OZOBOT · STREAM
APPROVED

Ozobot Bit Classroom Application:

Simple Harmonic Motion Simulation

Created by

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Topics

Physics, Simple Harmonic Motion, Displacement, Velocity, Acceleration, Period, Frequency, Angular Frequency, Amplitude, Calculus, Derivative, Graphing

Ages

Grades 11 and 12, College (all exercises)

Grades 7 through 10 (exercises 1-5 and 7 only)

Duration

60 minutes (all exercises)

30 minutes (exercises 1-5 and 7 only)

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Introduction

The study of *simple harmonic motion (SHM)* is central to any introductory course in physics. The force acting on the object that is oscillating is proportional to the displacement of the object from its equilibrium position but opposite in direction. Many oscillating objects do not move between fixed limits due to frictional forces that dissipate the energy of motion. A pendulum will eventually stop swinging and a piano string will stop vibrating. Motions such as these are said to be *damped* harmonic motions.

Although we can't eliminate friction from the damped harmonic motion of large objects, we can introduce energy into the system in a way that compensates for the energy loss from friction. The falling weight in a pendulum clock and the main spring of a watch supply energy in a way that makes them behave as if they were undamped.

In this classroom application, Ozobot Bit simulates simple harmonic motion. Ozobot's battery introduces energy into the system in a way that makes Ozobot behave as if it were undamped. As a result, Ozobot will oscillate between fixed limits as long as its battery has sufficient power.

The Map for this Ozobot Bit Application

Unlike most of the other classroom applications, you will need to construct the "map" yourself, as the map cannot possibly be printed on a single 8½" x 11" sheet of paper in a way that will permit simulating SHM. You will need (preferably) black and red Ozobot markers, or Sharpie Chisel Tip markers, or Crayola ColorMAX markers and a piece of 2½" wide adding machine tape that is 170 cm long. You will also need a meter stick for making measurements. Figure 1 shows a picture of the completed map with Ozobot located in the center section of the map. Although it is not easy to see in the picture, the long piece of adding machine tape has been taped at each end to the floor to keep it from moving while Ozobot Bit is in motion.

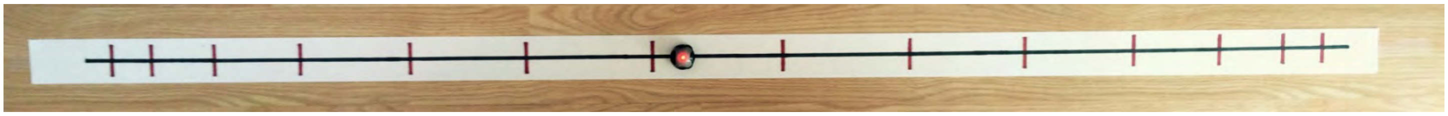


Figure 1

You need to draw the black line upon which Ozobot Bit travels *exactly 163 cm long*. To keep the line very straight, you are encouraged to use a straight edge of some sort and keep the marker's chisel tip edge in contact with the straight edge. There should be about 3 cm of white space at the ends of the black line, giving Ozobot Bit space to turn around while he is executing SHM.

There are a total of 14 red hash marks. They are drawn perpendicular to the long black line, are 1½" long, and are centered on the black line. Each of the red hash marks should be centered at exactly the following locations along the black line, as measured in ***cm*** from the left end of the black line:

3.5	8.5	16.5	27.5	41.5	56.5	73.0	90.0	106.5	121.5	135.5	146.5	154.5	159.5

You will notice in Figure 1 that the 14 hash marks have divided the black line into 15 segments. The hash lines are closer together near the ends of the black line and get further apart as Ozobot moves toward the center of the black line. The program that you will be loading onto Ozobot Bit has been designed in such a way that the time for Ozobot to move from one hash mark to the next is always very close to 2 seconds. In other words, the hash marks are *equally spaced in time* but not distance. You can use a stop watch to time a few, and you should find that the time is always very nearly 2 seconds.

