Chapter 7
Transmission Lines

In the mid-1980s or so, connecting conductors were of no consequence; i.e., the voltage and current at the input to the line are almost identical to the voltage and current at the output of the line. Today this is no longer true. As clock and data speeds continue to increase, seemingly without bound, these interconnect conductors will have a significant effect on the signal transmission and cannot be ignored.

— Dr. Clayton R. Paul

7.1 What Is a Transmission Line

A transmission line is a series of conductors, often but not necessarily two, used to guide electromagnetic energy from one place to the other [3]. It’s that simple. The more complicated part is the math behind it (Maxwell’s Equations; see Appendix E) because we no longer consider a transmission line a lumped-element $RLC$ network (see Fig. 7.9). Rather more, we consider the signal conductor as a transmission line through which an electromagnetic field is moved from one point to another. Transmission lines are characterized or described by their characteristic impedance $Z_0$ [Ω] and distributed parameter model (see Fig. 7.2).

Figure 7.1 shows some common transmission line geometries:

- **Coax.** The electromagnetic energy propagates through the dielectric (mostly PTFE, because of its low loss and stable $\epsilon_r$ for many frequencies) between the center conductor and the inside surface of the outer conductor (shield) of a coaxial cable.
- **Microstrip.** Transmission line where the signal conductor is on the top or bottom layer of a PCB with an adjusted return path conductor (e.g., ground plane or power supply plane).
- **Stripline.** Transmission line where the signal conductor is embedded between two signal return path conductors of a PCB (e.g., ground or power supply plane).
Fig. 7.1 Transmission line examples

- **Balanced line.** Two conductors of the same size and shape with equal impedance to ground and all other conductors (e.g., ethernet cable).
- **Waveguide.** A waveguide consists of a single hollow conductor used to guide the electromagnetic energy. Waveguides are used in the gigahertz frequency range, and they cannot pass DC signals.

### 7.2 When to Consider Transmission Lines

Every signal interconnection is a transmission line. However, it is not necessary to treat every signal path as a transmission line. Two different rules of thumb—if a conductor should be treated as transmission line or not—are explained in Sect. 7.2.1 (frequency-domain) and Sect. 7.2.2 (time-domain).

#### 7.2.1 Rule of Thumb for $l_{\text{critical}}$ in the Frequency Domain

A common rule of thumb when working in the frequency domain is [3]:

- **Interconnection length $l \geq (\lambda_{\text{min}}/10).** Consider the signal path as a transmission line to minimize signal distortions and ringing due to reflections and to minimize radiated emissions and electromagnetic interference (EMI).

- **Interconnection length $l < (\lambda_{\text{min}}/10).** Consider the signal path as a simple conductor. If an interconnection length $l$ [m] is short with respect to the signal wavelength $\lambda$ [m], it is good practice that the interconnection is considered as a simple conductor with lumped-element parameters (e.g., resistance $R$ [Ω] with series inductance $L$ [H]).

When determining the shortest wavelength $\lambda_{\text{min}}$ [m] in a digital signal (e.g., clock), it is necessary to know the maximum frequency $f_{\text{max}}$ [Hz] of the signal by considering the rising and falling time (rather than the fundamental frequency). The rule of thumb for calculating the bandwidth—or the highest significant sine wave frequency $f_{\text{max}}$ [Hz]—of a rectangular digital signal and the corresponding minimum wavelength $\lambda_{\text{min}}$ [m] [1]:
7.2 When to Consider Transmission Lines

\[ f_{\text{max}} = \text{bandwidth} = \frac{0.35}{t_{10-90\%}} \]  
(7.1)

\[ \lambda_{\text{min}} = \frac{v}{f_{\text{max}}} \]  
(7.2)

where,

\( f_{\text{max}} \) = highest significant sine wave frequency harmonic in a digital signal (what significant means is described in Sect. 5.3) in [Hz]

\( t_{10-90\%} \) = rise and/or fall time (whichever is smaller) from 10\% to 90\% of the slope of a digital signal in [sec]

\( \lambda_{\text{min}} \) = wavelength of the highest significant harmonic frequency in [m]

\( v \) = propagation velocity of the signal along the transmission line in [m/sec]

The frequency domain approximation for the critical length \( l_{\text{critical,fd}} \) [m] can be calculated with respect to the digital signal rise/fall time \( t_{10-90\%} \):

\[ l_{\text{critical,fd}} = \frac{\lambda_{\text{min}}}{10} = \frac{v}{10 \cdot f_{\text{max}}} = \frac{v \cdot t_{10-90\%}}{10 \cdot 0.35} = \frac{c \cdot t_{10-90\%}}{3.5 \cdot \sqrt{\varepsilon'_{r,\text{eff}}}} \]  
(7.3)

where,

\( c = 1/(\sqrt{\mu_0\varepsilon_0}) = 2.998 \cdot 10^8 \text{ m/sec} \) = speed of light

\( v \) = propagation velocity of the signal along the signal line in [m/sec]

\( f_{\text{max}} \) = highest significant sine wave frequency harmonic in a digital signal (what significant means is described in Sect. 5.3) in [Hz]

\( \varepsilon'_{r,\text{eff}} \) = effective dielectric constant (relative permittivity) through which the electromagnetic wave is propagating

### 7.2.2 Rule of Thumb for \( l_{\text{critical}} \) in the Time Domain

There is also a rule of thumb for the time domain [3]:

- \( t_{10-90\%} \leq 2 \cdot t_{pd} \). If the rise or fall time \( t_{10-90\%} \) [sec] of a digital signal is smaller than twice the propagation delay \( t_{pd} \) [sec] (along the signal line), then the signal path should be considered as a transmission line, in order to minimize signal distortions and ringing due to reflections and in order to minimize radiated emissions and electromagnetic interference (EMI).

- \( t_{10-90\%} > 2 \cdot t_{pd} \). If the rise and fall time \( t_{10-90\%} \) [sec] of a digital signal are bigger than twice the propagation delay \( t_{pd} \) [sec] of the signal across the conductor, the signal path may be considered as a simple conductor.

If we multiply both sides of the equation of the time domain rule of thumb with the signal propagation velocity \( v \) [m/sec], we get:
\[ t_{10-90\%} \leq 2 \cdot t_{pd} \]
\[ t_{10-90\%} \cdot v \leq 2 \cdot t_{pd} \cdot v \]
\[ l_{10-90\%} \leq 2 \cdot l_{\text{critical},td} \]

where,

- \( t_{10-90\%} \) = rise and/or fall time (whichever is smaller) from 10\% to 90\% of the digital signal in [sec]
- \( t_{pd} \) = propagation delay of the signal along the signal line in [sec]
- \( l_{10-90\%} \) = distance which the signal travels along the signal line during the rise and/or fall time (whichever is smaller) \( t_{10-90\%} \) in [m]
- \( l_{\text{critical},td} \) = critical length of the time domain rule of thumb in [m]

The time domain approximation for the critical length \( l_{\text{critical},td} \) [m] can be calculated with respect to the digital signal rise/fall time \( t_{10-90\%} \):

\[ l_{\text{critical},td} = \frac{l_{10-90\%}}{2} = \frac{v \cdot t_{10-90\%}}{2} = \frac{c \cdot t_{10-90\%}}{2 \cdot \sqrt{\epsilon'_r \text{eff}}} \]  \( (7.4) \)

where,

- \( l_{10-90\%} \) = distance which the signal travels along the signal line during the rise and/or fall time (whichever is smaller) \( t_{10-90\%} \) in [m]
- \( v \) = propagation velocity of the signal along the signal line in [m/sec]
- \( l_{10-90\%} \) = rise and/or fall time (whichever is smaller) from 10 to 90\% of the digital signal in [sec]
- \( c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.998 \cdot 10^8 \text{ m/sec} \) = speed of light
- \( \epsilon'_r \text{eff} \) = effective dielectric constant (relative permittivity) through which the electromagnetic wave is propagating

