**Supplementary Methods**

**Model Analysis**

The equilibria of both models were determined analytically as were the systems’ Jacobian matrices. To determine model outcomes at a particular set of parameter values, we first checked for equilibria that were biologically feasible (non-negative and real). We then performed a linear stability analysis for all biologically feasible equilibria. If there was a single stable, biologically feasible equilibrium, then we characterized the model outcome based on the qualitative features of the equilibrium (i.e., prey extinct, sexual display fixed, etc.). If there were no stable, biologically feasible equilibria for a particular set of parameters we numerically integrated (or iterated, for the discrete model) the model for 1000 time units to 1) ensure trajectories remained well behaved and 2) characterize the qualitative features of trajectories (i.e., prey extinct, sexual display fixed, cycle measurements, etc.).

For the continuous model, slight modifications were needed for analytical tractability. Namely, we set \( V(\bar{z}) = \sigma^2 \) for analytical calculations (though always included the more complex dependence on trait value for numerical integration). Since our more complex function for \( V(\bar{z}) \approx \sigma^2 \) unless \( \bar{z} \) approaches 0, equilibrium outcomes can safely be extrapolated from this simpler model, with one exception: the loss of the sexual display. We impose a lower bound on \( \bar{z} \) of 0 by assuming a loss of genetic variation in this limit, though the selection gradient does not necessarily approach 0 as \( \bar{z} \) approaches 0. To determine whether the display is lost from the continuous model, we set \( \bar{z} = 0 \) and analyze the resulting model of only predator-prey dynamics without evolution. In this situation, there is only one equilibrium for which the predator may persist: \((N_f^*, N_m^*, P^*) = (\frac{m}{2b_c(c_c-mt_h)}, \frac{m}{2b_c(c_c-mt_h)}, \frac{c_rc_c(b_c(c_c-mt_h)-m)}{(b(c-mt_h))^2})\). If (1) this equilibrium is
stable in the three-dimensional system and (2) the selection gradient is negative when the
predator density is equal to $P^*$, then the display is lost and the eco-evolutionary dynamics can
approach a stable equilibrium of $(N_f^*, N_m^*, P^*, z^*) = 
\left( \frac{m}{2bc(ce-mt_h)}, \frac{m}{2bc(ce-mt_h)}, \frac{cr_c(b(c-mt_h)-m)}{b(c-mt_h)^2}, 0 \right)$. 

We rely on the numerical procedure described above because the full generality of the
equilibrria and their stability conditions are too complex to be meaningfully interpreted. Still,
some special cases are illuminating. In addition to cases where the display is lost in the
continuous model, we identified six equilibria in the continuous model (though not all of these
will be biologically feasible for specific parameter values). Two of these equilibria are complex
and include both non-zero predator and prey densities and non-trivial (not 0 or 1) trait values.
The remaining four include extinction of the predator. One of these, $(N_f^*, N_m^*, P^*, z^*) = 
(0,0,0,1)$, also includes extinction of the prey, and is always unstable. Another equilibrium
includes no females but the presence of males and is thus always infeasible. The last two
equilibria are potentially feasible with the display at its sexually selected optimum and the prey
at non-zero densities. In practice, we assume an equal sex ratio in the absence of predation and
check if the per capita predator growth rate is positive to assess the stability of predator
extinction. In particular, this means that predator extinction is stable when $m > \frac{c(bc+sc/z)}{1+bc+ct_h+st_h/z^2}$
which is simply that the predator mortality rate is greater than the rate that prey biomass is
converted into predator biomass.

In the discrete model, we identified nine equilibria (though not all of these will be
biologically feasible for specific parameter values). Three of the equilibria include the predator at
non-zero densities and polymorphism in the display trait, two include the predator at non-zero
densities and the display at fixation, and two include the predator at non-zero densities and the
display trait having been lost from the population. Only the remaining two equilibria with the
 predator extinct can be analyzed simply. They are loss and fixation of the display trait with prey
density at carrying capacity: \((N^*_1, N^*_2, P^*) = \left(\frac{r_d-2}{r_d}, 0, 0\right)\) and \((N^*_1, N^*_2, P^*) = \left(0, \frac{r_d-2}{r_d}, 0\right)\). The
first equilibrium (loss of the display trait) is always unstable. The second equilibrium (fixation of
the display trait) is stable whenever \(2 < r_d < 6\) (as is required for stable prey growth) and the
eigenvalue \(c_d \left(\frac{(r_d-2)(b_d+s_d/2)}{r_d}\right)\) (which is simply the per capita predator growth rate) is less than 1 in
magnitude. Bistability is sometimes observed in the discrete model. In practice, we found that
this was only the case with the predator-extinction equilibrium. We always report an equilibrium
with non-zero predator densities when this is also stable (see Fig 4).

\(\text{Fisher process model analysis}\)

Although a straightforward extension of our discrete model, the Fisher process model
includes an additional two state variables that greatly diminish the analytical tractability of the
model. In the Fisher process model, it is not possible (even numerically) to solve for all
equilibria. As such, we focus on interesting cases from the discrete model to assess how
including the coevolution of female preference alters our results. For each parameter
combination we considered, we use initial conditions of 0.5 for the initial frequency of T2 and
assume that the T and P loci are initially in linkage equilibrium. We iterated the model for 2000
time steps for initial P2 frequencies of 0.05, 0.5, and 0.95. We assessed the qualitative features of
trajectories and final time points to characterize model outcomes. Specifically, we checked
whether the trajectory remained well behaved, the predator had gone extinct (using a threshold of
\(10^{-10}\)), and the display trait had gone to fixation (using a threshold of \(10^{-10}\) for the density of prey
carrying the T1 allele). For all well-behaved trajectories, we further checked whether the
trajectories displayed either ecological (just predator/prey density) or evolutionary (allele
frequency) cycles in the final 500 time steps. We considered a variable to be cycling if 1) it
changed directions at least twice in the final 500 time steps and 2) the range of oscillations was at
least 0.01. We note that this procedure will undoubtedly mark regions as “cycling” that are
experiencing damped oscillations toward a stable equilibrium. Thus, we will interpret these
regions as long transient cycles that may not be sustained.

References
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