Appendix S2: The iterative expression for state variable and variance.

Here we will give a brief deviation about the iterative process.

After given the a priori estimation $\hat{x}(t|t)$ for $x(t)$, and $P(t|t)$, a measurement update $\hat{x}(t|t+1)$ by giving $y(t+1)$ could be calculated if applying the corresponding relations (17) on (20) with the definition in (21) and determined $\hat{\lambda}$. Then the left side of equation (20) becomes

$$\left[ Q + D^T \hat{W} D \right] \hat{x} = \left[ P^{-1}(t|t) + \hat{\lambda} E_d^T E_d + D^T \hat{W} D \right] [\hat{x}(t|t+1) - \hat{x}(t|t)] \tag{1}$$

and the right side becomes

$$D^T \hat{W} y + \hat{\lambda} E_d^T E_d = D^T \hat{W} [y(t+1) - D\hat{x}(t|t)] - \hat{\lambda} E_d^T E_d \hat{x}(t|t) \tag{2}$$

where

$$\hat{W} = \left( R - \hat{\lambda}^{-1} M M^T \right)^{-1} \tag{3}$$

If let $\hat{R} = \hat{W}^{-1} = R - \hat{\lambda}^{-1} M M^T$, we can get

$$\left[ P^{-1}(t|t) + \hat{\lambda} E_d^T E_d + D^T \hat{R}^{-1} D \right] [\hat{x}(t|t+1) - \hat{x}(t|t)] = D^T \hat{R}^{-1} [y(t+1) - D\hat{x}(t|t)] - \hat{\lambda} E_d^T E_d \hat{x}(t|t)$$

By setting

$$P^{-1}(t|t) = P^{-1}(t|t) + \hat{\lambda} E_d^T E_d$$
$$P^{-1}(t+1|t+1) = \hat{P}^{-1}(t|t) + D^T \hat{R}^{-1} D$$

we will have the time update $\hat{x}(t+1)$ from $\hat{x}(t|t)$ as

$$\hat{x}(t+1) = \left[ I - \hat{\lambda} P(t+1|t+1) E_d^T E_d \right] \hat{x}(t|t)$$

and then if we set

$$e(t+1) = y(t+1) - D\hat{x}(t|t)$$
$$P(t+1) = \hat{P}(t|t)$$

the measurement update $\hat{x}(t|t+1)$ could be

$$\hat{x}(t|t+1) = P(t+1|t+1) D^T \hat{R}^{-1} e(t+1) + \hat{x}(t+1)$$

here we can obtain a form for iteration of $P(t+1|t+1)$ if let

$$P(t+1) = \hat{P}(t|t) = \left[ P^{-1}(t|t) + \hat{\lambda} E_d^T E_d \right]^{-1}$$
$$R_e(t+1) = \left[ \hat{R} + D P(t+1) D^T \right]^{-1}$$

that is

$$P(t+1|t+1) = \left[ P^{-1}(t+1) + D^T \hat{R}^{-1} D \right]^{-1} = P(t+1) - P(t+1) D^T R_e^{-1}(t+1) D P(t+1)$$

For the final state update, since for static reconstruction problem we have the state equation as (12), we can get

$$\hat{x}(t+1|t+1) = \hat{x}(t|t+1) = P(t+1|t+1) D^T \hat{R}^{-1} e(t+1) + \hat{x}(t+1)$$ \tag{4}$$

then an iterative process from known $\{\hat{x}(t|t), P(t|t)\}$ and $\{y(t+1)\}$ to $\{\hat{x}(t+1|t+1)\}$ was given as above.