### 7.2.3 Critical Length \( l_{\text{critical}} \)

The two rules above lead to similar values for the critical length \( l_{\text{critical}} \) [m], where the frequency domain \( l_{\text{critical},fd} \) is smaller than the time domain \( l_{\text{critical},td} \). It is therefore recommended to go with the rule of thumb of the frequency domain: Consider an interconnection as a transmission line if it is longer than:

\[ l_{\text{critical}} = \frac{\lambda_{min}}{10} \]  \( (7.5) \)

where,

- \( \lambda_{min} \) = wavelength of the highest significant harmonic frequency in the signal (what significant means is described in Sect. 5.3) in [m]
7.3 Characteristic Impedance $Z_0$

7.3.1 Characteristic Impedance $Z_0$ of Any Transmission Line

The characteristic impedance $Z_0$ [$\Omega$] is the most important property of a transmission line. The characteristic impedance $Z_0$ [$\Omega$] cannot be measured with a simple DC ohmmeter, and it is defined as the ratio of voltage $V^+$ [V] to current $I^+$ [A] of a single traveling wave along a transmission line [5]:

$$Z_0 = \frac{V^+}{I^+}$$

(7.6)

where

$V^+$ = voltage of the forward wave (the $^+$ emphasizes that voltage and current travel in the same direction) in complex phasor form in [V]

$I^+$ = current of the forward wave (the $^+$ emphasizes that voltage and current travel in the same direction) in complex phasor form in [A]

Figure 7.2 shows the equivalent circuit model for a transmission line with two conductors. The so-called distributed parameter model divides the transmission line into infinitely small segments of length $dz$ in [m]. The parameters of these segments are defined per-unit-length (e.g., per [m]):

$$R' = \text{Resistance per-unit-length} \ [\Omega/m]$$

(7.7)

$$L' = \text{Inductance per-unit-length} \ [H/m]$$

(7.8)

$$C' = \text{Capacitance per-unit-length} \ [F/m]$$

(7.9)

$$G' = \text{Conductance per-unit-length} \ [S/m]$$

(7.10)

The formula for the characteristic impedance $Z_0$ [$\Omega$] of a transmission line—based on the distributed parameter model—is defined as [3]:

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

(7.11)

Fig. 7.2 Distributed parameter model of a transmission line [3]
where,
\[ \omega = 2\pi f = \text{angular frequency in [rad/sec]} \]
\[ j = \sqrt{-1} = \text{imaginary unit} \]

The following subsections present formulas for the calculation of the characteristic impedance \( Z_0 \) [\( \Omega \)] for typical electrical transmission lines (the formulas are approximations; in case you need accurate results, you should use a solver that applies Maxwell’s Equations presented in Appendix E):

- **Lossless transmission lines:** Sect. 7.3.2.
- **Parallel wires:** Sect. 7.3.3.
- **Twisted pairs:** Sect. 7.3.4.
- **Coaxial cables:** Sect. 7.3.5.
- **Microstrip lines:** Sect. 7.3.6.
- **Coplanar waveguide with reference plane:** Sect. 7.3.7.
- **Centered striplines:** Sect. 7.3.8.

### 7.3.2 Characteristic Impedance \( Z_0 \) of Lossless Transmission Lines

In practice, it is often adequate to describe transmission lines as lossless (\( R' = 0, \ G' = 0 \)). In that case, the transmission line model can be simplified—like shown in Fig. 7.3—and the calculation of the characteristic impedance is reduced to:

\[
Z_0 = \sqrt{\frac{L'}{C'}}
\]  
(7.12)

where,
\( L' = \text{inductance per-unit-length [H/m]} \)
\( C' = \text{capacitance per-unit-length [F/m]} \)

**Fig. 7.3** Distributed parameter model of a loss less transmission line [3]
7.3 Characteristic Impedance $Z_0$

**Fig. 7.4** Cross section of two parallel wires or twisted pair

### 7.3.3 Characteristic Impedance $Z_0$ of Parallel Wires

The characteristic impedance $Z_0$ [Ω] of two parallel wires with identical diameter $D$ [m]—according to Fig. 7.4—can be calculated as [6]:

$$Z_0 = \frac{\eta_0}{\pi \sqrt{\epsilon'_r,\text{eff}}} \cosh^{-1} \left( \frac{d}{D} \right)$$  (7.13)

$$\epsilon'_r,\text{eff} = \epsilon'_r + 0.25 \left( \epsilon'_r - \epsilon'_r' \right)$$  (7.14)

where,

$\eta_0 = 377$ Ω = intrinsic impedance of free-space (vacuum) [Ω]

$d$ = distance from center to center of the two wires in [m]

$D$ = diameter of the two wires in [m]

$\epsilon'_r,\text{eff}$ = effective dielectric constant (relative permittivity) through which the electromagnetic wave is propagating

$\epsilon'_r'$ = relative permeability of the insulation around the conductors

$\epsilon'_r''$ = relative permeability of the medium around the insulated conductors

### 7.3.4 Characteristic Impedance $Z_0$ of Twisted Pairs

The characteristic impedance $Z_0$ [Ω] of a twisted pair with identical diameter $D$ [m]—according to Fig. 7.4—can be calculated as [6]:

$$Z_0 = \frac{\eta_0}{\pi \sqrt{\epsilon'_r,\text{eff}}} \cosh^{-1} \left( \frac{d}{D} \right)$$  (7.15)

$$\epsilon'_r,\text{eff} = \epsilon'_r' + q \left( \epsilon'_r - \epsilon'_r'' \right)$$  (7.16)
\[ q = 0.25 + 0.0004\theta \] (7.17)
\[ T = \frac{\tan(\theta)}{\pi d} \] (7.18)
\[ \theta = \tan^{-1}(T\pi d) \] (7.19)

where,

\( \eta_0 = 377 \Omega \) = intrinsic impedance of free space (vacuum) [\( \Omega \)]
\( d \) = distance from center to center of the two wires in [m]
\( D \) = diameter of the two wires in [m]
\( \epsilon'_{\text{eff}} \) = effective dielectric constant (relative permittivity) through which the electromagnetic wave is propagating
\( \epsilon'_{\text{r}_1} \) = relative permeability of the insulation around the conductors
\( \epsilon'_{\text{r}_2} \) = relative permeability of the medium around the insulated conductors
\( \theta \) = angle between the twisted pair’s center line and the twist in [rad]
\( T \) = twists per length in [1/m]

### 7.3.5 Characteristic Impedance \( Z_0 \) of Coaxial Cables

The characteristic impedance \( Z_0 \) [\( \Omega \)] of a round coaxial cable—according to Fig. 7.5—can be calculated as [6]:

\[ Z_0 = \frac{\eta_0}{2\pi \sqrt{\epsilon'_{\text{r}}} \ln \left( \frac{D_{\text{coax}}}{D_{\text{core}}} \right)} \] (7.20)

where,

\( \eta_0 = 377 \Omega \) = intrinsic impedance of free space (vacuum) [\( \Omega \)]
\( D_{\text{coax}} \) = inner diameter of the coaxial cable shield in [m]
\( D_{\text{core}} \) = outer diameter of the core wire of the coaxial cable in [m]
\( \epsilon'_{\text{r}} \) = dielectric constant of the dielectric between core and shield
7.3 Characteristic Impedance $Z_0$

Fig. 7.6 Microstrip line

7.3.6 Characteristic Impedance $Z_0$ of Microstrip Lines

The characteristic impedance $Z_0$ [Ω] of a microstrip line—according to Fig. 7.6—can be calculated as [6]:

$$Z_0 = \frac{\eta_0}{2\pi \sqrt{2} \sqrt{\varepsilon_r' + 1}}$$  \hspace{1cm} (7.21)

$$\ln \left( 1 + \frac{4h}{w'} \left( \frac{14 + 8/\varepsilon_r'}{11} \frac{4h}{w'} + \sqrt{\left( \frac{14 + 8/\varepsilon_r'}{11} \right)^2 + \left( \frac{4h}{w'} \right)^2 + \frac{1}{2} \frac{1 + 1/\varepsilon_r'}{\pi^2} } \right) \right)$$  \hspace{1cm} (7.22)

$$w' = w + \Delta w'$$  \hspace{1cm} (7.23)

$$\Delta w' = \Delta w \left( \frac{1 + 1/\varepsilon_r'}{2} \right)$$  \hspace{1cm} (7.24)

$$\Delta w = t \frac{1}{\pi} \ln \left( \frac{4e}{\sqrt{\left( \frac{t}{h} \right)^2 + \left( \frac{1/\pi}{w'/t+1,1} \right)^2} } \right)$$  \hspace{1cm} (7.25)

where,

$\eta_0 = 377$ Ω = intrinsic impedance of free space (vacuum) [Ω]

