S3 Text. Expression of the spatial orientation of the directional ear.

There are many ways to express the elements of SO(3) using three parameters. We adopt the roll–pitch–yaw expression in this article. Let us consider a sequence of rotations from the original orientation shown in Fig 1B:

1. Rotate around the yaw axis by an angle of $\theta_e$;
2. Rotate around the pitch axis by an angle of $\varphi_e$ (elevation is taken in the positive direction, so $\varphi_e$ is the inverse of the usual pitch angle);
3. Rotate around the roll axis by an angle of $\psi_e$.

We then have the rotation matrix $L = R_z(\theta_e) R_y(-\varphi_e) R_x(-\psi_e)$, where $R_\alpha(\zeta)$ denotes a matrix that express a rotation around the $\alpha$-axis by an angle of $\zeta$. 