A Lesson in Probability and Statistics

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Topics

Probability and Statistics

Robotics

Computer Science

Ages

Grades 7+

Duration:

approx. 50 minutes

A LESSON IN PROBABILITY AND STATISTICS

TEACHER'S GUIDE

What will students learn?

- *How can you program Ozobot to follow a predetermined path when it approaches an intersection?*
- *How is Ozobot programmed by default to respond when encountering an intersection?*
- *What is an OzoPill Counter?*
- *What is the meaning of each of the three OzoPill Counter codes: Enable, Pill+1 and Pill-1?*
- *What visual feedback does Ozobot provide when encountering each of these codes?*
- *What is the default value for the pill code counter when Ozobot is powered up?*
- *What is the maximum value the pill counter code can ever have?*
- *What happens to Ozobot when the pill counter reaches the value zero (0)?*
- *What is meant by the concept of probability?*
- *What are the theoretical probabilities for specific events while Ozobot traverses its map?*
- *How can you perform an experiment and collect statistics to support the theoretical probabilities?*

Explanation of OzoCodes Used in This Lesson

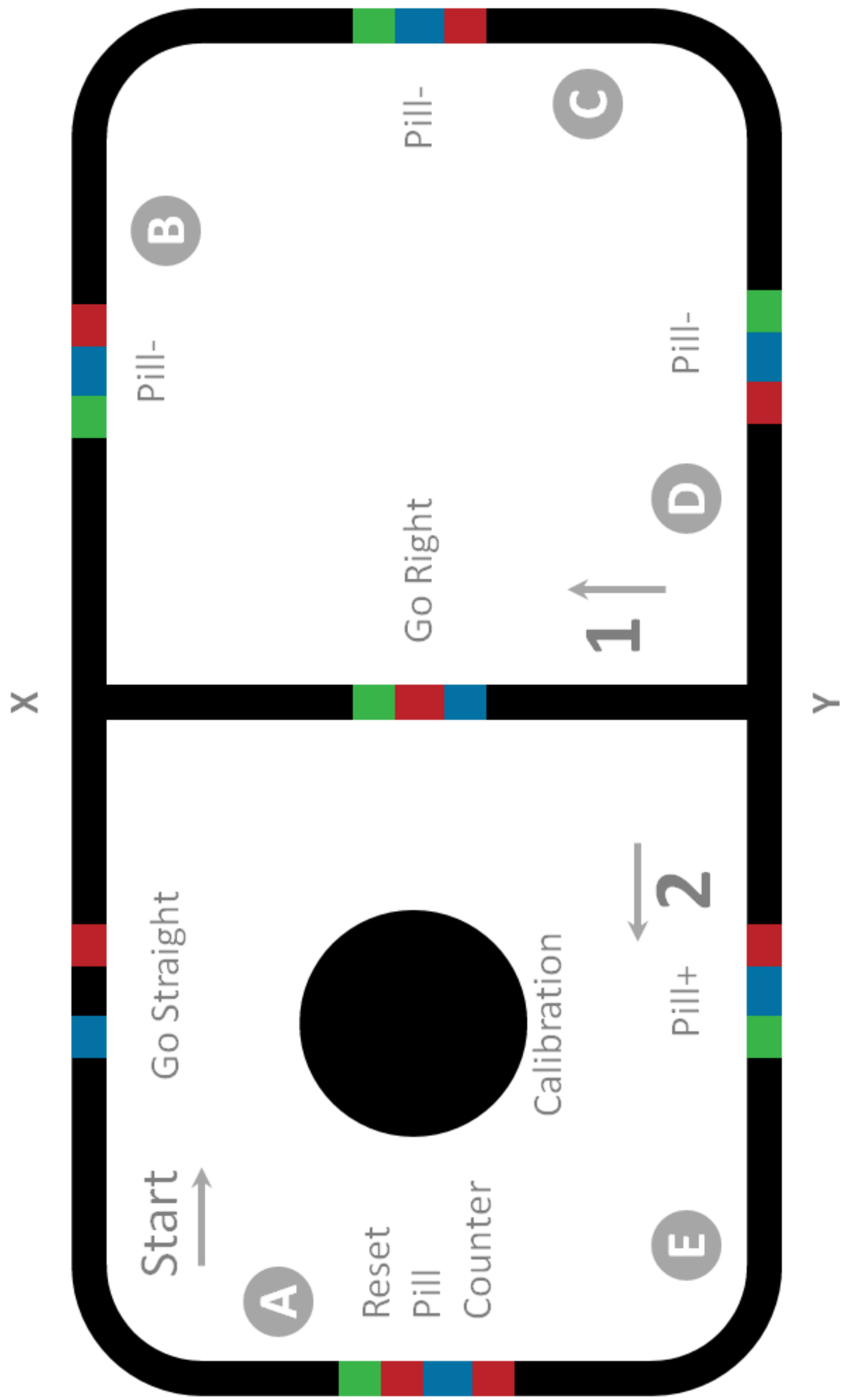
-  *The **Go Straight** OzoCode tells Ozobot to continue going straight when it approaches the intersection following the code. This code is a **one-directional** code (not a symmetric code), so the order of the three colors in the code is important—Ozobot should be moving toward the blue first.*
-  *The **Go Right** OzoCode tells Ozobot to turn right when it approaches the intersection following the code. Like the Go Straight code, this code is one-directional, so the order of the three colors in this code is important—Ozobot should be moving toward the blue first.*