Question: *Knowing that the lines are equally spaced in time but not distance, where would you expect Ozobot Bit to be traveling the fastest? The slowest?*

Running the OzoBlockly Program

1. Make sure that Ozobot Bit is calibrated on paper before running the program.
2. Make sure that Ozobot Bit has clean wheels and plenty of battery charge.
3. Make sure that the adding machine tape that is taped to the floor is free of dust or other debris that it may have gotten from being on the floor. You don't want this to get caught up inside of Ozobot Bit as he moves on the tape.
4. Ozobot bit should be loaded with the program *SimpleHarmonicMotion.ozocode*.
5. Ozobot Bit should always be placed at the right-side end of the long black line, about a centimeter in from the end, and facing the opposite end before starting the program.
6. Start Ozobot Bit by double-pressing the start button.
7. Ozobot Bit will show a RED LED and proceed to execute SHM, back-and-forth on the paper tape, until you remove him or he runs out of battery charge.
8. You should notice that he gradually speeds up as he approached the center of the tape, and slows down while approaching the ends of the tape. He will make a military style "about face" at each end.

Student Exercise #1: *The first thing that you will do is determine the period, **T**, of Ozobot's SHM. The **period** is defined as the time in seconds for one complete cycle. For a swinging pendulum, this would be the time for one complete back-and-forth swing. So if you were to imagine that Ozobot is a pendulum, you need to determine the time from when Ozobot Bit just begins turning around at one end of the tape until he just begins turning around again **at that same end of the tape**. What is Ozobot Bit's period **T**?*

Student Exercise #2: *The frequency of an object executing SHM is the number of complete vibrations per unit time and is given by* $\mathbf{v}$ *(Greek letter "Nu"). The frequency is the reciprocal of the period, i.e.,* $\mathbf{v = 1/T}$ *and has the dimensions of reciprocal time. Determine the frequency of Ozobot Bit's SHM.*

Student Exercise #3: *The* ***angular frequency***, $\boldsymbol{\omega}$ *(Greek letter "Omega"), of SHM is defined by* $\boldsymbol{\omega = 2\pi v}$*. As you can see, it differs from the frequency* $\mathbf{v}$ *by a factor of* 2π*. Since* $\mathbf{v = 1/T}$*, an alternative way of calculating* $\boldsymbol{\omega}$ *is by using the equation* $\boldsymbol{\omega = 2\pi/T}$*. Determine the angular frequency of Ozobot Bit's SHM.*

Student Exercise #4: *Imagine a steel ball pendulum hanging from a string and oscillating back-and-forth. Gradually, due to frictional losses, it comes to rest. Its resting point is its* ***equilibrium*** *position. Considering this analogy, what would the equilibrium position be for Ozobot Bit as it executes its SHM?*

Student Exercise #5: *The* ***amplitude, A****, of SHM is the maximum displacement from the equilibrium position. What is the amplitude of Ozobot Bit as it executes SHM?*

Student Exercise #6: *Simple harmonic motion is sinusoidal in time and can be characterized by the equation*

$$\mathbf{x = A \cos(\omega t)},$$

where $\mathbf{x}$ *is the displacement from equilibrium at time* $\mathbf{t}$*. Using your answers to exercises 3 and 5 along with a spreadsheet program such as Excel, compute displacements for one complete Ozobot Bit cycle of 60 seconds, by intervals of 2 seconds each. (Remember that 2 seconds is the time between red hash marks on Ozobot's map.) Displacement to the right of Ozobot's equilibrium point is positive (+) while displacement to the left is negative (-), as shown in Figure 2. In your spreadsheet program, make a graph of displacement* $\mathbf{x}$ *versus time* $\mathbf{t}$*.*

