Introduction

The study of the behavior of light is integral to any introductory course in physics. Phenomena involving light include, but are not limited, to reflection, refraction, interference, diffraction, polarization, and a variety of quantum effects. One of the first properties of light that is studied is related to our everyday observation that a light appears less bright as we move away from it. The exact relationship between "brightness" and distance from the light source is the topic of this Ozobot Bit classroom application.

There are a variety of ways by which the brightness of light can be measured. One very common way is to use a light sensor to measure what is known as **illumination**, a more scientific term for brightness. The SI (International System of Units) unit for illumination (also called illuminance) is the **lux**. The essentially obsolete incandescent light used the watt as a measure of brightness, but this included not only the light energy, but the large amount of heat energy dissipated by the bulbs. With the widespread current use of CFLs (**C**ompact **F**luorescent **L**ights) and LEDs (**L**ight **E**mitting **D**iodes), ratings are generally described by the use of **lumens**, a true measure of *light output*. One lux is equal to one lumen per square meter.

That's a lot of terminology! With our Ozobot Bit classroom application, we will simply refer to the unit of measure of illumination as ***AIL***, short for **A**rbitrary **I**llumination **U**nit). *Ozobot bit will act as our **simulated light sensor**, allowing us to measure illumination from a "simulated" light source*. The Ozobot Bit simulation avoids many of the complications encountered when performing the experiment with real apparatus:

- It is difficult to obtain a reasonably small, point source of light.
- It is not easy to arrange the light source and light sensor in a way that the light sensor can be placed at various distances from the light source.
- Some LED sources of light decrease in intensity as they warm up, resulting in an increase in the value of the exponent for the inverse "square" law.
- There should be no obstructions between the light source and light sensor.
- Reflective surfaces near the light source should be removed.

The Inverse Square Law of Light

Geometric arguments can be used to explain the inverse square law of light, as doubling the distance from the light source increases the area by four times through which the light is shining. Tripling the distance increases the area by nine times, and so on. Suppose that ***I*** is the illumination and ***D*** is the distance between the light source and the light sensor. In mathematical terms, this inverse square relationship can be expressed as:

$$I \propto \frac{1}{D^2} \quad \text{or} \quad I = \frac{k}{D^2} \quad \text{or} \quad ID^2 = k$$

where k is the constant of proportionality.

The Map for this Classroom Application

For reference while discussing this application, Figure 1 shows a small version of the Ozobot Bit map that students will be using. A full page version that can be printed for use with Ozobot Bit appears on the last page of this document.

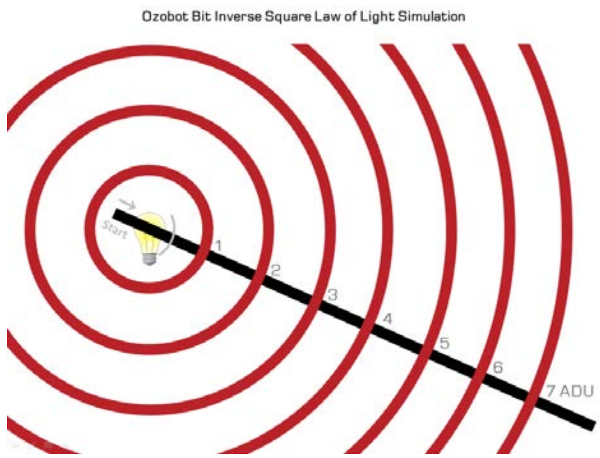


Figure 1

The simulated light source is shown as a light bulb at the center of a series of concentric circles that are centered on the filament of the light bulb. The smallest circle is 1 ADU (**A**rbitrary **D**istance **U**nit) from the light bulb's center. As you can see, the circles range from 1 ADU to a maximum of 7 ADUs from the center of the bulb. When Ozobot Bit is started, he should be placed at the location labeled "Start" in the direction indicated by the gray arrow, and his leading edge on the curved gray line. Ozobot Bit will act as though he is a light sensor, stopping at each of the circles and blinking his LED to indicate the value of the illumination in AIUs at each distance from the light source. After blinking the illumination value for a distance of 7 ADUs, Ozobot Bit will stop and turn off.