-  ***The Enable OzoPill Counter code, a one-directional code, is designed to set or reset the so-called pill counter. When Ozobot is powered-on the pill counter is set at its default value of 5. Similarly, when Ozobot passes over this code, the pill counter is reset to its default value of 5. After passing over an Enable OzoPill Counter code, Ozobot will flash white 5 times with a solid green background light.***
-  ***The OzoPill-1 code tells Ozobot to decrease the pill counter by 1 after passing over this code. It will count down by 1 each time it passes over this code until the pill counter has reached the value 0 (zero). Upon reaching the value 0, Ozobot executes a "done" maneuver, disabling line-following and blinking red. Ozobot can then be reset by turning it off and then on again. This reset will set the value of the pill counter back to the default value of 5. Note that Ozobot will flash white the current value of the pill counter with solid red background light after passing over an OzoPill-1 code, providing visual feedback for the user.***
-  ***The OzoPill+1 code, whose colors are reversed from the OzoPill-1 code, tells Ozobot to increase the pill counter by 1 after passing over this code. However, it is important to note that this code will never allow the pill counter to exceed the value of 5. This may help to explain why the OzoCodes Color Code Reference Chart describes the counters as "from five down to stop." Note that Ozobot will flash white the current value of the pill counter with solid green background light after passing over an OzoPill+1 code, providing visual feedback for the user.***

The OzoMap for This Lesson

The OzoMap on page 3 can be duplicated for use by students with their Ozobots. Printing the OzoMap on a bright white card stock type paper is encouraged as it has proven to provide extremely accurate reading of the color codes by Ozobot.

After explaining the OzoCodes used in this lesson, a detailed discussion of this OzoMap with the students is important as it will promote an understanding of Ozobot's interaction with the OzoMap and its OzoCodes.



Student/Teacher OzoMap Discussion Questions

The answer to each of the questions for teacher reference is shown in italic in square brackets [ ] following each question.

