Ozobot Bit Classroom Application:

Determining the Value of π with Ozobot and a Stopwatch

Created by

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Topics

Pi, Math, science, experiment, error analysis, percentage error, critical thinking

Ages

Grades 7 – 12

Duration

35-55 minutes

Lesson:

Determining the Value of π with Ozobot and a Stopwatch

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Introduction

There has probably been no number that has fascinated mankind over the ages more than pi. Symbolized by the Greek letter **π**, ***pi*** is defined as the ***ratio of the circumference of a circle to its diameter***. The ancients may have observed that a cart moves a distance forward that is about three times the diameter of its wheels for each rotation of the wheels, giving an approximate value of 3 to pi. Since then, pi, proven to be an irrational number, has been computed to more than a trillion digits by supercomputers. Pi to nine decimal places is 3.141592653. Knowing this we can celebrate the "ultimate pi day" once every century on 3/14/15 at 9:26:53 o'clock. Many lovers of math and science wore shirts commemorating this day on March 3, 2015—the next chance won't come until March 3, 2115.

Circles are one of the most common forms in nature—from cart wheels, to ripples on a pond, to the shape of planets, moons and stars. This fact suggests that pi is likely of high interest to intelligent civilizations that may exist throughout the universe. It is no wonder that we here on Earth have a fascination with this number!

What better number could there be to investigate with Ozobot than the number **π**? In this lesson students will use Ozobot and a stopwatch along with several OzoMaps of circles and squares to determine an approximate value for π. In the process, students will learn how to compute percent error and think critically about the possible causes for error in their computed values for pi, employing many STEM and NGSS (Next Generation Science Standards) practices.

The Ozobot Equation for Pi

An approximation for the value of **π** will be determined by the use of a stopwatch. You will time two events—the time, T_C, for Ozobot to make one lap around a circle and the time, T_S, to make one lap around a square whose sides are the length of the diameter, D, of the circle. See figure 1.

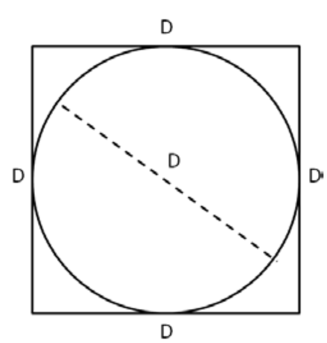

Figure 1

By the definition of **π**, if ***C*** is the circumference of a circle with diameter ***D***, then **π** = ***C/D***. If we keep the velocity of Ozobot the same on the circle and square, then the distance traveled is proportional to the time required for Ozobot to make one lap. In other words, we can use times instead of distances in our equation for **π**. Since T_S is the time for *four* diameters, the time for *one* diameter would be $T_S/4$. Therefore, our final Ozobot equation for approximating the value of π is

$$\pi = \frac{C}{D} = \frac{T_C}{T_S/4} = \frac{4T_C}{T_S} \qquad \textit{Equation 1}$$

Ozobot's approximation for **π** can be calculated by multiplying the time for Ozobot to traverse the circle once by 4, and then dividing this product by the time for Ozobot to traverse the square once.

Students can then compute the percentage error in the value they obtained for **π** by using the usual equation for calculating percent error:

$$\%\ Error = \frac{|Experiment\ value - Actual\ value|}{Actual\ Value} \text{ X } 100\% \qquad \textit{Equation 2}$$

Preparation for the Lesson

1. Make copies of the OzoMaps on the last three pages of this document for each lab group. One of the maps contains a small circle and a small square with sides equal in diameter to the small circle. The other two maps contain a large circle and a large square with sides equal in diameter to the large circle. The short gray lines on the OzoMaps are reference lines for starting and stopping their stopwatches.
2. Using a poster board or very large sheet of paper, prepare a **super large** circle and **super large** square with sides equal in diameter to the super large circle. This super large circle and square should be at least twice the size of the large circle and large square. If the teacher prefers, the student lab groups could construct these super large circles and squares as part of their work. (The reason for different sizes is that students will likely find that **the percentage error in π decreases as the circle and corresponding square gets larger**. One of their challenges is to come up with possible explanations for this decrease in percentage error.)
3. Make a copy of the data tables on page 4 of this document for each of the student lab groups.
4. Have an Ozobot that is fully charged, calibrated, and has clean wheels ready for each of the lab groups. Either Ozobot 1.0 or Ozobot Bits can be used.
5. Each lab group will need a stop watch to measure Ozobot lap times on the circles and squares, preferably one that reads to hundredths of a second. Alternatively, they could use a stop watch app on their cell phones.
6. To start their Ozobots, students should press the start button ***once***. DO NOT double-press the start button.