$h$ = distance of the PCB trace above the reference (ground) plane in [m]

$w$ = width of the PCB trace in [m]

$w'$ = corrected PCB trace width due to thickness $t$ in [m]

$t$ = PCB trace thickness in [m]

$\varepsilon_r'$ = dielectric constant of the PCB dielectric material

7.3.7 Characteristic Impedance $Z_0$ of Coplanar Waveguide with Reference Plane

The characteristic impedance $Z_0$ [Ω] of a coplanar waveguide with reference plane (ground) according to Fig. 7.7 can be calculated as [6]:

$$Z_0 = \frac{\eta_0}{2\pi \sqrt{2} \sqrt{\varepsilon_r' + 1}}$$  \hspace{1cm} (7.21)

$$\ln \left( 1 + \frac{4h}{w'} \left( \frac{14 + 8/\varepsilon_r'}{11} \frac{4h}{w'} + \sqrt{\left( \frac{14 + 8/\varepsilon_r'}{11} \right)^2 + \left( \frac{4h}{w'} \right)^2 + \frac{1}{2} \frac{1 + 1/\varepsilon_r'}{\pi^2} } \right) \right)$$  \hspace{1cm} (7.22)

$$w' = w + \Delta w'$$  \hspace{1cm} (7.23)

$$\Delta w' = \Delta w \left( \frac{1 + 1/\varepsilon_r'}{2} \right)$$  \hspace{1cm} (7.24)

$$\Delta w = t \frac{1}{\pi} \ln \left( \frac{4e}{\sqrt{\left( \frac{t}{h} \right)^2 + \left( \frac{1/\pi}{w'/t+1,1} \right)^2} } \right)$$  \hspace{1cm} (7.25)
Fig. 7.7 Coplanar waveguide with reference plane (ground) or microstrip line with signal side reference plane

Fig. 7.8 Stripline

\[
Z_0 = \frac{\pi 60 \, \Omega}{\sqrt{\frac{\varepsilon'_{r,\text{eff}}}{K(k) K(k')} + \frac{K(k_1)}{K(k_1')}}} \quad (7.26)
\]

\[
k = \frac{w}{w + 2s} \quad (7.27)
\]

\[
k_1 = \frac{\tanh \left( \frac{\pi w}{4h} \right)}{\tanh \left( \frac{\pi (w + 2s)}{4h} \right)} \quad (7.28)
\]

\[
k' = \sqrt{1 - k^2} \quad (7.29)
\]

\[
k'_1 = \sqrt{1 - k_1^2} \quad (7.30)
\]

\[
\varepsilon'_{r,\text{eff}} = \frac{1 + \varepsilon_r K(k') K(k_1)}{1 + \frac{K(k')}{K(k)} \frac{K(k_1)}{K(k')}} \quad (7.31)
\]

where,

\(K(x)\) = elliptic integral of the first kind of \(x\) (\(x\) stands for \(k, k', k_1, k'_1\))

\(h\) = distance between the two reference (ground) planes in [m]

\(w\) = width of the PCB trace in [m]

\(s\) = space between the PCB trace and the coplanar reference plane in [m]

\(\varepsilon'_{r,\text{eff}}\) = effective dielectric constant of the PCB dielectric material

### 7.3.8 Characteristic Impedance \(Z_0\) of Centered Striplines

The characteristic impedance \(Z_0\) [\(\Omega\)] of a stripline—which is located in the middle of two reference planes according to Fig. 7.8—can be calculated as [6]:
7.4 Per-Unit-Length Inductance $L'$ and Capacitance $C'$

Electrically short transmission lines ($l < \lambda/10$) can be modeled as lumped-element $RLC$ networks (Fig. 7.9). The following subsections present formulas for the calculation of the per-unit-length inductance $L'$ [H/m] and the per-unit-length capacitance $C'$ [F/m] for typical electrical transmission lines:

- **Two-wire lines**: Sect. 7.4.1.
- **Wires above ground plane**: Sect. 7.4.2.
- **Coaxial cables**: Sect. 7.4.3.
- **PCB traces**: Sect. 7.4.4.

\begin{align}
Z_0 &= \frac{60 \Omega}{\sqrt{\epsilon_r'}} \ln \left( \frac{4h}{\pi K_1} \right), \text{ for } \frac{w}{h} \leq 0.35 \tag{7.32}
\end{align}

\begin{align}
Z_0 &= \frac{94.15 \Omega}{w/h} + \frac{1}{\pi \sqrt{\epsilon_r'}} \ln \left( \frac{w}{h} \right), \text{ for } \frac{w}{h} > 0.35 \tag{7.33}
\end{align}

\begin{align}
K_1 &= \frac{w}{2} \left[ 1 + \frac{t}{\pi w} \left( 1 + \ln \left( \frac{4\pi w}{t} \right) + 0.51\pi \left( \frac{t}{w} \right)^2 \right) \right] \tag{7.34}
\end{align}

\begin{align}
K_2 &= \frac{2}{1 - t/h} \ln \left( \frac{1}{1 - t/h} + 1 \right) - \left( \frac{1}{1 - t/h} - 1 \right) \ln \left( \frac{1}{(1 - t/h)^2} - 1 \right) \tag{7.35}
\end{align}

where,

- $h$ = distance between the two reference (ground) planes in [m]
- $t$ = thickness of the PCB trace in [m]
- $w$ = width of the PCB trace in [m]
- $\epsilon_r'$ = dielectric constant of the PCB dielectric material
7.4.1 $L'$ and $C'$ of Two-Wire Lines

For a setup with two parallel wires (e.g., ribbon cable with identical diameter $D$ [m]), where the current flows in the opposite direction, the per-unit-length inductance $L'$ [H/m] and the per-unit-length capacitance $C'$ [F/m] are given as [4]:

$$L'_{TwoWireLine} \approx \frac{\mu'_{r,eff} \mu_0}{\pi} \cosh^{-1} \left( \frac{d}{D} \right)$$  \quad (7.36)$$

$$C'_{TwoWireLine} \approx \frac{\pi \epsilon'_{r,eff} \epsilon_0}{\cosh^{-1} \left( \frac{d}{D} \right)}$$  \quad (7.37)$$

where,

$cosh^{-1} = $ area hyperbolic cosine (inverse hyperbolic cosine) function $[1, +\infty]$  
$\mu'_{r,eff} =$ effective relative permeability of the environment of the two wires and therefore the material through which the magnetic flux flows  
$\mu_0 = 12.57 \cdot 10^{-7}$ H/m = permeability of vacuum, absolute permeability  
$\epsilon'_{r,eff} =$ effective relative permittivity (dielectric constant) of the environment of the two wires and, therefore, the material through which the electric field lines are formed  
$\epsilon_0 = 8.854 \cdot 10^{-12}$ F/m = permittivity of vacuum, absolute permittivity  
$d =$ distance from center to center of the two wires in [m]  
$D =$ diameter of the two wires in [m]

The effective relative permeability $\mu'_{r,eff}$ is often equal 1.0, because the insulation material of cables and the environment around the lines is usually non-magnetic (e.g., plastics, aluminum, or copper).

The calculation of the effective relative permeability $\epsilon'_{r,eff}$ is often complicated because there are different dielectric materials involved (e.g., cable insulation and air). The approximate effective dielectric constant $\epsilon'_{r,eff}$ for a pair of wire—like shown in Fig. 7.10—is given in as [6]:

**Fig. 7.10** Two-wire line
7.4 Per-Unit-Length Inductance $L'$ and Capacitance $C'$

Table 7.1  Unit-per-length inductance $L'$ and capacitance $C'$ of two-wire lines with PVC insulation around each wire

<table>
<thead>
<tr>
<th>AWG</th>
<th>d [m]</th>
<th>$\mu_{\text{reff}}$ [1]</th>
<th>$\epsilon_{r1}$ [1]</th>
<th>$\epsilon_{r2}$ [1]</th>
<th>$\epsilon_{\text{reff}}$ [1]</th>
<th>$L'_{\text{loop}}$ [H/m]</th>
<th>$C'$ [F/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AWG6</td>
<td>0.0081</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1.5</td>
<td>0.52 $\mu$H/m</td>
<td>32.1 pF/m</td>
</tr>
<tr>
<td>AWG20</td>
<td>0.0026</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1.5</td>
<td>0.73 $\mu$H/m</td>
<td>22.7 pF/m</td>
</tr>
<tr>
<td>AWG32</td>
<td>0.00075</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1.5</td>
<td>0.90 $\mu$H/m</td>
<td>21.0 pF/m</td>
</tr>
</tbody>
</table>

(a) PVC insulates wires, side-by-side without cable jacket.

