S4 Text. Modification to Poisson likelihood for censored observations

First, consider the model of decreasing mobility between February 2 – April 4. The number of trips between zip codes $i$ and $j$ (in either direction) in age group $a$ at time $t$ is denoted $Y_{i,ja,t}$. For simplicity we can considered a fixed time and age group and just write $Y_{ij}$ for this variable. This is the sum of trips from $i$ to $j$ and $j$ to $i$, which we denote $Y_{i,j}$ and $Y_{j,i}$ respectively.

Let $\lambda$ be the mean of the Poisson process implied by some parameter values (again just for notational convenience), so the likelihood when $Y_{ij}$ is known exactly is

$$ Y_{ij} \sim \text{Poisson}(\lambda) $$

or in other words

$$ p(Y_{ij}|\lambda) $$

where $p$ is the probability mass function of a Poisson distribution with mean $\lambda$.

There are two possibilities for when the total $Y_{ij}$ isn’t known exactly, either one of $Y_{i,j}$ and $Y_{j,i}$ are below 50 (but not both) or both are below 50. Suppose just one is below 50 and let it be $Y_{i,j}$ be known (otherwise switch the indices). The total $Y_{ij}$ could therefore be any value between $Y_{i,j}$ and $Y_{i,j} + 49$ and so the likelihood is

$$ p(Y_{i,j} \leq Y_{ij} \leq Y_{i,j} + 49|\lambda) = \sum_{k=0}^{49} p(Y_{i,j} + k|\lambda) $$

Similarly, if both are below 50 then $Y_{ij}$ could take any value between 0 and 99, yielding a likelihood

$$ p(0 \leq Y_{ij} \leq 99|\lambda) = \sum_{k=0}^{99} p(k|\lambda) $$

Likelihoods were modified in a similar way for the model of mobility between June 1 – August 31, where the uncertainty (and therefore the number of values summed over) accumulates for each week that is censored.