Lab Group Data Collection

Based on all of the previous discussion, data collection should be pretty straight forward. Have the students get lap times beginning with the small circle/square, recording their T_C and T_S times for five trials in the top data table of their data table sheet, and computing and recording the average for the five trials in the data table. Repeat for the large and super large circles/squares.

Finally, they should compute their experimental values for **π** using equation 1 and then compute the corresponding percentage errors using equation 2, recording their results in the table at the bottom of the data table sheet.

Discussion of Results

The most important thing about this experiment is ***not*** the value that student groups obtain for **π**. They may even be a little disappointed that they don't get results equal to 3.14***. The most important result is that the percentage error in the value of Pi gets smaller as the size of the circle/square gets larger.*** Have a discussion with students that can bring out their critical thinking ability to come up with possible explanations for this. Perhaps a list could be made on the white board of their explanations. Which explanations are most plausible? This is a perfect application of both STEM and NGSS practices!

Data Table: Ozobot Lap Times

	Time for One Lap (seconds)					
Trial #	Small Circle T_C	Small Square T_S	Large Circle T_C	Large Square T_S	Super Large Circle T_C	Super Large Square T_S
1						
2						
3						
4						
5						
Average						

Data Table: Experimental Values for Pi and Percentage Error in the Value for Pi

Circle/Square	$\pi = \frac{4T_C}{T_S}$ (Equation 1)	$\% \ Error = \frac{\lvert Experiment\ value - Actual\ value \rvert}{Actual\ Value}$ X 100% (Equation 2)
Small		
Large		
Super Large		

For the Teacher: Typical Results

Data Table: Ozobot Lap Times*

	Time for One Lap (seconds)					
Trial #	Small Circle T_C	Small Square T_S	Large Circle T_C	Large Square T_S	Super Large Circle T_C	Super Large Square T_S
1	8.87	10.00	16.60	19.93	38.66	47.32
2	8.62	10.13	16.34	20.05	38.51	47.85
3	8.70	10.16	16.37	19.51	38.33	47.81
4	8.88	10.18	16.41	19.31	38.68	47.27
5	8.76	10.12	16.57	19.53	38.78	47.03
Average	8.77	10.12	16.46	19.67	38.59	47.46

* Times will vary some from one Ozobot to another.

* The Super Large Circle/Square used had D = 37 cm.

Data Table: Experimental Values for Pi and Percentage Error in the Value for Pi+

Circle/Square	$\pi = \frac{4T_C}{T_S}$ (Equation 1)	$\% \, Error = \frac{\lvert Experiment \, value - Actual \, value \rvert}{Actual \, Value}$ X 100% (Equation 2)
Small	3.47	10.5%
Large	3.35	6.7%
Super Large	3.25	3.5%

+ The number 3.14 was used as the "actual" value for Pi in these calculations.

Note to teacher: The data and results for the super large square/circle are *really helpful* for driving the point that *increasing size of the circle/square decreases the percentage error in the value for Pi*. However, if the extra time required to make the super large circle/square and time required for students to collect that data is beyond your time limits, you can skip the super large size circle/square. The reason you can skip super large is that there should be pretty consistent results from student group to group using only the small and large sizes.