(b) PVC insulates wires, side-by-side with PVC cable jacket.

Fig. 7.11  Wire above ground plane

\[
\epsilon'_{r,\text{eff}} \approx \epsilon'_{r2} + 0.25 (\epsilon'_{r1} - \epsilon'_{r2}) \tag{7.38}
\]

where,

$\epsilon'_{r1}$ = relative permeability of the insulation around the conductors

$\epsilon'_{r2}$ = relative permeability of the medium around the insulated conductors

Based on Eqs. 7.36 and 7.37, Table 7.1a lists examples of per-unit-length inductance $L'$ and per-unit-length capacitance $C'$ of a two-wire line without an additional cable jacket ($\epsilon_{r2} = 1$). On the other hand, Table 7.1b lists the values for $L'$ and $C'$ with a two-wire line and a PVC cable jacket around the wires ($\epsilon_{r2} = 3$).

### 7.4.2 $L'$ and $C'$ of Wires Above Ground Plane

In the case of a wire above an infinite ground plane (Fig. 7.11), the per-unit-length loop inductance $L'$ [H/m] and the per-unit-length capacitance $C'$ [F/m] are given as [4]:

\[ L_{\text{WireAboveGndPlane}}' \approx \frac{\mu_{r,\text{eff}}' \mu_0}{2\pi} \cosh^{-1}\left(\frac{2d}{D}\right) \]  
\[ C_{\text{WireAboveGndPlane}}' \approx \frac{2\pi \varepsilon_{r,\text{eff}}'}{\cosh^{-1}\left(\frac{2d}{D}\right)} \]

where,
\[ \cosh^{-1} = \text{area hyperbolic cosine (inverse hyperbolic cosine) function \([1, +\infty]\)} \]
\[ \mu_{r,\text{eff}}' = \text{effective relative permeability of the environment of the wire and therefore the material through which the magnetic flux flows} \]
\[ \mu_0 = 12.57 \cdot 10^{-7} \, \text{H/m} = \text{permeability of vacuum, absolute permeability} \]
\[ \varepsilon_{r,\text{eff}}' = \text{effective relative permittivity (dielectric constant) between the wire and the ground plane and therefore the material through which the electric field lines are formed} \]
\[ \varepsilon_0 = 8.854 \cdot 10^{-12} \, \text{F/m} = \text{permittivity of vacuum, absolute permittivity} \]
\[ d = \text{distance from the center of the wire to the surface of the ground plane [m]} \]
\[ D = \text{diameter of the wire in [m]} \]

The effective relative permeability \[ \mu_{r,\text{eff}}' \] is usually equal 1.0, because all the media around the conductor are non-magnetic (e.g. plastics, copper), whereas the effective relative permittivity \[ \varepsilon_{r,\text{eff}}' \] between the wire and the ground plane is very difficult to calculate, as it depends very much on the distance of the wire to the ground plane, the thickness of the insulation around the wire, and the media between the wire and the ground plane. Table 7.2 gives an idea about the ranges of the per-unit-length inductance \[ L' \, [\text{H/m}] \] and the per-unit-capacitance \[ C' \, [\text{F/m}] \] of a single wire above a ground plane in case the wire is close to the ground plane or far away.

<table>
<thead>
<tr>
<th>AWG</th>
<th>d [m]</th>
<th>( \mu_{r,\text{eff}}' )</th>
<th>( \varepsilon_{r1} )</th>
<th>( \varepsilon_{r2} )</th>
<th>( \varepsilon_{r,\text{eff}}' )</th>
<th>( L'_{\text{loop}} ) [H/m]</th>
<th>C' [F/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AWG6</td>
<td>0.00405</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1.5</td>
<td>0.26\mu\text{H/m}</td>
<td>64.2pF/m</td>
</tr>
<tr>
<td>AWG20</td>
<td>0.0013</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1.5</td>
<td>0.36\mu\text{H/m}</td>
<td>45.5pF/m</td>
</tr>
<tr>
<td>AWG32</td>
<td>0.00038</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1.5</td>
<td>0.40\mu\text{H/m}</td>
<td>42.0pF/m</td>
</tr>
</tbody>
</table>

(a) PVC insulated wire laying directly on ground plane.

<table>
<thead>
<tr>
<th>AWG</th>
<th>d [m]</th>
<th>( \mu_{r,\text{eff}}' )</th>
<th>( \varepsilon_{r1} )</th>
<th>( \varepsilon_{r2} )</th>
<th>( \varepsilon_{r,\text{eff}}' )</th>
<th>( L'_{\text{loop}} ) [H/m]</th>
<th>C' [F/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AWG6</td>
<td>0.01</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.45\mu\text{H/m}</td>
<td>24.6pF/m</td>
</tr>
<tr>
<td>AWG20</td>
<td>0.01</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.78\mu\text{H/m}</td>
<td>14.3pF/m</td>
</tr>
<tr>
<td>AWG32</td>
<td>0.01</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1.06\mu\text{H/m}</td>
<td>10.1pF/m</td>
</tr>
</tbody>
</table>

(b) PVC insulated wire 10mm above ground plane.
7.4 Per-Unit-Length Inductance $L'$ and Capacitance $C'$

For a coaxial cable (Fig. 7.12), the per-unit-length loop inductance $L'$ [H/m] and the per-unit-length capacitance $C'$ [F/m] are given as [4]:

$$L'_{coax} \approx \frac{\mu_r \mu_0}{2\pi} \ln\left(\frac{D_{coax}}{D_{core}}\right)$$  \hspace{1cm} (7.41)$$

$$C'_{coax} \approx \frac{2\pi \epsilon_r \epsilon_0}{\ln\left(\frac{D_{coax}}{D_{core}}\right)}$$  \hspace{1cm} (7.42)$$

where,

$\mu_r = \text{relative permeability of the coaxial cable dielectric material in [1]}$
$\mu_0 = 12.57 \cdot 10^{-7} \text{ H/m} = \text{permeability of vacuum, absolute permeability}$
$\epsilon_r = \text{relative permittivity of the coaxial cable dielectric material in [1]}$
$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m} = \text{permittivity of vacuum, absolute permittivity}$

$D_{coax} = \text{inner diameter of the coaxial cable shield in [m]}$
$D_{core} = \text{outer diameter of the core wire of the coaxial cable in [m]}$

Table 7.3 gives an idea about the ranges of the per-unit-length inductance $L'$ [H/m] and the per-unit-capacitance $C'$ [F/m] of typical coaxial cables.
7.4.4 $L'$ and $C'$ of PCB Traces

It is not common to specify the per-unit-length inductance $L'$ [H/m] and the per-unit-length capacitance $C'$ [F/m] for PCB traces. Instead, it is more common to specify the characteristic impedance $Z_0$ [Ω] of the transmission line (PCB trace), which can be expressed as:

$$Z_0 = \sqrt{\frac{L'}{C'}} \quad (7.43)$$

Another fundamental parameter is the propagation velocity $v$ [m/sec] of a signal along a transmission line:

$$v = \frac{1}{\sqrt{L'C'}} = \frac{c}{\sqrt{\mu'_r, eff \epsilon'_r, eff}} \quad (7.44)$$

$$v = \frac{c}{\sqrt{\epsilon'_r, eff}}, \text{ because usually } \mu'_r, eff = 1.0 \quad (7.45)$$

Thus, we can write:

$$L' = \frac{Z_0}{v} = \frac{Z_0 \sqrt{\epsilon'_r, eff}}{c} \quad (7.46)$$

$$C' = \frac{1}{vZ_0} = \frac{\sqrt{\epsilon'_r, eff}}{cZ_0} \quad (7.47)$$

where,

$L'$ = per-unit-length inductance of the transmission line (e.g., PCB trace) [H/m]

$C'$ = per-unit-length capacitance of the transmission line (e.g., PCB trace) [F/m]

$c = 1/(\sqrt{\mu_0 \epsilon_0}) = 2.998 \cdot 10^8$ m/sec = speed of light

$\mu'_r, eff$ = effective magnetic relative permeability through which the electromagnetic wave is propagating

$\epsilon'_r, eff$ = effective dielectric constant (relative permittivity) through which the electromagnetic wave is propagating

Tables 7.4, 7.5 and, 7.6 present some example values of per-unit-length inductance $L'$ [H/m] and per-unit-length capacitance $C'$ [F/m] for microstrip lines, striplines, and coplanar waveguides with reference plane. All values are approximations.