The value of the illumination could be anywhere between 0 and 999. Suppose that the illumination is 572 AIUs. Ozobot bit would then blink red 5 times, green 7 times, and blue 2 times, leaving a small time interval between each of the colors to allow recording the value:

In the event that any of the digits is zero, Ozobot Bit will blink WHITE once. Therefore, if the illumination is 040 (forty), then the blinking sequence would be:

Running the Ozobot Bit Program

1. Load the OzoBlockly program *LightIllumination.ozocode*.
2. Make sure that Ozobot Bit is calibrated on paper before running the program.
3. Make sure that Ozobot Bit has clean wheels and plenty of battery charge.
4. Place Ozobot Bit at the location labeled "Start", facing the direction shown by the gray arrow, and with Ozobot's leading edge on the curved gray line. Ozobot Bit will display an AQUA LED while moving.
5. Start Ozobot Bit by double-pressing the start button.
6. Ozobot Bit will then behave as described in the previous section of this document. After displaying the illumination for the 7 ADU circle, he will stop and power down. Each time the student lab group runs the program, the results will be slightly different as the program has some randomness built-in.

Student Exercises:

Student Exercise #1 (Grades 7-12): *Have each of the lab groups run their OzoBlockly program while explaining to them how to determine the illumination in AIUs. There is no need for them to record this data, as the purpose of this exercise is only to make sure that they know how to determine the illumination for each of the seven distances from the light source.*

Student Exercise #2 (Grades 7-12): *Now that the student groups know how to determine the illumination, ask them to collect illumination data for three runs and record their data in the data table at the top of page 5. To reduce the effects of randomness, have them compute the average illumination of the three runs for each of the distances from the light source. (The right-most column of the data table, ID^2, will be used in student exercise 5.)*

Student Exercise #3 (Grades 7-12): *Ask the lab groups to plot the average illumination and corresponding distance from the light source on the graph at the bottom of page 5. The students will clearly observe that the points do not fall on a straight line. Ask them to use their best judgment in drawing a curved line that fits the data well even though it may not lie on every point. Then ask the students to explain* ***in their own words*** *what the relationship seems to be between illumination and distance from the light source.*

Student Exercise #4 (Grades 7-12): *Explain the concepts of* ***interpolation*** *(estimating between known data points) and* ***extrapolation*** *(estimating beyond the ends of the known data points) to the class.*

a) Which (interpolation or extrapolation) would you be doing if you were to estimate the illumination when the distance is 3.5 ADUs? What is the value of your estimate in AIUs?
b) Which (interpolation or extrapolation) would you be doing if you were to estimate the illumination when the distance is 0.5 ADUs? What is the value of your estimate in AIUs?
c) Which (interpolation or extrapolation) would you be doing if you were to estimate the illumination when the distance is 10 ADUs? What is the value of your estimate in AIUs?
d) Ask the students if they feel more confident in their interpolated illuminations or their extrapolated illuminations. Why?

Student Exercise #5 (Grades 7-12): *The idea that "as the distance increases, the illumination decreases quite rapidly" may suggest what is referred to as an* ***inverse square*** *relationship between the variables. Such a relationship is described mathematically in the following way:*

$$\mathrm{I} \propto \frac{1}{\mathrm{D}^2} \quad \text{or} \quad \mathrm{I} = \frac{\mathrm{k}}{\mathrm{D}^2} \quad \text{or} \quad \mathrm{ID}^2 = \mathrm{k}$$

The symbol ∝ is read "is proportional to" and k is the constant of proportionality. In the case of our experiment, we can test this by finding the product of illumination ***I*** *by the square of the distance,* $\boldsymbol{D^2}$. *If this product is nearly the same for each (**I, D**) data pair, we have pretty convincing evidence that* ***illumination*** *is* ***inversely*** *proportional to the* ***square of the distance****. Ask the students to determine the* $\boldsymbol{ID^2}$ *products and record their data in the right-most column of their data tables. They should find the products all quite similar in value, in which case they should determine the average of these* $\boldsymbol{ID^2}$ *products and record this average in the bottom-right cell of the data table.*

Student Exercise #6 (Grades 7-12): *Now that we have an equation relating illumination and distance, e.g.,* $\boldsymbol{ID^2} \approx 989$, *we should be able to make much more accurate interpolation and extrapolation predictions than we did back in student exercise 4. Using your equation, determine the illumination for distances of (a) 3.5 ADUs, (b) 0.5 ADUs, and (c) 10 ADUs.*