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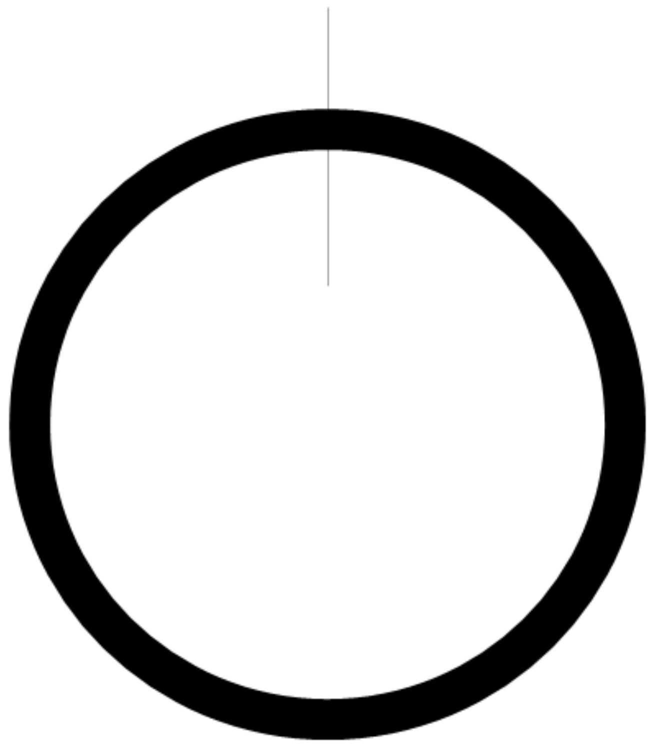
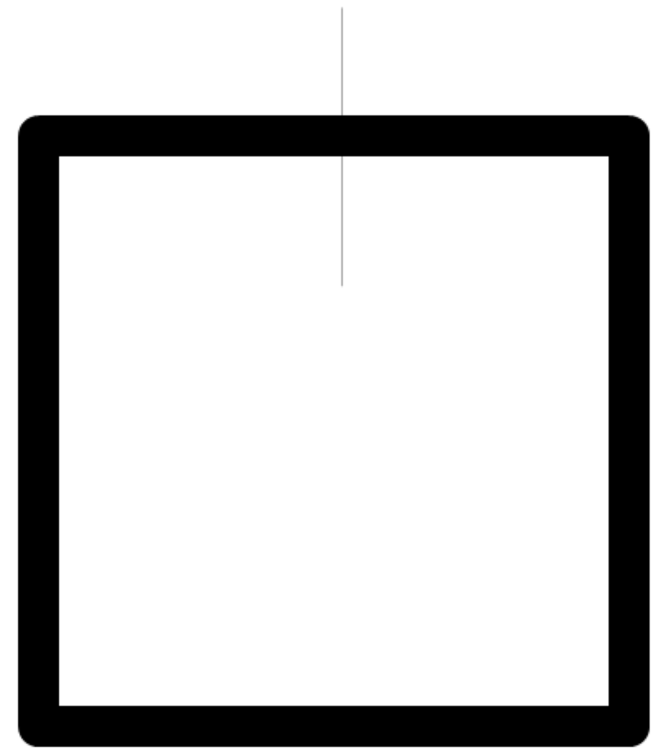
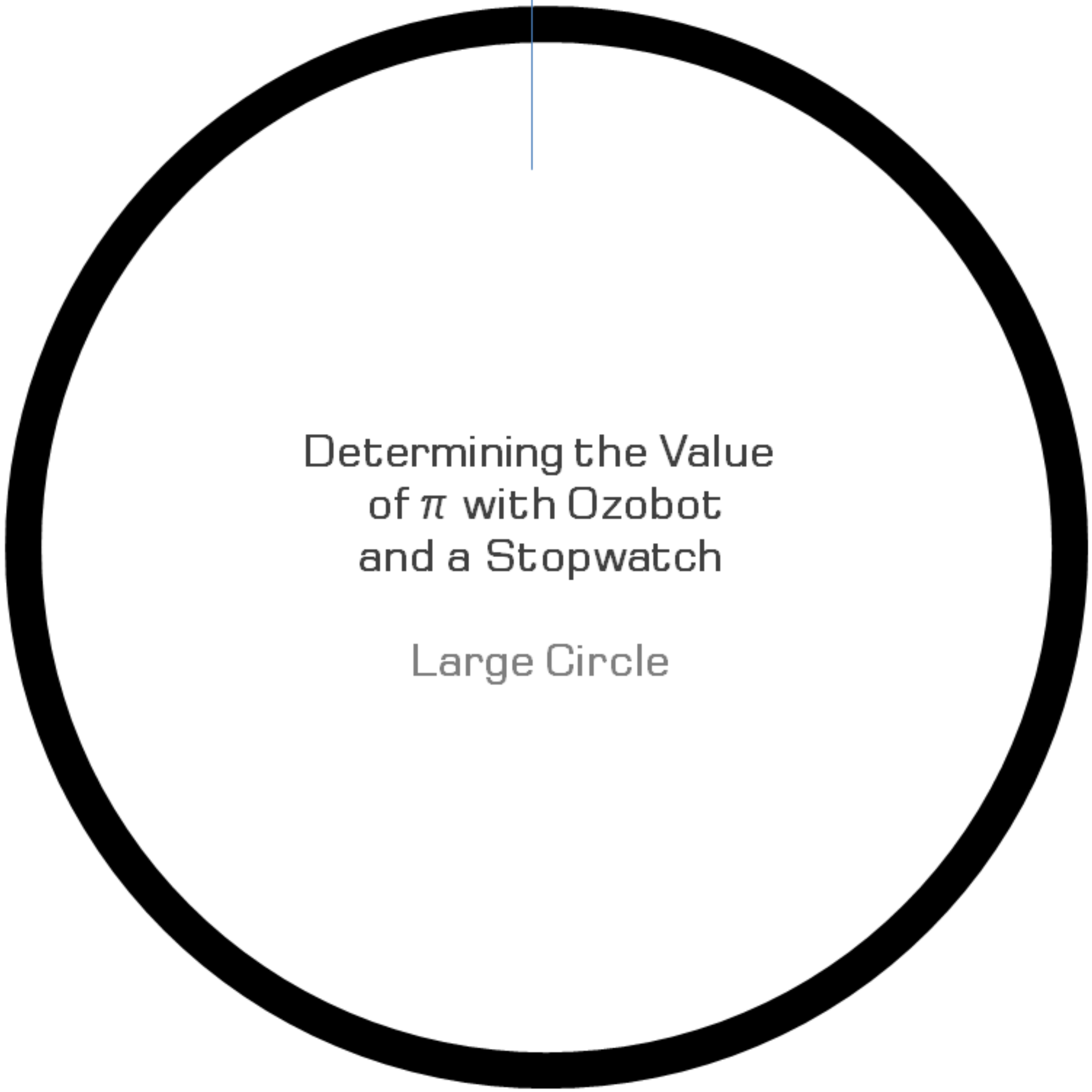