Formulas for calculating the characteristic impedance $Z_0$ of some selected PCB structures can be found in the next Chap. 11.1.7.2.
Table 7.4 Unit-per-length inductance $L'$ and capacitance $C'$ of microstrip lines shown in Fig. 7.6. 
$t$ = thickness of the PCB trace. $h$ = distance of the PCB trace above the reference plane. $w$ = width of the PCB trace

<table>
<thead>
<tr>
<th>$t$ [m]</th>
<th>$h$ [m]</th>
<th>$w$ [m]</th>
<th>$\varepsilon_r$ [1]</th>
<th>$\varepsilon_{r,eff}$ [1]</th>
<th>$L_{loop}'$ [H/m]</th>
<th>$C'$ [F/m]</th>
<th>$Z_0$ [$\Omega$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.50E-05</td>
<td>0.0015</td>
<td>0.0001</td>
<td>4.5</td>
<td>2.9</td>
<td>0.90µH/m</td>
<td>33.6pF/m</td>
<td>157</td>
</tr>
<tr>
<td>3.50E-05</td>
<td>0.0015</td>
<td>0.001</td>
<td>4.5</td>
<td>3.2</td>
<td>0.49µH/m</td>
<td>71.4pF/m</td>
<td>83</td>
</tr>
<tr>
<td>3.50E-05</td>
<td>0.0015</td>
<td>0.01</td>
<td>4.5</td>
<td>3.8</td>
<td>0.13µH/m</td>
<td>320pF/m</td>
<td>20</td>
</tr>
<tr>
<td>3.50E-05</td>
<td>0.0001</td>
<td>0.0001</td>
<td>4.5</td>
<td>3.2</td>
<td>0.38µH/m</td>
<td>94.6pF/m</td>
<td>63</td>
</tr>
<tr>
<td>3.50E-05</td>
<td>0.0001</td>
<td>0.01</td>
<td>4.5</td>
<td>3.9</td>
<td>0.09µH/m</td>
<td>462pF/m</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 7.5 Unit-per-length inductance $L'$ and capacitance $C'$ of coplanar waveguides with a reference plane shown in Fig. 7.7. $h$ = distance between the two reference (ground) planes. $w$ = width of the PCB trace. $s$ = space between the PCB trace and the coplanar reference plane

<table>
<thead>
<tr>
<th>$h$ [m]</th>
<th>$w$ [m]</th>
<th>$s$ [m]</th>
<th>$\varepsilon_r$ [1]</th>
<th>$\varepsilon_{r,eff}$ [1]</th>
<th>$L_{loop}'$ [H/m]</th>
<th>$C'$ [F/m]</th>
<th>$Z_0$ [$\Omega$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0015</td>
<td>0.0001</td>
<td>0.01</td>
<td>4.5</td>
<td>2.82</td>
<td>0.85µH/m</td>
<td>38.8pF/m</td>
<td>152.0</td>
</tr>
<tr>
<td>0.0015</td>
<td>0.001</td>
<td>0.01</td>
<td>4.5</td>
<td>2.95</td>
<td>0.43µH/m</td>
<td>75.3pF/m</td>
<td>76.0</td>
</tr>
<tr>
<td>0.0015</td>
<td>0.01</td>
<td>0.001</td>
<td>4.5</td>
<td>3.66</td>
<td>0.13µH/m</td>
<td>325pF/m</td>
<td>19.6</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.0001</td>
<td>0.01</td>
<td>4.5</td>
<td>3.5</td>
<td>0.48µH/m</td>
<td>80.5pF/m</td>
<td>77.5</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.01</td>
<td>0.0001</td>
<td>4.5</td>
<td>4.1</td>
<td>0.10µH/m</td>
<td>450pF/m</td>
<td>15.0</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.01</td>
<td>0.0001</td>
<td>4.5</td>
<td>4.47</td>
<td>0.005µH/m</td>
<td>10'010pF/m</td>
<td>0.7</td>
</tr>
<tr>
<td>0.0015</td>
<td>0.0001</td>
<td>0.0001</td>
<td>4.5</td>
<td>2.75</td>
<td>0.43µH/m</td>
<td>62.1pF/m</td>
<td>89.0</td>
</tr>
<tr>
<td>0.0015</td>
<td>0.001</td>
<td>0.0001</td>
<td>4.5</td>
<td>2.8</td>
<td>0.25µH/m</td>
<td>123pF/m</td>
<td>45.5</td>
</tr>
<tr>
<td>0.0015</td>
<td>0.01</td>
<td>0.0001</td>
<td>4.5</td>
<td>3.42</td>
<td>0.10µH/m</td>
<td>378pF/m</td>
<td>16.4</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>4.5</td>
<td>3.09</td>
<td>0.39µH/m</td>
<td>87.5pF/m</td>
<td>67.0</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.001</td>
<td>0.0001</td>
<td>4.5</td>
<td>3.87</td>
<td>0.09µH/m</td>
<td>455pF/m</td>
<td>14.4</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.01</td>
<td>0.0001</td>
<td>4.5</td>
<td>4.44</td>
<td>0.005µH/m</td>
<td>10'018pF/m</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 7.6 Unit-per-length inductance $L'$ and capacitance $C'$ of striplines shown in Fig. 7.8. $t$ = thickness of the PCB trace. $h$ = distance between the two reference planes. $w$ = width of the PCB trace

<table>
<thead>
<tr>
<th>$t$ [m]</th>
<th>$h$ [m]</th>
<th>$w$ [m]</th>
<th>$\varepsilon_r$ [1]</th>
<th>$L_{loop}'$ [H/m]</th>
<th>$C'$ [F/m]</th>
<th>$Z_0$ [$\Omega$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.50E-05</td>
<td>0.0015</td>
<td>0.0001</td>
<td>4.5</td>
<td>0.84µH/m</td>
<td>77.7pF/m</td>
<td>91.0</td>
</tr>
<tr>
<td>3.50E-05</td>
<td>0.0015</td>
<td>0.001</td>
<td>4.5</td>
<td>0.27µH/m</td>
<td>186pF/m</td>
<td>38.1</td>
</tr>
<tr>
<td>3.50E-05</td>
<td>0.0015</td>
<td>0.01</td>
<td>4.5</td>
<td>0.043µH/m</td>
<td>1164pF/m</td>
<td>6.1</td>
</tr>
<tr>
<td>3.50E-05</td>
<td>0.0001</td>
<td>0.0001</td>
<td>4.5</td>
<td>0.13µH/m</td>
<td>382pF/m</td>
<td>18.5</td>
</tr>
<tr>
<td>3.50E-05</td>
<td>0.0001</td>
<td>0.001</td>
<td>4.5</td>
<td>0.019µH/m</td>
<td>2588pF/m</td>
<td>2.7</td>
</tr>
<tr>
<td>3.50E-05</td>
<td>0.0001</td>
<td>0.01</td>
<td>4.5</td>
<td>0.002µH/m</td>
<td>24'650pF/m</td>
<td>0.3</td>
</tr>
</tbody>
</table>
7.5 Propagation Constant $\gamma$

The propagation constant $\gamma$ [1/m]—also called propagation factor—describes the attenuation and phase shift of the signal as it propagates through the transmission line. To be even more precise: the propagation constant of a sinusoidal electromagnetic wave is a measure of the change undergone by the amplitude and phase of the wave as it propagates in a given direction (Fig. 7.13).

