Analysis strategy

We started by considering the time series referring to the entire population, without age stratification, and we assessed which formulation of model (1) fitted our data best in terms of AIC over all the study period. We first tested the usefulness of an overdispersion parameter by comparing distributional assumptions (Poisson vs negative binomial). Then, considering the mean is decomposed as \( \mu_t = \text{pop}_t \nu_t + \lambda Y_t - 1 + \tau X_t - 1 \), we compared different expressions for the endemic component \( \nu_t \). Yearly seasonality for weekly data is described through the general formulation

\[
\log(\nu_t) = \alpha + \sum_{s=1}^{S} \gamma_s \sin\left(\frac{2\pi st}{52}\right) + \delta_s \cos\left(\frac{2\pi st}{52}\right)
\]

where \( \alpha \) is an intercept and \( \gamma_s \) and \( \delta_s \) quantify the amplitude of the sine-cosine waves. We first assessed the optimal number \( S \) of harmonic functions to be included, and in a second stage we checked whether replacing the sine-cosine waves with rainfall and temperature information better characterised differences across winters. Finally, we extended the model to include multiple lags for covariates, relaxing the assumption of temporal dependence limited to one week, as suggested by [1]. Denoting by \( Q \) the number of lags considered, the mean was written as

\[
\mu_t = \text{pop}_t \nu_t + \lambda \sum_{q=1}^{Q} w_q(y) Y_{t-q} + \tau \sum_{q=1}^{Q} w_q(x) X_{t-q}
\]

where \( w_q(y) \) and \( w_q(x) \) are normalized lag weights defined according to a geometric structure, with parameters \( p_x \) and \( p_y \) shaping the respective exponential decays, i.e.

\[
w_q(y) = \frac{p_y(1 - p_y)^{q-1}}{\sum_{q=1}^{Q} p_y(1 - p_y)^{q-1}}
\]

To avoid overfitting, we compared the performance of our different model formulations in terms of one-step-ahead forecasts. Such performance is usually measured in terms of scoring rules, where a score is intended as a penalty \( s(P, x) \), that is a function of the observed counts \( x \) and predictive distribution \( P \). When dealing with count data, the logarithmic score \( \log(s(P, x)) = -\log P_x \) is a standard choice: it is simply a log transformation of the probability mass \( P \) at the observed count \( x \) [2,3]. Hence, for each choice of lags and endemic variables, we computed the logarithmic scores for a set of forecasts and we took their mean as a summary measure.

The same model assessment procedure was replicated when looking at age-specific counts: the model in equation (3) was simultaneously fitted to the five age groups, initially assuming that all the coefficients could be age-specific. We first performed model selection using AIC to evaluate whether some coefficients could be shared across age groups: model fit was assessed in a sequential way, testing at each stage which of the components led to a larger AIC reduction when associated with non age-specific coefficients.

References