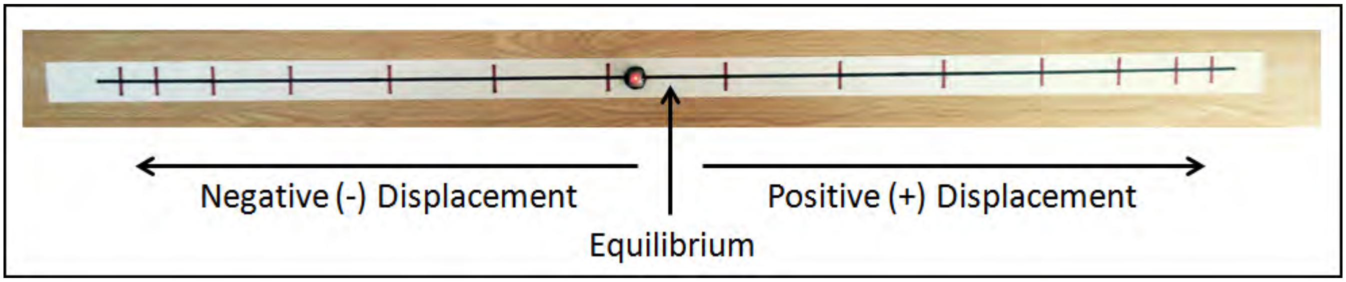


Figure 2

Student Exercise #7: *Now it's time once again to make a measurement with Ozobot Bit executing its SHM. You want to determine Ozobot Bit's* ***maximum speed****. You have probably come to the conclusion by now, that this must occur between the two red hash marks at the center of the black line on which Ozobot executes SHM. Using a stop watch, measure this time in seconds. It might be a good idea to take a few measurements and use the average. Measure the distance in cm between the centers of the two red hash marks. Compute the average speed in this interval using the fact that speed = distance / time.*

Student Exercise #8: *Now let's compare this maximum speed to that predicted by the equation for SHM velocity. Since* $\mathbf{x = A \cos(\omega t)}$*, calculus tells us that* $\mathbf{v = dx/dt = -\omega A \sin(\omega t)}$*. The maximum speed occurs twice each cycle (when the sin* (ωt) *is 1 or -1) and has a value of* $\boldsymbol{\omega A}$*. Using your answers to exercises 3 and 5, compute the maximum speed.*

Student Exercise #9: *Now let's compute the maximum acceleration. Since* $\mathbf{v = -\omega A \sin(\omega t)}$*, and calculus tells us that the acceleration* $\mathbf{a = dv/dt = -\omega^2 A \cos(\omega t)}$*, then the maximum acceleration occurs twice each cycle. This is when* $\mathbf{\cos(\omega t)}$ *is 1 or -1, which occurs when t = 0 and when* $\mathbf{t = \pi/\omega}$ *= 3.14159 / 0.105s*^{2} *= 30 seconds.* ***The maximum acceleration occurs when Ozobot Bit is at either end of the black line.*** *What is the magnitude* $\mathbf{\omega^2 A}$ *of this maximum acceleration?*

Student Exercise #10: *Let's pretend that Ozobot Bit is executing its SHM as an actual simple pendulum hanging from a massless string with the 60 second period that we have already determined. Physics tells us that the period of a simple pendulum, when its amplitude is small compared to its length, is given by the equation:*

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where ***L*** *is the length of the pendulum from its pivot point to the center of mass of the pendulum bob, and* ***g*** *is the acceleration due to gravity. (Note that the period is independent of the mass of the bob.) What would the length* ***L*** *have to be for such a pendulum?*

Student Exercise #11: *Using the equation for velocity (exercise #8) and the equation for acceleration (exercise #9), return to your spreadsheet and create graphs of velocity vs. time and acceleration vs. time for one complete Ozobot Bit cycle of 60 seconds, by intervals of 2 seconds each.*

Student Exercise #12: *Referring to your graphs of displacement, velocity, and acceleration vs. time from exercises #6 and #11 as well as figures 2 and 3, identify the points (A, B, C, D, E) from figure 3 for which each of the following statements is true for Ozobot Bit's simple harmonic motion. Remember that Ozobot Bit is started at time 0 at point E.*