Small Circle



Small Square with
Sides Equal in Diameter
To the Small Circle


Determining the Value
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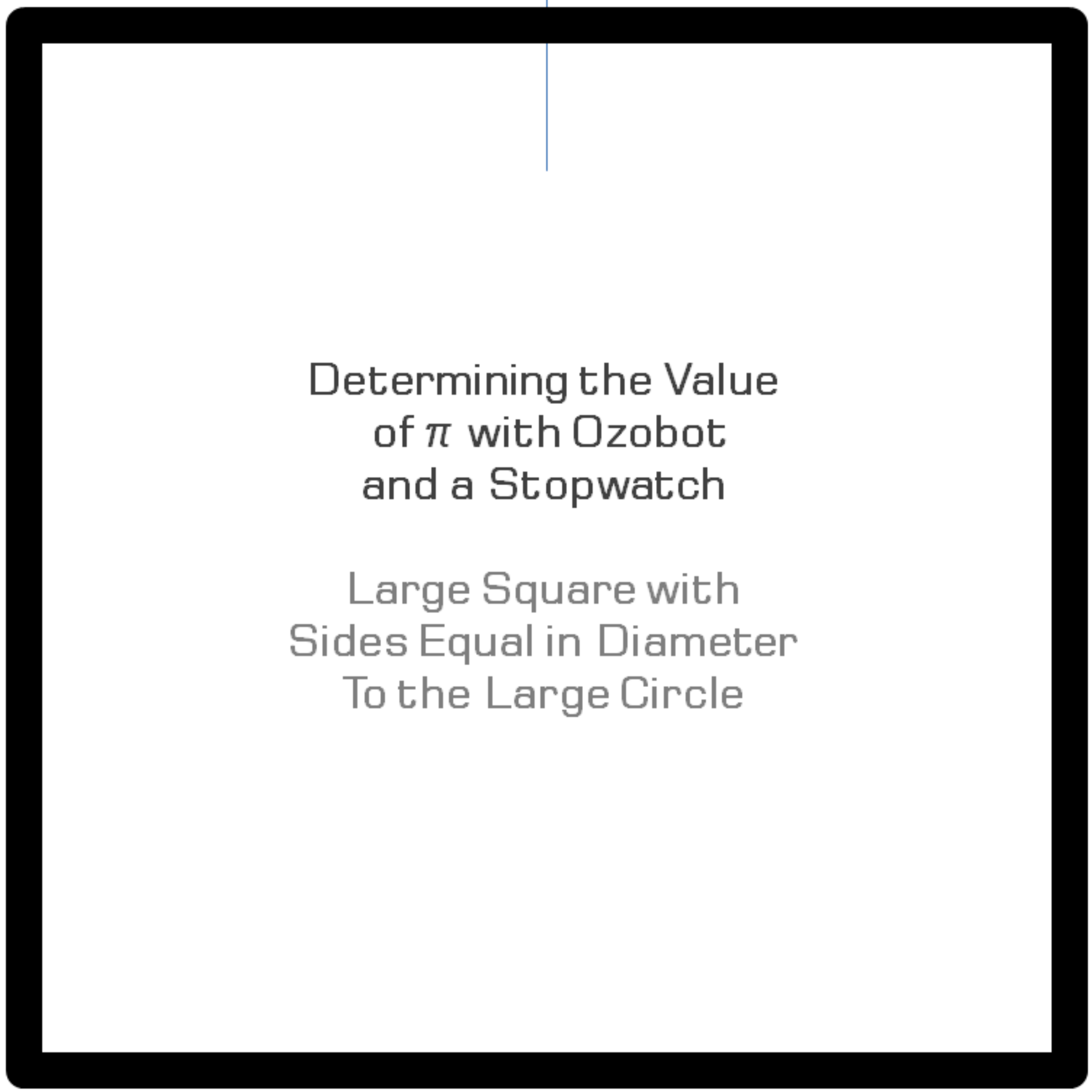
Large Circle


Determining the Value
of π with Ozobot
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Large Square with
Sides Equal in Diameter
To the Large Circle