# Ozobot Bit Classroom Application: Demonstrations of Triangular and Square Numbers


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**Topics**

Mathematics

Triangular numbers

Square numbers

Pattern recognition

Recursion

Proofs

**Ages**

Grades 7-12

**Duration**

40 minutes

# Ozobot Bit Classroom Application: Demonstrations of Triangular and Square Numbers

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***Introduction***

A great majority of mathematicians now define mathematics as *the science of patterns*. These patterns can exist in a variety of forms including counting, reasoning, shape, and motion, to mention just a few. If you are a math teacher who teaches sequences, you are probably familiar with the concepts of triangular and square numbers. Study of these two number sequences by students can really bring home the idea that mathematics is indeed *the science of patterns*. In this classroom application, Ozobot Bit is used to visually enhance these patterns in a way that will motivate your students to understand the concepts underlying triangular and square numbers.

***Triangular Numbers***

A triangular number counts the objects required to construct a triangle. For reference while discussing triangular numbers, Figure 1 shows a small version of the Ozobot Bit map that students will use to investigate triangular numbers. A larger version appears on page 3 that can actually be used with Ozobot Bit while running the OzoBlockly program *TriangularNumbers.ozocode*.

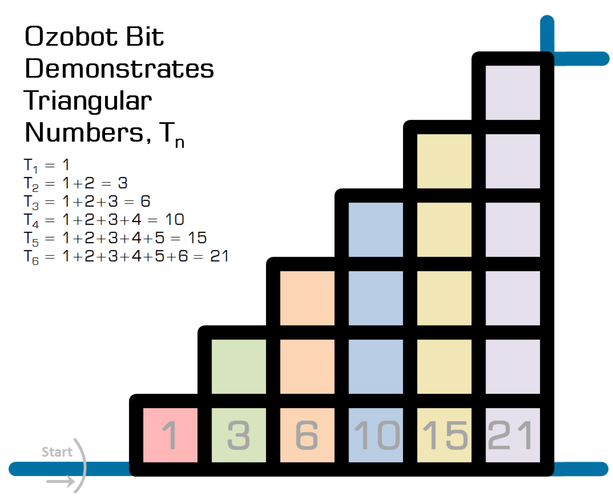



*Figure 1*

When Ozobot Bit is placed at the "Start" position facing the direction shown by the arrow and then started by double-clicking the start button, he will blink white once, indicating that he is about to demonstrate $T_1$. His light will turn red and he will then traverse the lone red block on the far left, return to the start position, make an about face, and then blink white twice, indicating that he is about to demonstrate $T_2$. His light will turn green and he will then traverse the triangle formed by the red block and the two green blocks. He will continue this procedure until he has demonstrated the first six triangular numbers $T_1$ through $T_6$, and then start the whole process all over again, until he runs out of juice or you stop him. While traversing the map, he will show an LED color that closely matches the color of the blocks that he will be adding to the previous triangle to make a new triangle.

***Classroom Exercise #1:*** *By observing Ozobot Bit's demonstration and the equations shown on the map for $T_1$ through $T_6$, ask the students what $T_n$ represents. [Answer: $T_n$ represents the sum of the first n positive integers.]*

***Classroom Exercise #2:*** *Ask the students to determine the value of $T_7$, the next number in the sequence. [If they understand the pattern, they should see that T7 = T6 + 7 = 21 + 7 = 28.] Ozobot Bit demonstrated the recursive relationship that $T_n = T_{n-1} + n$.*

***Classroom Exercise #3:*** *Now see if they can come up with an equation for $T_n$ in terms of n itself. To get them to think about this, have then again consider $T_7$ by writing it down **twice**, once forward and once backward, and then adding the numbers in each column:*

$$\begin{array}{c} 1 + 2 + 3 + 4 + 5 + 6 + 7 \\ \underline{7 + 6 + 5 + 4 + 3 + 2 + 1} \\ 8 + 8 + 8 + 8 + 8 + 8 + 8 \end{array}$$

*The total sum is 7 * 8 = 56. Why is the total 56, and not 28, as was obtained in Classroom Exercise #2? The series was added twice, so we have to divide by 2 to get the correct answer 28. The value of $T_7$ is (7*8)/2 = 28. In this example, n is 7, so in general:*

$$T_n = \frac{n(n+1)}{2}$$

***Classroom Exercise #4:*** *There is a story that Karl Friedrich Gauss, an exceptional German mathematician circa 1800, and his classmates were once asked by a teacher to add the numbers up from 1 to 100, in order to keep them busy for a long time. Gauss returned with the correct answer after a very short time had passed, using the technique that was outlined in Classroom Exercise #3. What answer did Gauss give his teacher? [(100*101)/2 = 5050].*

***Classroom Exercise #5***: *A **fully-connected network** of n computing devices is a network in which each device is connected by a cable to each and every other device in the network. By observing patterns with 2, 3, 4, 5, and 6 device networks, see if the students can show that the number of cables required can be expressed as a triangular number. [The number of cables required for a fully connected network with n devices is $T_{n-1}$.] This is equivalent to asking how many hand-shakes would be required if everyone in a room were to shake hands once with every other person in the room.*

