Supporting Text - Modeling HIV-1 Infection in the Brain

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\textbf{Derivation of }R_0\textbf{ }

Our model possesses a unique infection-free equilibrium (IFE), \((T_*, 0, M_*, 0, M_B*, 0, 0, 0)\), where

\[
T_* = \frac{\lambda}{d},
\]

\[
M_* = \frac{\lambda_M (\psi + d_M)}{(\varphi + d_M)(\psi + d_M) - \psi \varphi},
\]

\[
M_B* = \frac{\lambda_M \varphi}{(\varphi + d_M)(\psi + d_M) - \psi \varphi}.
\]

Following the next generation matrix method, we linearize the five model equations corresponding to infection classes, \(i.e., T^*, M^*, M_B^*, V, \) and \(V_B\), about the IFE, and introduce the following matrices:

\[
F = \begin{bmatrix}
0 & 0 & 0 & \frac{\beta \lambda}{d} & 0 & 0 \\
0 & 0 & 0 & \frac{\beta M \lambda_M (\psi + d_M)}{(\varphi + d_M)(\psi + d_M) - \psi \varphi} & 0 & 0 \\
0 & 0 & 0 & 0 & \beta M \lambda_M \varphi & 0 \\
p & p_M & 0 & 0 & 0 & 0 \\
p & p_M & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

and

\[
V = \begin{bmatrix}
\delta & 0 & 0 & 0 & 0 \\
0 & \varphi + \delta_M & -\psi & 0 & 0 \\
0 & -\varphi & \psi + \delta_M & 0 & 0 \\
0 & 0 & 0 & c & 0 \\
0 & 0 & 0 & 0 & c \\
\end{bmatrix},
\]

where \(F\) represents a matrix of new infections and/or viral production in the linearized system and \(V\) represents a matrix of the transfer of cell or virus into and out of the
compartment. The basic reproduction number is then given by the spectral radius of $FV^{-1}$. That is,

$$R_0 = \frac{1}{2\varphi\psi AD} \cdot \left[ \sqrt{2} \varphi\psi AD \left( AB + CE + \sqrt{(AB)^2 + HC^2 + C(B + C)G + BC\Psi} \right) \right],$$

where

- $A = \frac{\varphi^2 + \delta_M \varphi + d_M \varphi + 2\varphi\psi + \psi^2 + \delta_M \psi + d_M \psi + d_M \delta_M}{\varphi\psi}$,
- $B = p\beta\lambda d_M \varphi\psi$,
- $C = p_M\beta_M \lambda_M \delta d_M \psi$,
- $D = d_M c_M \delta \delta_M$,
- $E = \varphi^2 + 2\delta_M \varphi + \varphi\psi + d_M \psi + d_M \delta_M$,
- $G = \varphi^4 + (\varphi\psi)^2 + 2d_M\psi(\varphi^2 + \delta_M \varphi + d_M \delta_M)$, $(\varphi\psi)^2$,
- $H = 1 + \psi^2 - 2d_M \delta_M \varphi^2 + 2d_M \varphi\psi^2$,

and

$$\Psi = \frac{1}{(\varphi\psi)^2} \left( -3\varphi^4 - 2\varphi^2\psi + (d_M + \delta_M)(-2\varphi^3 + 2d_M \psi(\psi + \delta_M)) + 2(\varphi + d_M)(\psi^3 + d_M \delta_M) + (\varphi\psi)^2 + 2d_M \delta_M(\psi^2 + d_M \delta_M) \right).$$