Student Exercise #7 (Grades 11-12, Using Spreadsheet Software such as Excel): *When experimental data suggests an inverse relationship between the two variables* ***A*** *and* ***B****, there are many possibilities, the most common being:*

(a) A is inversely proportional to B, an ***inverse first power*** *relationship.*
(b) A is inversely proportional to B^2*, an* ***inverse square*** *relationship.*
(c) A is inversely proportional to B^3*, an* ***inverse cube*** *relationship.*

In student exercise 5, we found that the relationship between illumination ***I*** *and distance* ***D*** *is an inverse square power relationship. But how do you proceed if you are not sure what the power is? With spreadsheet software such as Excel, there is a nice way to determine the power.*

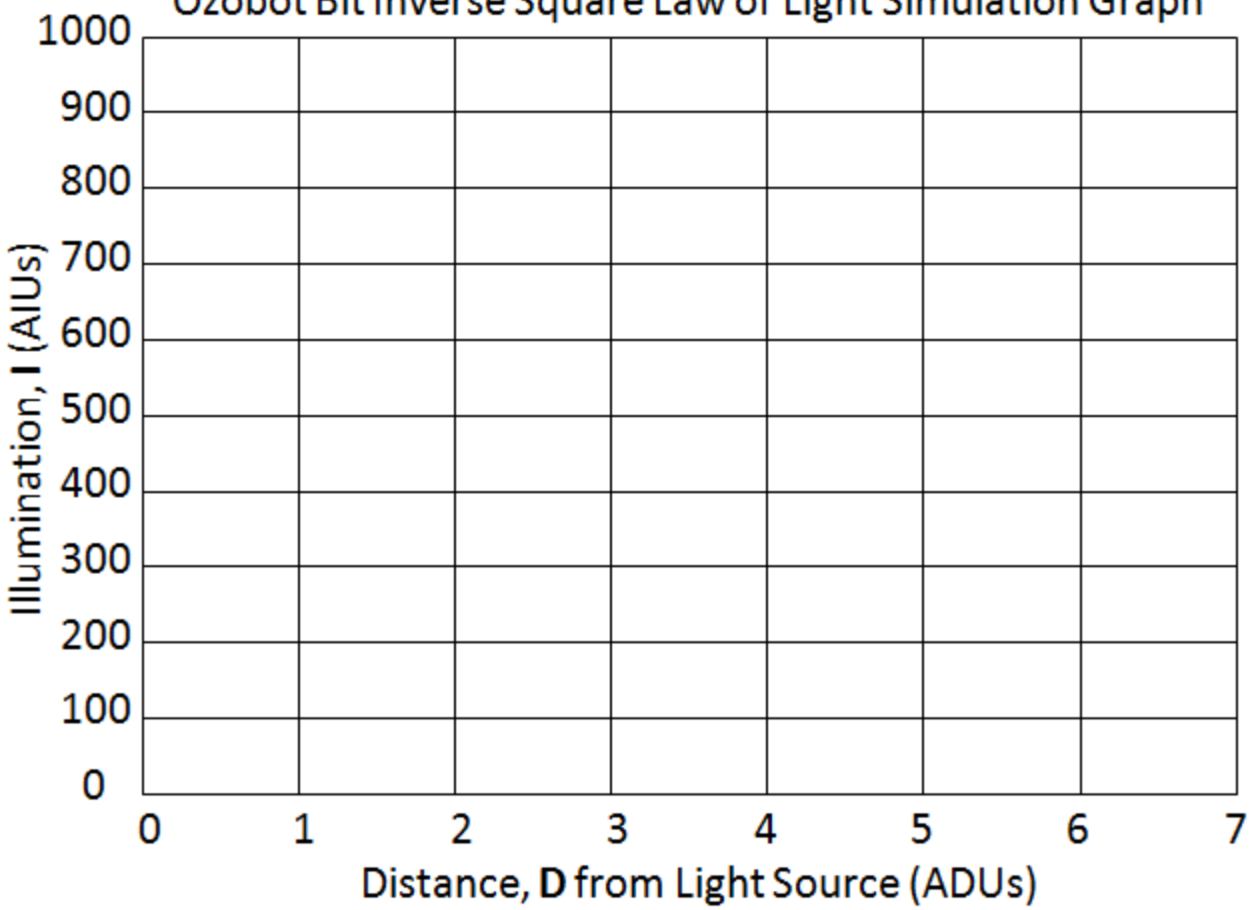
Copy your distance and average illumination data pairs into two columns in Excel, and then create an Excel scatter graph of ***I*** *(on the y-axis) vs.* ***D*** *(on the x-axis). Click on one of the data points and select "Add Trendline". Select "Power" as the "Trend/Regression Type", and also check the box for "Display Equation on chart". You should get an equation similar to* $\boldsymbol{y = 991.11x^{-2.002}}$. *Since the power, -2.002, of x (distance) is very close to -2, you can feel confident that the relationship between pressure and volume is an inverse square power relationship.*