1. **What is the effect of the Go Straight code following Ozobot's power-up at the start?** ***[When Ozobot reaches the intersection labeled X, Ozobot will continue to go straight and not turn right. The code in effect cancels Ozobot's default behavior of independently and randomly choosing which direction to take this time when it approaches the intersection.]***
2. **What is the value of the pill counter after Ozobot's power-up at the start?** ***[At power-up, the pill counter always defaults at its highest possible value, namely 5.]***
3. **After going straight through intersection X, what is the value of Ozobot's pill counter at points B, C, and D in the map?** ***[Each of the three OzoPill-codes decreases the pill counter by 1. Therefore, the values of the pill counter at points B, C, and D are 4, 3, and 2, respectively.]***
4. **When Ozobot reaches the intersection labeled Y, how does it decide whether to go the direction labeled 1 or the direction labeled 2?** ***[Ozobot makes independent decisions using algorithms involving random logic that have been programmed into Ozobot.]***
5. **What is the probability that Ozobot will go the direction labeled 1? labeled 2?** ***[Through its random logic, Ozobot will go direction 1 about half the time and direction 2 the other half. Here is where the teacher can explain a little about the concept of probability.*** ***Probability*** ***is usually expressed by mathematicians and statisticians by*** ***numbers between 0 and 1 inclusive****, with 1 representing absolute certainty. Therefore, Ozobot will follow direction 1 with a probability of 0.5 and will follow direction 2 with a probability of 0.5.]***
6. **What is the effect of the Go Right code on Ozobot in the event that Ozobot takes the direction labeled 1? [When Ozobot reaches the intersection labeled X, Ozobot will turn to the right with a probability of 1 (absolute certainty)** ***The code in effect cancels Ozobot's default behavior of independently and randomly choosing which direction to take this time when it approaches the intersection.]*.**

7. ***Assuming that Ozobot has taken direction 1, what will the value of the pill counter be at points B and C? [When Ozobot was at point D, its pill counter had the value 2. Therefore, at points B and C, the pill counter will have the values 1 and 0, respectively, since it has passed two additional OzoPill- codes.]***
8. ***What will Ozobot now do upon reaching point C? [Since the pill counter has reached the value 0, Ozobot will execute a "done" maneuver, disabling line-following and blinking red.]***
9. ***Now let's suppose that instead of following direction 1 at intersection Y, that Ozobot follows direction 2. What will be the value of the pill counter when Ozobot reaches point E? How does Ozobot respond visually to the OzoPill+ code? [After reaching point E, the value of the pill counter will be increased to the value 3, as it had the value 2 back when it was at point D. Ozobot flashes white three times (since the pill counter has the value 3) with solid green background light in response to the OzoPill+ code.]***
10. ***What will be the value of the pill counter when Ozobot reaches point A? [Before reaching the Reset Pill Counter code, the pill counter has the value 3. After reaching point A, the value of the pill counter will be reset to the default value of 5, and Ozobot will be in the same state as though it had just started, continuing with another loop.]***

Probability Considerations

Of particular interest in this OzoMap is to determine the probability that Ozobot (a) will not return to the start position before executing a done maneuver, (b) will return to the start position exactly once before executing a done maneuver, (c) will return to the start position exactly twice before executing a done maneuver, (d) will return to the starting position exactly three times before executing a done maneuver, etc.

Study of these probabilities is closely related to one of the major features of Ozobot—it makes independent routing decisions at intersections based upon algorithms involving random logic. To say that Ozobot makes independent routing decisions means that the decision that it makes at one intersection "has nothing to do" with the decision that it makes later at the same intersection or at

another intersection. Moreover, it turns out that when two or more events are independent, the probability that all of the events occur is obtained by multiplying their separate probabilities. This probability rule applies to two consecutive tosses of a coin, to two consecutive tosses of a die, and to Ozobot's random decisions upon entering intersections.

Now let's see if we can determine the probabilities proposed at the beginning of this section.