# Ozobot Bit Demonstrates Triangular Numbers, $T_n$

$T_1 = 1$

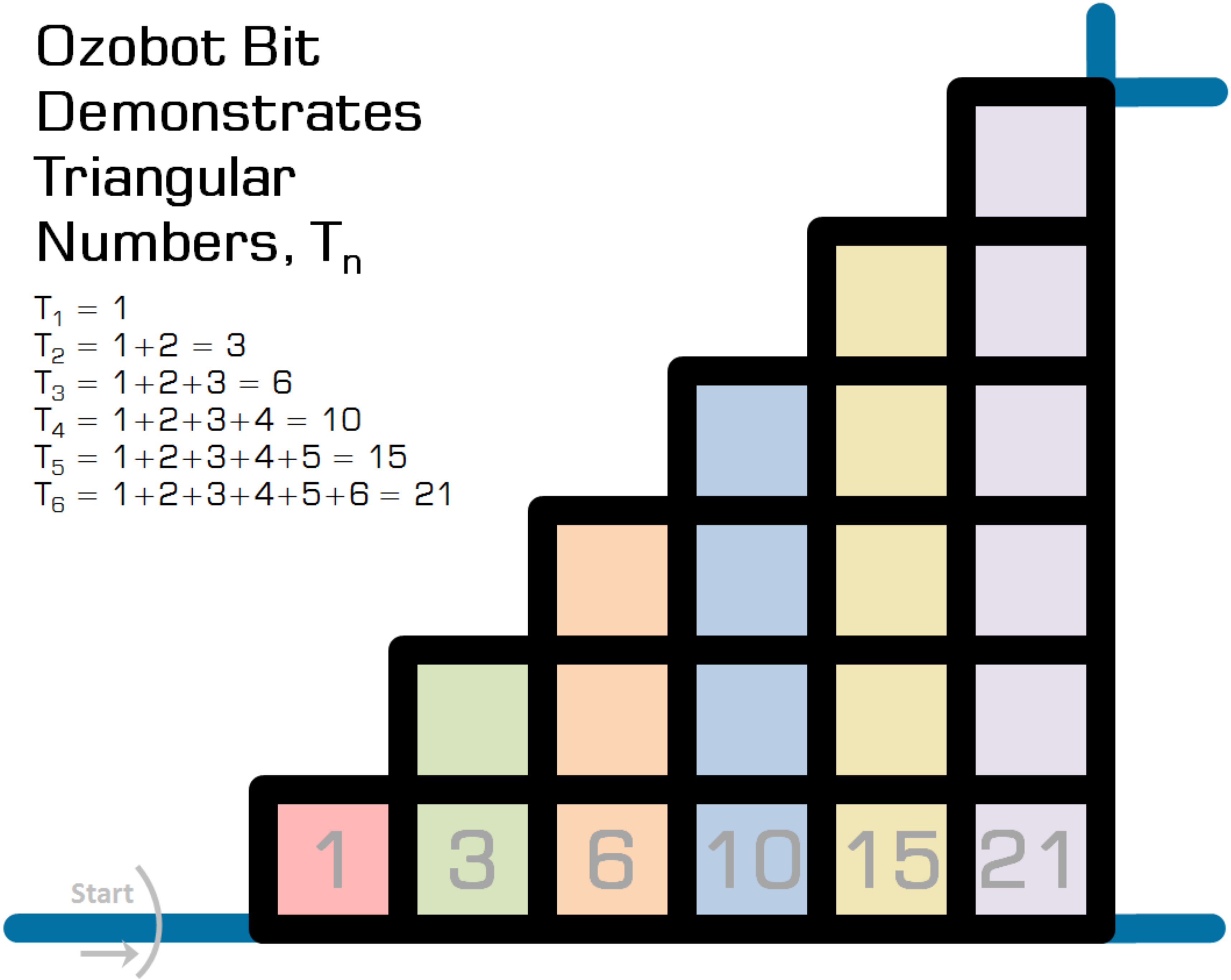
$T_2 = 1+2 = 3$

$T_3 = 1+2+3 = 6$

$T_4 = 1+2+3+4 = 10$

$T_5 = 1+2+3+4+5 = 15$

$T_6 = 1+2+3+4+5+6 = 21$



***Square Numbers***

A positive integer that is multiplied by itself, *i.e.*, squared, is called a *square number*. A square number counts the number of blocks required to make a square. For reference while discussing square numbers, Figure 2 shows a small version of the Ozobot Bit map that students will use to investigate square numbers. A larger version appears on page 6 that can actually be used with Ozobot Bit while running the OzoBlockly program *SquareNumbers.ozocode*.