Student Exercise #8 (Grades 11-12, Using Vernier Software and Technology's Logger Pro Software): *If you have access to Logger Pro, this can be used instead of spreadsheet software such as Excel. Enter your distance and average illumination data from student exercise 2 into a new Logger Pro file. Using the "Analyze" option in Logger Pro and then the "Curve Fit" option, use "Define Function" supplying the equation* ***A*x^C*** *on your data. You should find that the value of the constant* ***C*** *is very close to -2, providing strong evidence that the relationship between illumination and distance is an inverse square power relationship. Also, the value of the constant* ***A*** *should be in the ballpark of the average value of the* $\boldsymbol{ID^2}$ *products in your original data table.*

Ozobot Bit Inverse Square Law of Light Simulation Data Table

Distance, **D** from Light Source (ADUs)	Illumination, **I** (AIUs)				Product of Illumination times Distance Squared **ID²**
	Run 1	Run 2	Run 3	Average	
1					
2					
3					
4					
5					
6					
7					
				Average **ID²** →	

Ozobot Bit Inverse Square Law of Light Simulation Graph



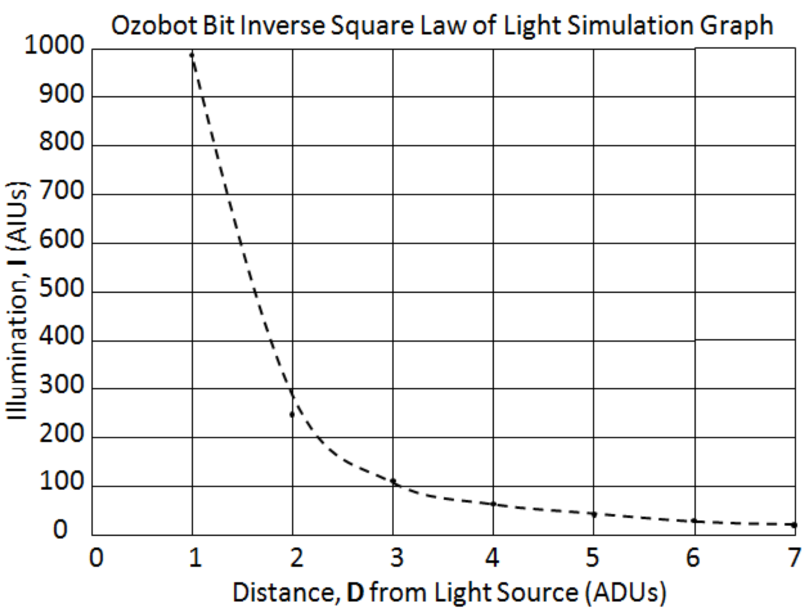
Answers to Exercises

1. *See the section entitled "The Map for This Application".*
2. *Here is a typical data table that might be obtained by a lab group:*

Ozobot Bit Inverse Square Law of Light Simulation Data Table

Distance, **D** from Light Source (ADUs)	Illumination, **I** (AIUs)				Product of Illumination times Distance Squared **ID²**
	Run 1	Run 2	Run 3	Average	
1	991	984	994	989.7	
2	240	246	254	246.7	
3	103	114	114	110.3	
4	66	66	57	63	
5	34	36	45	38.3	
6	30	29	25	28	
7	21	21	18	20	
				Average **ID²** →	

3. *A typical lab group's graph might appear as follows:*



Typical responses regarding the relationship between illumination I and distance D might be:

When one gets big the other gets smaller.
As the distance increases, the illumination decreases. As the illumination increases, the distance decreases.
As the distance increases, the illumination decreases quite rapidly.