Let’s imagine a sinusoidal voltage, current, electric field, or magnetic field which propagates in the direction of $z$ [m] and which has amplitude of $A_0$ at its source and an amplitude of $A_z$ at the distance $z$ [m] from the source. Then $A_z$ can be written as:

$$A_z(z) = A_0 \cdot e^{-\gamma z}$$

(7.48)

where,

$A_z = \text{complex phasor of a sinusoidal voltage [V], current [A], electric field [V/m], or magnetic field [A/m] at the distance } z \text{ away from the source}$

$A_0 = \text{complex phasor of a sinusoidal voltage [V], current [A], electric field [V/m], or magnetic field [A/m] at the source}$

$\gamma = \text{the complex propagation constant in [1/m]}$

$z = \text{distance traveled along a transmission line in [m]}$

The propagation constant $\gamma$ [1/m] can be calculated based on the per-unit-length parameters of a transmission line [2]:

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = j\omega \sqrt{\frac{\epsilon}{\mu}}$$

(7.49)

where,

$R' = \text{resistance per-unit-length in [}\Omega/\text{m]}$

$L' = \text{inductance per-unit-length in [H/m]}$

$C' = \text{capacitance per-unit-length in [F/m]}$

Fig. 7.13 Attenuation of a sinusoidal signal along a transmission line with attenuation factor $\alpha$ [1/m]
7.5 Propagation Constant $\gamma$

$G' = \text{conductance per-unit-length in [S/m]}

\omega = 2\pi f = \text{angular frequency of the signal in [rad/sec]}

\mu = \mu' - j\mu'' = \text{complex permeability of the media through which the electromagnetic wave is propagating in [H/m]}

\epsilon = \epsilon' - j\epsilon'' = \text{complex permittivity of the media through which the electromagnetic wave is propagating in [F/m]}

More details about the complex permittivity and permeability can be found in Appendix Sects. D.5 and D.6.

Another notation of $\gamma$ is given as [2]:

$$\gamma = \alpha + j\beta$$  \hspace{1cm} (7.50)

where,

$\alpha = \text{attenuation constant (or attenuation factor) in [1/m]}

\beta = \text{phase constant (or phase factor) in [rad/m]}

This means that the transmission line Eq. 7.48 can be rewritten as:

$$A_z = A_0 \cdot e^{-\alpha z} e^{-j\beta z}$$  \hspace{1cm} (7.51)

In a general form, the attenuation constant $\alpha$ [1/m] and the phase constant $\beta$ [rad/m] can be calculated like this [2]:

$$\alpha = \omega \sqrt{\frac{(\epsilon'\mu' - \epsilon''\mu'')}{2}} \cdot \left( \sqrt{1 + \left( \frac{\epsilon'\mu'' + \epsilon''\mu'}{\epsilon'\mu' - \epsilon''\mu''} \right)^2} - 1 \right)$$  \hspace{1cm} (7.52)

$$\beta = \frac{2\pi}{\lambda} = \omega \sqrt{\frac{(\epsilon'\mu' - \epsilon''\mu'')}{2}} \cdot \left( \sqrt{1 + \left( \frac{\epsilon'\mu'' + \epsilon''\mu'}{\epsilon'\mu' - \epsilon''\mu''} \right)^2} + 1 \right)$$  \hspace{1cm} (7.53)

where,

$\omega = 2\pi f = \text{angular frequency of the signal in [rad/sec]}

\epsilon' = \text{real part of the complex permittivity (}\epsilon = \epsilon' - j\epsilon''\text{) in [F/m]}

\epsilon'' = \text{imaginary part of the complex permittivity (}\epsilon = \epsilon' - j\epsilon''\text{) in [F/m]}

\mu' = \text{real part of the complex permeability (}\mu = \mu' - j\mu''\text{) in [H/m]}

\mu'' = \text{imaginary part of the complex permeability (}\mu = \mu' - j\mu''\text{) in [H/m]}

The magnetic loss $\mu''$ can often be neglected ($\mu'' = 0$) and the dielectric loss $\epsilon''$ can be written as [6]:

$$\epsilon = \epsilon' - j\epsilon'' = \epsilon' - j\frac{\sigma}{\omega}$$  \hspace{1cm} (7.54)
\[ \epsilon'' = \frac{\sigma}{\omega} \] (7.55)

where,

\[ \epsilon'' = \text{dielectric loss} = \text{energy dissipated when a medium is under the influence of an external electric field} = \text{imaginary part of the complex permittivity} (\epsilon = \epsilon' - j\epsilon'') \text{ in [F/m]} \]

\[ \sigma = \text{specific conductance of the medium where the wave is propagating through in [S/m]} \]

With \( \mu'' = 0 \) and \( \epsilon'' = \sigma/\omega \), the attenuation constant \( \alpha \) [1/m] and the phase constant \( \beta \) [rad/m] can be calculated like this [6]:

\[
\alpha = \omega \sqrt{\frac{\mu'\epsilon'}{2} \left( \sqrt{1 + \left( \frac{\sigma}{\omega\epsilon'} \right)^2} - 1 \right)} \] (7.56)

\[
\beta = \omega \sqrt{\frac{\mu'\epsilon'}{2} \left( \sqrt{1 + \left( \frac{\sigma}{\omega\epsilon'} \right)^2} + 1 \right)} \] (7.57)

For a lossless line (\( \sigma = 0 \)) we get [2]:

- No attenuation \( \alpha \) [1/m]:
  \[ \alpha = 0 \] (7.58)

- Phase shift \( \beta \) [rad/m]:
  \[ \beta = \omega \sqrt{R'C'} = \omega \sqrt{\epsilon'\mu'} \] (7.59)

where,

\[ \omega = 2\pi f = \text{angular frequency of the signal in [rad/sec]} \]

\[ \epsilon' = \epsilon'_{r}\epsilon_0 = \text{ability to store energy in a medium when an external electric field is applied} = \text{real part of the complex permittivity} (\epsilon = \epsilon' - j\epsilon'') \text{ in [F/m]} \]

\[ \mu' = \mu'_{r}\mu_0 = \text{ability to store energy in a medium when an external magnetic field is applied} = \text{real part of the complex permeability} (\mu = \mu' - j\mu'') \text{ in [H/m]} \]
7.6 Input Impedance of Transmission Lines

7.6.1 Input Impedance of Any Transmission Line

The characteristic impedance $Z_0$ [Ω] of a transmission line—as already discussed in Chap. 7.3—is the ratio of the amplitude of a single voltage wave to its current wave. Since most transmission lines also have a reflected wave, the characteristic impedance is generally not the impedance that is measured on the line. The impedance $Z_{in}(l)$ [Ω] (Fig. 7.14) measured at a given distance $l$ [m] from the load impedance $Z_L$ [Ω] can be expressed as [6]:

$$Z_{in}(l) = \frac{V(l)}{I(l)} = \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} = Z_0 \frac{Z_L \cosh(\gamma l) + Z_0 \sinh(\gamma l)}{Z_0 \cosh(\gamma l) + Z_L \sinh(\gamma l)}$$

(7.60)

where,

$Z_{in}(l) =$ impedance measured at distance $l$ from the load $Z_L$ in [Ω]
$V(l) =$ voltage at distance $l$ from the load $Z_L$ in [V]
$I(l) =$ current at distance $l$ from the load $Z_L$ in [V]
$Z_0 =$ the complex characteristic impedance of the transmission line in [Ω]
$Z_L =$ the complex load impedance in [Ω]
$\gamma =$ $\alpha + j\beta =$ the complex propagation constant in [1/m]
$l =$ the distance from where the input of the transmission line to the load in [m]

7.6.2 Input Impedance of a Lossless Transmission Line

For a lossless transmission line, the propagation constant is purely imaginary [6]:

$$\alpha = 0$$

(7.61)

$$\gamma = j\beta = \frac{2\pi j}{\lambda}$$

(7.62)

and the input impedance to a lossless transmission line can be calculated as:
\[ Z_{in}(l) = \frac{Z_0 Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} = \frac{Z_0 Z_L \cos(\beta l) + jZ_0 \sin(\beta l)}{Z_0 \cos(\beta l) + jZ_L \sin(\beta l)} \quad (7.63) \]

where,

- \( Z_{in}(l) \) = impedance measured at distance \( l \) from the load \( Z_L \) in [\( \Omega \)]
- \( Z_0 \) = the complex characteristic impedance of the transmission line in [\( \Omega \)]
- \( Z_L \) = the complex load impedance in [\( \Omega \)]
- \( l \) = the distance from where the input of the transmission line to the load in [m]
- \( \beta = 2\pi/\lambda \) = phase constant (or phase factor) in [rad/m]