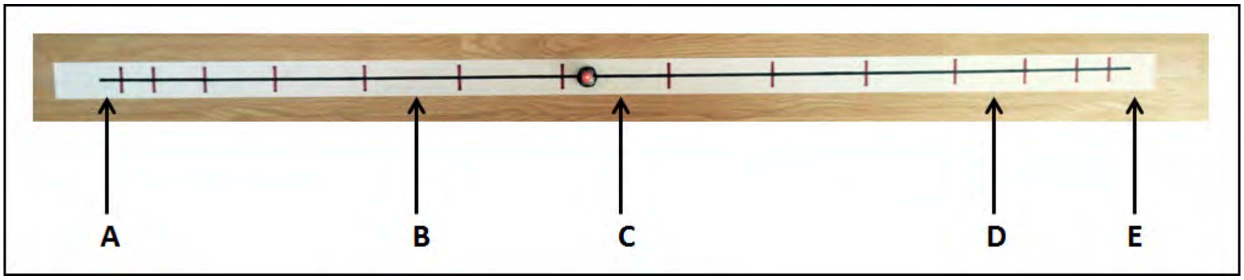
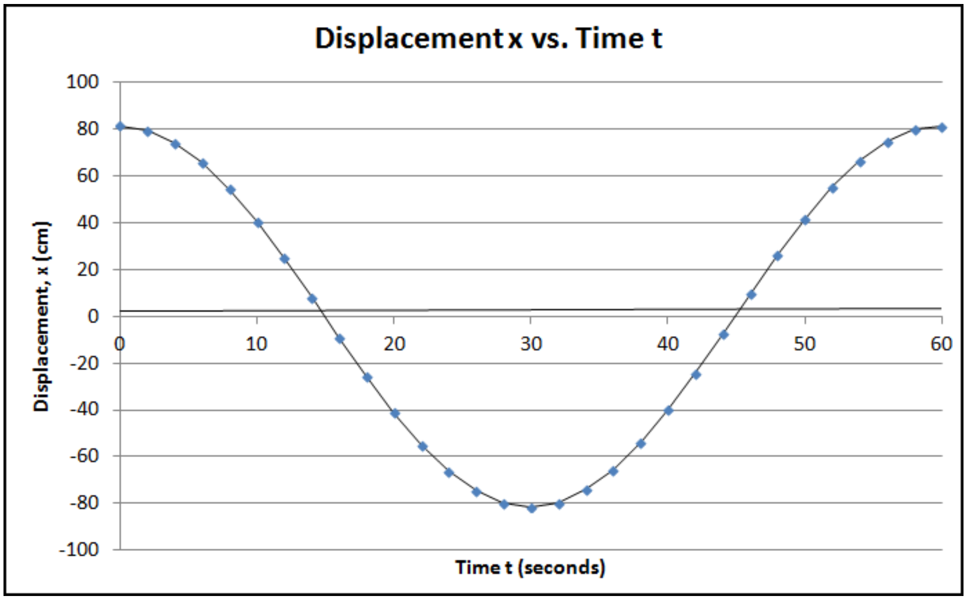


Figure 3

1. *The displacement is zero.*
2. *The speed is zero.*
3. *The displacement is negative.*
4. *The acceleration is zero.*
5. *The magnitude of the velocity is at a maximum.*
6. *The magnitude of the acceleration is at a maximum.*
7. *The acceleration is at a maximum negative value.*
8. *The displacement is positive but not at a maximum value.*

Student Exercise #13: *What is the mathematical relationship between the displacement, velocity, and acceleration versus time graphs?*

Answers to Exercises:

1. *Typical values will be very close to 60 seconds. So Ozobot bit has a long period as far as pendulums are concerned. This is good for our experiments, as we can make measurements that would require much more expensive electronic equipment in we were using a real pendulum!*
2. *Typical answers will be close to 1/60 = 0.0167 s^{-1}.*
3. *Typical answers will be close to 2π/T = 0.105 s^{-1}.*
4. *Ozobot Bit's equilibrium position would be at the center of the long black line.*
5. *The amplitude **A** = 163 cm / 2 = 81.5 cm.*
6. 

7. *Typical answers will be close to 17 cm / 2 seconds = 8.5 cm/sec.*
8. *Typical values for maximum speed will be close to ωA = (0.105 s^{-1})(81.5 cm) = 8.6 cm/sec, in good agreement with the empirical calculation from exercise 7.*
9. *Typical values for the maximum acceleration **ω^2A** will be close to (0.105 $s^{-1})^2$ (81.5 cm) = 1 cm/s^2.*
10. *Solving the equation for L, and letting g = 9.8 m/s^2, we find that L = 894 meters. This string would be longer than the height of the tallest artificial structure in the world—the Burj Khalifa in Dubai, United Arab Emirates. This structure is 829.8 meters tall. It might also be difficult to find a massless string that long! The string would at least have to have a mass that is small compared to the bob hanging from it.*
11. *See the graphs on the next page. For completeness, we have included all three graphs: displacement, velocity, and acceleration versus time, in that order from top to bottom.*
12. *(1) C (2) A, E (3) A, B (4) C (5) C (6) A, E (7) E (8) D*
13. *In terms of calculus, we can say that the velocity is the derivative of displacement with respect to time; and that acceleration is the derivative of velocity with respect to time. In terms of coordinate geometry, we can say that the instantaneous slope of the displacement curve gives the velocity at that*

point in time; and that the instantaneous slope of the velocity curve gives the acceleration at that point in time.