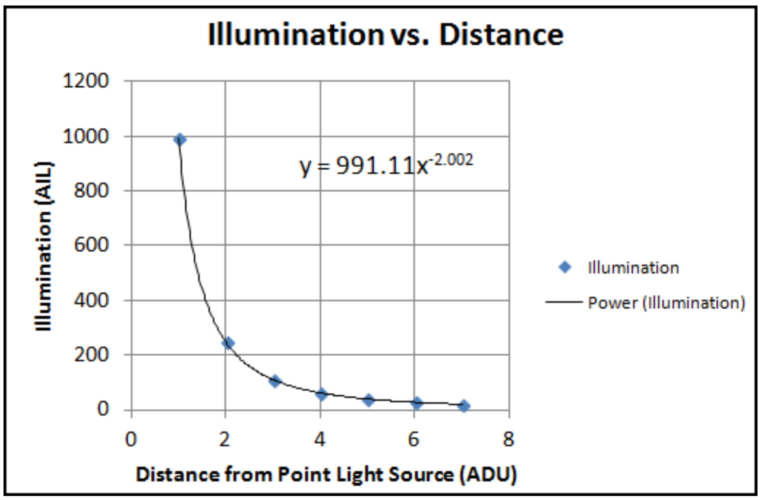
4. *(a) Interpolation. The illumination would be approximately 80 AIUs.*
   (b) Extrapolation. There will likely be very wide variances in lab group guesses—maybe from 2000 to 4000.
   (c) Extrapolation. Guesses might be between 8 and 12.
   (d) Most students will indicate that they are more confident in the interpolated values. The extrapolated values are beyond the ends of the graph, and it is difficult to know exactly how to extend the curve.

5. *Typical lab group data tables will appear as follows, with* $\boldsymbol{ID^2}$ *nearly the same, and having an average of about 989.*

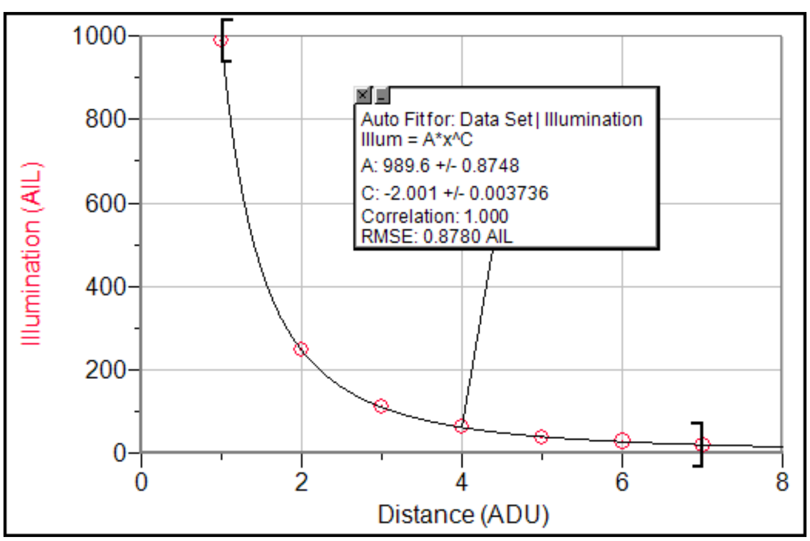
Ozobot Bit Inverse Square Law of Light Simulation Data Table

Distance, **D** from Light Source (ADUs)	Illumination, **I** (AIUs)				Product of Illumination times Distance Squared **ID²**
	Run 1	Run 2	Run 3	Average	
1	991	984	994	989.7	989.7
2	240	246	254	246.7	986.8
3	103	114	114	110.3	992.7
4	66	66	57	63	1008
5	34	36	45	38.3	957.5
6	30	29	25	28	1008
7	21	21	18	20	980
				Average **ID²** →	989

6. *(a) 81 AIU, (b) 3956 AIU, and (c) 9.9 AIU.*
7. *A typical Excel chart would look similar to this:*



8. *A typical Logger Pro chart would look like this:*



Ozobot Bit Inverse Square Law of Light Simulation