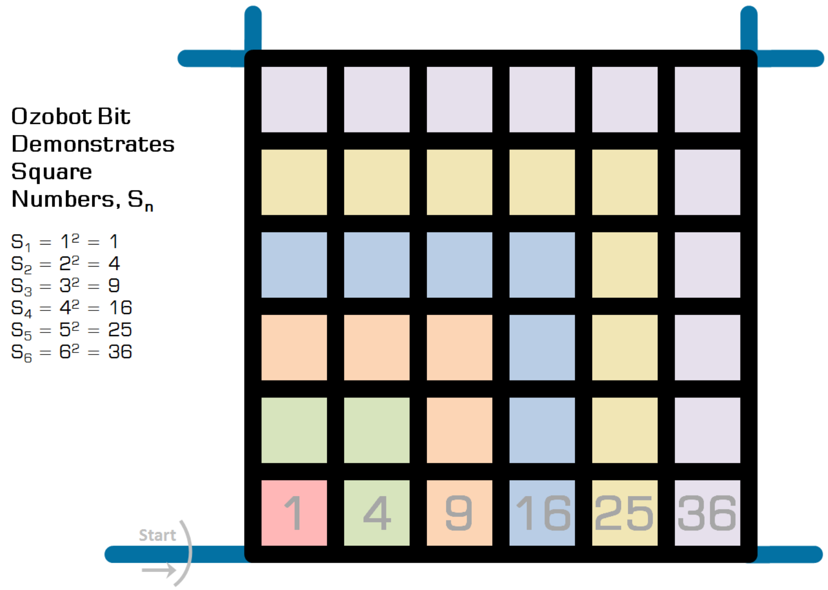



*Figure 2*

When Ozobot Bit is placed at the "Start" position facing the direction shown by the arrow and then started by double-clicking the start button, he will blink white once, indicating that he is about to demonstrate $S_1$. His light will turn red and he will then traverse the lone red block on the far left, return to the start position, make an about face, and then blink white twice, indicating that he is about to demonstrate $S_2$. His light will turn green and he will then traverse the square formed by the red block and the three green blocks. He will continue this procedure until he has demonstrated the first six square numbers $S_1$ through $S_6$, and then start the whole process all over again, until his battery is depleted or you stop him. While traversing the map, he will show an LED color that closely matches the color of the blocks that he will be adding to the previous square to make a new square.

***Classroom Exercise #6***: *Help students to identify the recursive relationship $S_n = S_{n-1} + (2n – 1)$ by studying the patterns suggested by Ozobot Bit's demonstration:*

*$S_2 = S_1 + 3$ (adding the 3 green blocks)*
*$S_3 = S_2 + 5$ (adding the 5 orange blocks)*
*$S_4 = S_3 + 7$ (adding the 7 blue blocks)*
*etc.*

***Classroom Exercise #7:*** *Have the students observe patterns to show that **every square number is the sum of two consecutive triangular numbers**. Figure 3 shows how $S_2 = T_2 + T_1$ and $S_3 = T_3 + T_2$. In general, students should observe the pattern that $S_n = T_n + T_{n-1}$.*

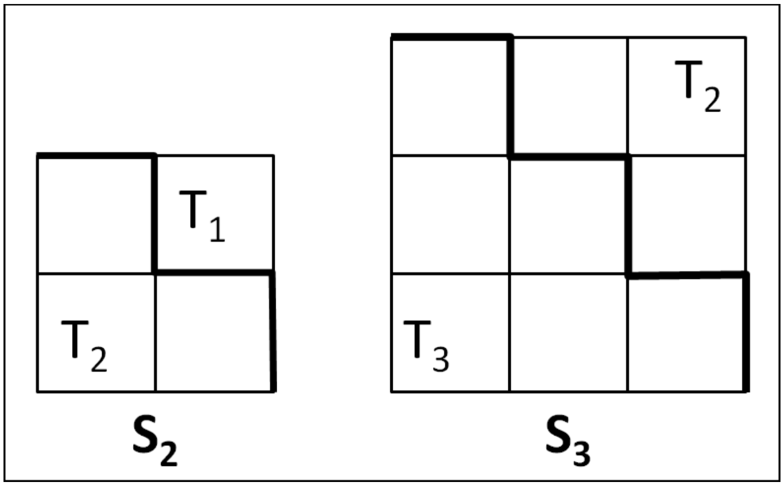



*Figure 3*

***Classroom Exercise #8:*** *Ask the students to prove that **squares of even numbers are even**; in fact such squares are divisible by 4. [Even numbers can be written as 2n, where n is any non-negative integer. So, squaring an even number, we have $(2n)^2 = 4n^2$, which is both even and divisible by 4. Note that Ozobot Bit's demonstration shows that $S_2$, $S_4$, and $S_6$ are all even and divisible by 4.]*

***Classroom Exercise #9:*** *Ask the students to prove that **squares of odd numbers are odd**. [Odd numbers can be written as 2n + 1 where n is any non-negative integer. Therefore, squaring an odd number, we have $(2n+1)^2 = 4n2 +4n + 1 = 2(2n^2 + 2n) + 1$, which is of the form 2k + 1 and therefore odd. Note that Ozobot Bit's demonstration shows that $S_1$, $S_3$, and $S_5$ are all odd.]*

# Ozobot Bit Demonstrates Square Numbers, $S_n$

$S_1 = 1^2 = 1$
$S_2 = 2^2 = 4$
$S_3 = 3^2 = 9$
$S_4 = 4^2 = 16$
$S_5 = 5^2 = 25$
$S_6 = 6^2 = 36$