### 7.6.3 Input Impedance of a Transmission Line At \( l = \lambda/2 \)

When the distance from the input of the transmission line to the load is a multiple of \( \lambda/2 \) (\( l = n\lambda/2 \)) and therefore \( \beta l = n\pi \) (where \( n \) is an integer), the input impedance to the transmission line \( Z_{in}(l) \) is equal the load impedance \( Z_L \) [6]:

\[ Z_{in}(l) = Z_L \quad (7.64) \]

### 7.6.4 Impedance of a Transmission Line At \( l = \lambda/4 \)

When the distance from the input of the transmission line to the load is a multiple of \( \lambda/4 \) (\( \beta l = n\pi/2 \)) and therefore \( l = n\lambda/4 \) (where \( n \) is an integer), the input impedance to the transmission line \( Z_{in}(l) \) is [6]:

\[ Z_{in}(l) = \frac{Z_0^2}{Z_L} \quad (7.65) \]

This means in case \( Z_L = 0 \), the transmission line input impedance becomes \( Z_{in} = \infty \) and vice versa.

### 7.6.5 Impedance of a Matched Transmission Line

In case the load impedance \( Z_L \) is equal the characteristic impedance \( Z_0 \) of the transmission line (in other words: the load is matched), we can write:

\[ Z_{in}(l) = Z_0 = Z_L \quad (7.66) \]
7.6.6 Impedance of a Shorted Transmission Line

For the case of a shorted load $Z_L = 0$, the input impedance $Z_{in}(l)$ is purely imaginary and a periodic function of position and wavelength $\lambda$ [m] (frequency $f$ [Hz]):

$$Z_{in}(l) = jZ_0 \tan(\beta l) = jZ_0 \tan\left(\frac{2\pi l}{\lambda}\right) \quad (7.67)$$

7.6.7 Impedance of an Open Transmission Line

For the case of an open load $Z_L = \infty$, the input impedance $Z_{in}(l)$ is purely imaginary and a periodic function of position and wavelength $\lambda$ [m] (frequency $f$ [Hz]):

$$Z_{in}(l) = -jZ_0 \cot(\beta l) = -jZ_0 \cot\left(\frac{2\pi l}{\lambda}\right) \quad (7.68)$$

7.7 High-Frequency Losses

The lossless line model is often accurate enough for frequencies up 100 MHz. However, above 100 MHz, the high-frequency losses may not be neglected anymore, and therefore the attenuation factor $\alpha$ [1/m] cannot be assumed to be zero. High-frequency losses are a result of [3]:

- **Ohmic Loss.** Ohmic losses result from the resistance of the conductors. Ohmic losses increase proportionally to $\sqrt{f}$ due to the skin effect (see Fig. 11.5).
  Note: The ohmic loss is a function of the frequency $f$ [Hz] and of the geometry of the conductor (diameter of a conductor and surface area).
- **Dielectric Loss.** Dielectric losses occur because dielectric materials absorb energy from the propagating electric field (heating the material; see Sect. 7.8). The dielectric loss is a function of frequency $f$ [Hz], the dissipation factor $\tan(\delta)$, and the dielectric constant $\epsilon'_r$.
  Note: The dielectric loss does not depend on the geometry of the transmission line, only on the dielectric material.
7.8 Loss Tangent \( \tan(\delta) \)

The electric and magnetic losses quantify a material’s inherent dissipation of electromagnetic energy (e.g., heat). The loss tangent \( \tan(\delta) \) (Fig. 7.15) is used to express how lossy a medium or a transmission line is: a small \( \tan(\delta) \) means low loss.

The electric loss tangent—often called dielectric loss tangent or dissipation factor \( D_f \)—is defined as [2]:

\[
\tan(\delta_e) = \frac{\epsilon''}{\epsilon'} = \frac{\text{loss current}}{\text{charging current}}
\]  

(7.69)

where,

\( \epsilon' = \epsilon_r \epsilon_0 = \text{ability to store energy in a medium when an external electric field is applied} = \text{real part of the complex permittivity (} \epsilon = \epsilon' - j\epsilon'' \text{) in [F/m]} \)

\( \epsilon'' = \epsilon_r'' \epsilon_0 = \text{dielectric loss = energy dissipated when a medium is under the influence of an external electric field = imaginary part of the complex permittivity (} \epsilon = \epsilon' - j\epsilon'' \text{) in [F/m]} \)

The electrical conductivity \( \sigma \) [S/m] of a dielectric material is defined as [2]:

\[
\sigma = \omega \epsilon''
\]  

(7.70)

and therefore the loss tangent can be written as function of \( \sigma \) [S/m]:

\[
\tan(\delta_e) = \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega \epsilon'}
\]  

(7.71)

The magnetic loss tangent \( \tan(\delta_m) \) is usually not of interest for traditional transmission lines because the materials involved are not magnetic. However, for the sake of completeness, here the definition of the magnetic loss tangent [2]:

Fig. 7.15 Charging and loss current density
\[ \tan(\delta_m) = \frac{\mu''}{\mu'} \] (7.72)

where,

\[
\mu' = \mu'_r \mu_0 = \text{ability to store energy in a medium when an external magnetic field is applied}
\]

\[
\mu'' = \mu''_r \mu_0 = \text{magnetic loss = the energy dissipated when a medium is under the influence of an external magnetic field}
\]

\[
\mu' = \mu' - j\mu'' \quad \text{in [H/m]}
\]

\[
\mu'' = \mu''_r \mu_0 \quad \text{in [F/m]}
\]

### 7.9 Balanced vs. Unbalanced Transmission Lines

EMC design engineers should be well aware of the concept and benefits of balanced transmission lines. Here the differences between balanced and unbalanced transmission lines:

- **Balanced.** A balanced transmission line consists of two conductors which have the same impedance along their line and the same impedance to ground and all other conductors (Fig. 7.16). Differential signals should be transmitted over balanced transmission lines.

- **Unbalanced.** In an unbalanced transmission line, the impedances of the forward and return current lines to ground are unequal. Single-ended signals are signals which are referenced to ground and should therefore be sent over unbalanced transmission lines (Fig. 7.17).

![Fig. 7.16 Balanced transmission line with common-mode noise](image1)

![Fig. 7.17 Unbalanced transmission line with noise on signal line](image2)
Balanced transmission lines are very robust against common-mode noise because common-mode signals will be canceled out at the receiver’s side. Common-mode noise coupling is explained in Sect. 12.4. However, to prevent differential-mode noise coupling, the two signal conductors must be routed close to each other (e.g., twisting them). Differential-mode noise coupling is explained in Sect. 12.3.

Generally, single-ended interfaces should be sent over unbalanced transmission lines, and differential signal interfaces should be sent over balanced transmission lines. Examples:

- **Unbalanced.** Suitable for single-ended signal interfaces.
  - PCB data lines. Microstrip lines (see Fig. 7.6), striplines (see Fig. 7.8), and coplanar waveguides (see Fig. 7.7)
  - Cables and wires. Coaxial cables (see Fig. 7.5) and multi-layer flat-ribbon or flat-flex cables with at least one solid ground plane

- **Balanced.** Suitable for differential and pseudo-differential signal interfaces.
  - PCB data lines. Microstrip lines and striplines routed as differential pairs
  - Cables and wires. Twisted pair (shielded or unshielded, see Fig. 7.4), twin-lead cables, flat-ribbon or flat-flex cables (one layer)

### 7.10 Single-Ended vs. Differential Interfaces

*Single-ended signal interfaces* consist of a single signal line and a reference potential. In contrast, *differential signal interfaces* consist of two complementary signals—a differential pair—and a reference potential (compare Figs. 7.18 and 7.19).

While Ethernet is a truly differential signal interface, most of the differential interfaces listed below are pseudo-differential, meaning that two single-ended digital signal sources are switched inverted relative to each other (around a common mode voltage level). The advantage of a pseudo-differential interface to a truly differential is that it doesn’t need a *balun* (transformer which converts between a balanced signal and an unbalanced signal; see Sect. 11.7). The disadvantage of a pseudo-differential interface over a truly differential interface is that due to

![Fig. 7.18 Single-ended signal](image-url)
slight timing differences at signal changes (from high to low and vice versa), a common-mode noise current is generated, which could lead to unintended radiated emissions (common-mode noise is the number one source for unintended radiation; see Sect. 9.9).