- ***What is the probability that Ozobot will not return to the start position before executing a done maneuver? [Ozobot will not return to the start position if the decision is made to go the direction labeled 1 when reaching intersection Y. The probability to go direction 1 is 0.5, as discussed earlier. So, in the long run, Ozobot can be expected to execute a done maneuver without returning to the start position about half the time.]***
- ***What is the probability that Ozobot will return to the start position exactly once before executing a done maneuver? [Ozobot will return to the start position exactly once before executing a done maneuver if:***
    - ***The random decision is made to go the direction labeled 2 the first time Ozobot reaches intersection Y, and***
    - ***The random decision is made to go the direction labeled 1 the second time Ozobot reaches intersection Y.***

  Since the probability for each of these two events is 0.5, and since they are independent, the probability that both events occur is the product of their probabilities: 0.5 x 0.5 = 0.5^2 = 0.25. So, in the long run, Ozobot can be expected to return to the start position exactly once before executing a done maneuver about a quarter of the time.]

- ***What is the probability that Ozobot will return to the start position exactly twice before executing a done maneuver? [Ozobot will return to the start position exactly twice before executing a done maneuver if:***
    - ***The random decision is made to go the direction labeled 2 the first time Ozobot reaches intersection Y, and***
    - ***The random decision is made to go the direction labeled 2 the second time Ozobot reaches intersection Y, and***

- *The random decision is made to go the direction labeled 1 the third time Ozobot reaches intersection Y.*

Since the probability for each of these three events is 0.5, and since they are independent, the probability that all three events occur is the product of their probabilities: 0.5 x 0.5 x 0.5 = 0.5^3 = 0.125. So, in the long run, Ozobot can be expected to return to the start position exactly twice before executing a done maneuver about one-eighth of the time.]

By now the pattern should be emerging and is summarized in the following table.

Number of Ozobot returns to start before executing a done maneuver	Probability		
0	0.5	1/2	0.5
1	0.5^2	1/4	0.25
2	0.5^3	1/8	0.125
3	0.5^4	1/16	0.0625
4	0.5^5	1/32	0.03125
5	0.5^6	1/64	0.015625
6	0.5^7	1/128	0.0078125

With each additional Ozobot return to start before executing a done maneuver, the probability decreases by half of its previous value.

Classroom Statistics Experiment

With the theoretical background now covered, the students should be ready and anxious to do an experiment to test the theoretical probabilities in the above table.

Each student group should start Ozobot at least 15 to 20 times, keeping track of the number of Ozobot returns to start before executing a done maneuver. Combining the data from all students groups will then provide a sample size large

enough to observe reasonable "long-term" trends. A typical data table with the combined class results could appear as follows:

Number of Ozobot returns to start before executing a done maneuver	Experimental Tallies	Count	Experimental Probability	Theoretical Probability
0	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	33	33/60 = 0.550	0.50000
1	𝍸 𝍸 𝍸 |	16	16/60 = 0.267	0.25000
2	𝍸 |	6	6/60 = 0.100	0.12500
3	||	2	2/60 = 0.033	0.06250
4	|	1	1/60 = 0.017	0.03125
5		0	0	0.01563
6	||	2	2/60 = 0.033	0.00781
	TOTAL	60		

The experimental and theoretical probabilities are reasonably close for 0 to 2 Ozobot returns, with the experimental results approximately halving as the number of returns to start goes up by one. Beyond 3 returns to start, the outliers in the small sample size provide for more erratic results.

STEM topics

- ***Interdisciplinary—robotics, probability and statistics working together***
- ***Computer Science—colored visual codes are used to program a line-following robot***

Materials

Ozobots (1 per every 3 to 4 students, fully charged)

One 8½" x 11" sheet of bright white cardstock for each student group with the OzoMap on page 3 printed on the cardstock

One copy of the blank data table on the next page for each student group and one copy for combining the student group results

Estimated time-frame

Approximately one 50-minute class period

Number of Ozobot returns to start before executing a done maneuver	Experimental Tallies	Count	Experimental Probability	Theoretical Probability
0				0.50000
1				0.25000
2				0.12500
3				0.06250
4				0.03125
5				0.01563
6				0.00781
	TOTAL			