Supporting information

**S1 Text. Environmental contamination.** The full model includes environmental contamination on a ward-level. The bacterial load at any given time \( t \) is based on the differential equation

\[
\frac{dE}{dt} = \nu \frac{I(t)}{N(t)} - \mu E(t)
\]  

(1)

Solving the above differential equation requires discretizing over \( t \), resulting in a finite number of time steps \( t_0, t_1, \ldots, t_N \). We then assume \( I(t) = I_t \) and \( N(t) = N_t \) to be constant within a time step and use it as initial conditions. Separating variables leads to

\[
\frac{dE}{\nu \frac{I_t}{N_t} - \mu E(t)} = dt
\]

and thus

\[
\int \frac{dE}{\nu \frac{I_t}{N_t} - \mu E(t)} = \int dt
\]

\[
\Rightarrow - \frac{1}{\mu} \log \left| \frac{\nu I_t}{N_t} - \mu E(t) \right| = t + C
\]

\[
\Rightarrow \log \left| \frac{\nu I_t}{N_t} - \mu E(t) \right| = -\mu(t + C)
\]

\[
\Rightarrow \left| \frac{\nu I_t}{N_t} - \mu E(t) \right| = \exp[-\mu(t + C)] = \exp(-\mu t) \exp(-\mu C)
\]

Now, two cases have to be distinguished.

1. Case: \( \nu \frac{I_t}{N_t} - \mu E(t) \geq 0 \)

\[
\Rightarrow -\mu E(t) = A_t \cdot \exp[-\mu t] - \nu \frac{I_t}{N_t}
\]

\[
\Rightarrow E(t) = -\frac{A_t}{\mu} \exp[-\mu t] + \nu \frac{I_t}{\mu N_t}
\]

\[
\Rightarrow E(t) = B_t \cdot \exp[-\mu t] + \nu \frac{I_t}{\mu N_t}
\]

2. Case: \( \nu \frac{I_t}{N_t} - \mu E(t) < 0 \)

\[
\Rightarrow \mu E(t) = A_t \cdot \exp[-\mu t] + \nu \frac{I_t}{N_t}
\]
\[ E(t) = \frac{A_t}{\mu} \exp[-\mu t] + \frac{\nu I_t}{\mu N_t} \]
\[ \Rightarrow E(t) = B_t \cdot \exp[-\mu t] + \frac{\nu I_t}{\mu N_t} \]

Determine \( B_{t_0} \) for initial condition \( E(t_0) = E_{t_0} \):
\[
E_{t_0} = E(t_0) = B_0 + \frac{\nu I_{t_0}}{\mu N_{t_0}}
\]
and therefore
\[
B_{t_0} = E_{t_0} - \frac{\nu I_{t_0}}{\mu N_{t_0}} \quad (2)
\]

For \( t_0 \leq t \leq t_1 \) the environmental load can be then computed by
\[
E(t) = \left( E_{t_0} - \frac{\nu I_{t_0}}{\mu N_{t_0}} \right) e^{-\mu t} + \frac{\nu I_{t_0}}{\mu N_{t_0}}
\]
\[
= E_{t_0} e^{-\mu t} + \frac{\nu I_{t_0}}{\mu N_{t_0}} (1 - e^{-\mu t}).
\]

For \( t_0 \leq t_i \leq t_N \) it holds
\[
B_{t_i} = \frac{E_{t_i} - \frac{\nu I_{t_i}}{\mu N_{t_i}}}{e^{-\mu t_i}} \quad (3)
\]
and therefore, it holds for \( \lfloor t \rfloor := \max\{t_0 \leq x \leq t_N \mid x \leq t\} \) and \( t \in \mathbb{R} \setminus \{t_0, t_1, \ldots, t_N\} \)
\[
E(t) = \frac{E_{\lfloor t \rfloor} - \frac{\nu I_{\lfloor t \rfloor}}{\mu N_{\lfloor t \rfloor}}}{e^{-\mu \lfloor t \rfloor}} \cdot e^{-\mu t} + \frac{\nu I_{\lfloor t \rfloor}}{\mu N_{\lfloor t \rfloor}}
\]
\[
= E_{\lfloor t \rfloor} e^{-\mu(t - \lfloor t \rfloor)} + \frac{\nu I_{\lfloor t \rfloor}}{\mu N_{\lfloor t \rfloor}} \left( 1 - e^{-\mu(t - \lfloor t \rfloor)} \right).
\]

and
\[
E(t_i) = E_{t_{i-1}} e^{-\mu(t_i - t_{i-1})} + \frac{\nu I_{t_{i-1}}}{\mu N_{t_{i-1}}} \left( 1 - e^{-\mu(t_i - t_{i-1})} \right)
\]
for \( 0 \leq i \leq N \).