- **Single-ended interfaces.** Single-ended signals should be sent over unbalanced transmission lines.
  - **Communication interfaces.** RS232, VGA video connectors, SCSI interfaces, VMEbus, PCI
  - **PCB data buses.** I²C, SPI
  - **Digital signaling.** TTL, CMOS

- **Differential signal interfaces.** Differential signals should be sent over balanced transmission lines.
  - **Communication interfaces.** Ethernet, USB, HDMI, DVI-D, Serial ATA, PCI Express
  - **Digital signaling.** LVDS, ECL, PECL, CML/SCL

### 7.11 Summary

- **Transmission line.** A transmission line is a series of conductors, often but not necessarily two, used to guide electromagnetic energy from one place to the other. Transmission line parameters are summarized in Table 7.7.
- **Critical length** $l_{\text{critical}}$. Reflections and ringing can occur if a signal interconnection is longer than the critical length, and impedance matching must be considered to avoid this. Good and poor impedance matching is presented in Table 6.2. Rule of thumb:

\[ l_{\text{critical}} \approx \frac{\lambda_{\text{min}}}{10} \]  

\[ (7.73) \]
Table 7.7 Transmission line parameters for any media, lossless media \((\sigma = 0)\), low-loss media good \((\varepsilon''/\varepsilon' = \sigma/(\omega\varepsilon') << 0.01)\) and good conductors \(\varepsilon''/\varepsilon' > 100\) \([5]\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Any media</th>
<th>Lossless medium ((\sigma = 0))</th>
<th>Low-loss medium ((\varepsilon''/\varepsilon' &lt;&lt; 1))</th>
<th>Good conductor ((\varepsilon''/\varepsilon' &gt;&gt; 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha \ [\text{m}^{-1}])</td>
<td>(\frac{\omega}{2} \sqrt{\frac{\varepsilon' \mu' - \varepsilon'' \mu''}{2} + \left(\frac{\varepsilon' \mu' + \varepsilon'' \mu''}{\varepsilon' \mu' - \varepsilon'' \mu''} - 1\right)})</td>
<td>0</td>
<td>(\sigma \frac{\mu}{2\sqrt{\varepsilon'}})</td>
<td>(\sqrt{\pi \mu \sigma})</td>
</tr>
<tr>
<td>(\beta \ [\text{rad/m}])</td>
<td>(\frac{2\pi}{\lambda} = \frac{\omega}{\mu} \sqrt{\frac{\varepsilon' \mu' - \varepsilon'' \mu''}{2} + \left(\frac{\varepsilon' \mu' + \varepsilon'' \mu''}{\varepsilon' \mu' - \varepsilon'' \mu''} + 1\right)})</td>
<td>(\omega \sqrt{\mu' \varepsilon'})</td>
<td>(\omega \sqrt{\mu' \varepsilon'})</td>
<td>(\pi \sqrt{\mu' \varepsilon'})</td>
</tr>
<tr>
<td>(\eta \ [\Omega])</td>
<td>(\frac{\mu' \omega + j\omega \mu''}{\sqrt{\varepsilon' \mu' + j\omega \varepsilon''}})</td>
<td>(\frac{\mu'}{\varepsilon'})</td>
<td>(\frac{\mu'}{\varepsilon'})</td>
<td>((1 + j) \frac{\pi \mu}{\varepsilon'})</td>
</tr>
<tr>
<td>(\lambda \ [\text{m}])</td>
<td>(\frac{2\pi}{\beta} = \frac{v}{f} = \frac{1}{\sqrt{\frac{\varepsilon' \mu' - \varepsilon'' \mu''}{2} + \left(\frac{\varepsilon' \mu' + \varepsilon'' \mu''}{\varepsilon' \mu' - \varepsilon'' \mu''} + 1\right)}})</td>
<td>(\frac{1}{f \sqrt{\mu' \varepsilon'}})</td>
<td>(\frac{1}{f \sqrt{\mu' \varepsilon'}})</td>
<td>(\frac{4\pi}{f \sqrt{\mu' \sigma}})</td>
</tr>
<tr>
<td>(v_p \ [\text{m/s}])</td>
<td>(\frac{\omega}{\beta} = \frac{v}{f} = \frac{1}{\sqrt{\frac{\varepsilon' \mu' - \varepsilon'' \mu''}{2} + \left(\frac{\varepsilon' \mu' + \varepsilon'' \mu''}{\varepsilon' \mu' - \varepsilon'' \mu''} + 1\right)}})</td>
<td>(\frac{1}{\sqrt{\mu' \varepsilon'}})</td>
<td>(\frac{1}{\sqrt{\mu' \varepsilon'}})</td>
<td>(\frac{4\pi f}{\mu' \sigma})</td>
</tr>
</tbody>
</table>

Notes:
- \(\varepsilon''/\varepsilon' = \sigma/\omega\), \(\omega = 2\pi f\). Low-loss medium in practice: \(\varepsilon''/\varepsilon' \ll 0.01\). Good conductor medium in practice: \(\varepsilon''/\varepsilon' > 100\)

where,
- \(l_{\text{critical}}\) = critical length in [m]
- \(\lambda_{\text{min}}\) = wavelength of the highest significant harmonic frequency in the signal in [m]

- **Characteristic impedance** \(Z_0\). The characteristic impedance \(Z_0\) \([\Omega]\) of a uniform transmission line is the ratio of the complex amplitudes of voltage \(V\) \([\text{V}]\) and current \(I\) \([\text{A}]\) of a single wave propagating along the line in the same direction (in the absence of reflections in the other direction).

\[
Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \tag{7.74}
\]
where,

- \( R' \) = resistance per-unit-length in [\( \Omega/m \)]
- \( L' \) = inductance per-unit-length in [H/m]
- \( C' \) = capacitance per-unit-length in [F/m]
- \( G' \) = conductance per-unit-length in [S/m]
- \( \omega = 2\pi f \) = angular frequency in [rad/sec]
- \( j = \sqrt{-1} \) = imaginary unit

For a lossless line (\( R' = 0, G' = 0 \)), the characteristic impedance is reduced to:

\[
Z_0 = \sqrt{\frac{L'}{C'}}
\] (7.75)

- **Rule of thumb for the per-unit-length loop self-inductance \( L' \) and capacitance \( C' \) for small signal cables (AWG6 to AWG32) and narrow signal lines on PCBs (0.1 mm to 0.25 mm):
  - \( L' = 1 \mu H/m = 1 nH/mm \)
  - \( C' = 50 pF/m = 50 fF/mm \)

- **Propagation constant \( \gamma \).** The propagation constant \( \gamma \) [1/m] of a sinusoidal electromagnetic wave is a measure of the change undergone by the amplitude and phase of the wave as it propagates along a transmission line in direction \( z \) [m].

\[
\gamma = \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')} = j\omega \sqrt{\frac{\epsilon}{\mu}}
\] (7.76)

where,

- \( \alpha \) = attenuation constant (or attenuation factor) in [1/m]
- \( \beta \) = phase constant (or phase factor) in [rad/m]
- \( j = \sqrt{-1} \) = imaginary unit
- \( R' \) = resistance per-unit-length in [\( \Omega/m \)]
- \( L' \) = inductance per-unit-length in [H/m]
- \( C' \) = capacitance per-unit-length in [F/m]
- \( G' \) = conductance per-unit-length in [S/m]
- \( \omega = 2\pi f \) = angular frequency of the signal in [rad/sec]
- \( \mu = \mu' - j\mu'' \) = complex permeability of the media through which the electromagnetic wave is propagating in [H/m]
- \( \epsilon = \epsilon' - j\epsilon'' \) = complex permittivity of the media through which the electromagnetic wave is propagating in [F/m]

- **Balanced vs. unbalanced.** Balanced transmission lines are more robust against EMI (common-mode interference) than unbalanced transmission lines.
- **Single-ended vs. differential.** Differential signals should be sent over balanced transmission lines and single-ended signals over unbalanced transmission lines.
References


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